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HANKEL KERNELS OF HIGHER WEIGHT FOR THE BALL

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The purpose of this note is to write down the general form of Hankel kernels for the complex unit ball $\mathbf{B}$ in $\mathbf{C}^d$. In the one dimensional case (unit disk Δ in $\mathbf{C}$) this was done in [JP] and our treatment below has been guided by the insights gained there, and later, in a slightly different context, in [P]. We begin by summarizing the relevant facts in the case of the disk in a form convenient for us.

1. The disk revisited

Let $A^{\alpha,2}$ be the Hilbert space of analytic functions on Δ which are square integrable with respect to the probability measure $d\mathcal{M}_a(z) = (\alpha + 1)(1 - |z|^2)^\alpha d\mathcal{E}(z)$ ($\mathcal{E}$ = Euclidean measure with an appropriate normalization, viz. $\int_\Delta d\mathcal{E}(z) = 1$). In [JP] we considered bilinear forms on $A^{\alpha,2}(\Delta)$ of the form

$$H(f_1, f_2) = \int\int_{\Delta\times\Delta} \overline{\mathcal{A}(z_1, z_2)} f_1(z_1) f_2(z_2)\, d\mathcal{M}_a(z_1)\, d\mathcal{M}_\alpha(z_2),$$

where the kernel $\mathcal{A}$ is given by

$$\mathcal{A}(z_1, z_2) = \frac{1}{2\pi i}\int_{\mathrm{T}} \frac{(z_1 - z_2)^s}{(w - z_1)^r (w - z_2)^r}\, b(w)\, dw,$$

where $r = \alpha + 2 + s$ ($s \in \mathbf{N}$). We write now this formula as

$$\mathcal{A}(z_1, z_2) = \frac{1}{2\pi i}\int_{\mathrm{T}} \frac{(z_1 - z_2)^s}{(1 - \bar{w}z_1)^r (1 - \bar{w}z_2)^r}\, b(w)\, \bar{w}^{2r} dw.$$

By partial integration one finds, quite generally, that

$$\int_\Delta \bar{f} g\, d\mathcal{M}_{m-1} = \frac{(m+1)!}{2\pi i}\int_{\mathrm{T}} \bar{f} b \bar{w}^{m+1} dw$$

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where $g = b^{(m)}$. (Alternative proof: Just check it for the monomials. The (limiting) case $m = 0$ is identity, while $m = 1$ is essentially Green's formula.) If $m = 2r - 1$ this gives

$$\mathscr{A}(z_1, z_2) = \frac{1}{(2r)!}\int_\Delta \frac{(z_1 - z_2)^s (1 - |w|^2)^s}{(1 - z_1\bar{w})^r (1 - z_2\bar{w})^r} g(w)\, d\mathscr{M}_\beta(w)$$

where $\beta = 2(\alpha + 2) + s - 2$. (Notice the splitting of the $(1 - |w|^2)$-factor, which is convenient for our purposes.) It is often convenient to set $\nu = \alpha + 2$, $\mu = \beta + 2$. Then the last relation reads $\mu = 2\nu + s$.

Remark. By partial integration

$$0 = \int_\Delta \frac{d}{dz}(f\bar{g}(1 - |z|^2)^{\alpha+1})\, d\mathscr{E}(z)$$

$$= \int_\Delta f'\bar{g}(1 - |z|^2)^{\alpha+1} d\mathscr{E}(z) - \int_\Delta \frac{(\alpha + 1)zf}{1 - |z|^2}\bar{g}(1 - |z|^2)^{\alpha+1} d\mathscr{E}(z)$$

so that $f \mapsto f'$ and $f \mapsto P_{\alpha+1}\left(\frac{(\alpha + 1)zf}{1 - |z|^2}\right)$ are adjoint operators in $A^{\alpha+1}(\Delta)$, $P_{\alpha+1}$ being the corresponding orthogonal projection. Therefore, if $s = 1$, the bilinear form can be written in the form

$$H(f_1, f_2) = \text{const}\cdot\int_\Delta (f_1'f_2 - f_1f_2')\bar{g}(1 - |z|^2)^s d\mathscr{M}_\beta(z).$$

In a similar way we recover, if $s > 1$, quite generally the *transvectant* (German: *Überschiebung*). This replaces, in one stroke, the rather clumsy attempts in [JP], Section 3.4.

2. The ball

Let $\mathbf{B}$ denote the unit ball in $\mathbf{C}^d$. The measure $\mathscr{M}_\alpha$ $(\alpha > -1)$ and the Hilbert space $A^{\alpha,2}(\mathbf{B})$ are defined in an analogous way as for the disk (Section 1).

Let us begin with the case $\boxed{s = 1}$.

"Ansatz".

$$\mathscr{A}(z_1, z_2) = \mathscr{A}_g(z_1, z_2) = \int_{\mathbf{B}} \sum_{i=1}^d A^i(z_1, z_2, w) g_i(w)\, d\mathscr{M}_\beta(w)$$

where $\beta = 2(\alpha + 1 + d) + 1 - (d + 1)$. It will be convenient to put $\nu = \alpha + d + 1$,

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$\mu = \beta + d + 1$. Then the preceding relation reads $\mu = 2\nu + 1$. (This agrees with the convention made (for general s) in Section 1 if $d = 1$.)

Let

$$\begin{pmatrix} \overset{d\times(d+1)}{D} \\ \overset{1\times(d+1)}{E} \end{pmatrix} \in \mathrm{SU}(d, 1),$$

writing $\phi z = \dfrac{D(z)}{E(z)}$ with $D(z) = D\begin{pmatrix} z \\ 1 \end{pmatrix}$, $E(z) = E\begin{pmatrix} z \\ 1 \end{pmatrix}$. Then

$$f(z) \mapsto f(\phi z)(E(z))^{-\nu} \equiv \tilde{f}(z),$$

$$\mathscr{A}(z_1, z_2) \mapsto \mathscr{A}(\phi z_1, \phi z_2)(E(z_1))^{-\nu}(E(z_2))^{-\nu} \equiv \tilde{\mathscr{A}}(z_1, z_2).$$

We want the g_i to transform as components of a weighted differential form:

$$g_i(w) \mapsto \sum_{j=1}^{d} g_j(\phi w)\,\frac{\partial \phi^j(w)}{\partial w^i}\,(E(w))^{-2\nu} \equiv \tilde{g}_i(w).$$

This requires that

$$\text{(1)} \qquad \mathscr{A}^i(\phi z_1, \phi z_2, \phi w) = \sum_{j=1}^{d} \mathscr{A}^j(z_1, z_2, w)\,\frac{\partial \phi^i(w)}{\partial w^j} \times \\ \times (E(z_1))^{\nu}(E(z_2))^{\nu}(\overline{E(w)})^{\mu}E(w).$$

Indeed, if (1) is fulfilled we find, using $d\mathscr{M}_\beta(\phi w) = |E(w)|^{-2\mu}d\mathscr{M}_\beta(w)$,

$$\begin{aligned} \tilde{\mathscr{A}}(z_1, z_2) &= \int_{\mathbf{B}} \sum_{i=1}^{d} A^i(\phi z_1, \phi z_2) g_i(w)\, d\mathscr{M}_\beta(\phi w) \cdot (E(z_1))^{-\nu}(E(z_2))^{-\nu} \\ &= \int_{\mathbf{B}} \sum_{i=1}^{d}\sum_{j=1}^{d} A^j(z_1, z_2, w)\,\frac{\partial \phi^i(w)}{\partial w^j}\,(E(z_1))^{\nu}(E(z_2))^{\nu}(\overline{E(w)})^{\mu}\,E(w) \\ &\qquad \times g_i(w)\,|E(w)|^{-2\mu}\,d\mathscr{M}_\beta(w) \cdot (E(z_1))^{-\nu}(E(z_2))^{-\nu} \\ &= \int_{\mathbf{B}} \sum_{j=1}^{d} A^j(z_1, z_2, w)\,\tilde{g}_j(w)\,d\mathscr{M}_\beta(w). \end{aligned}$$

EXAMPLE. If $d = 1$ this is fulfilled with

$$\mathscr{A}(z_1, z_2, w) = \frac{(z_1 - z_2)(1 - |w|^2)}{(1 - z_1\bar{w})^{\alpha+2+1}(1 - z_2\bar{w})^{\alpha+2+1}},$$

in full agreement with Section 1.

NEW "ANSATZ". $\mathscr{A}^i(z_1, z_2, 0) = z_1^i - z_2^i$.

Clearly (1) is then fulfilled if ϕ fixes the origin 0 (a rotation). Next we apply (1) to the fundamental symmetry interchanging w and 0. For simplicity we first assume $w = (t,0,\ldots,0)$, $t > 0$. Then (see [R]; i, j denote indices > 1)

$$\phi(z) = \left(\frac{t - z^1}{1 - tz^1}, -(1 - t^2)^{1/2}\frac{z^i}{1 - tz^1}\right),$$

$$\frac{\partial\phi}{\partial z^1} = \left(\frac{t^2 - 1}{(1 - tz^1)^2}, \frac{(1 - t^2)^{1/2}tz^i}{(1 - tz^1)^2}\right),$$

$$\frac{\partial\phi}{\partial z^j} = \left(0, -(1 - t^2)^{1/2}\frac{\delta_j^i}{1 - tz^1}\right),$$

$$E(z) = \iota\frac{1 - tz^1}{(1 - t^2)^{1/2}} \Rightarrow E(\phi^{-1}z) = \iota\frac{1 - t\dfrac{t - z^1}{1 - tz^1}}{(1 - t^2)^{\frac{1}{2}}} = \iota\frac{(1 - t^2)^{\frac{1}{2}}}{1 - tz^1},$$

$$E(0) = \frac{\iota}{(1 - t^2)^{\frac{1}{2}}},$$

where we in the last implication used $\phi^2 = \mathrm{id}$ and the notation $\iota = (-1)^{\frac{d+1}{2}}$. Writing $A = (A^1,\ldots,A^d)$ we find

$$A(z_1, z_2, w) = (A^i(z_1, z_2, w))$$

$$= \left(\sum_{j=1}^{d} A^j(\phi^{-1}z_1, \phi^{-1}z_2, 0)\frac{\partial\phi^i(0)}{\partial w^j}\right) \times (E(\phi^{-1}z_1))^{\nu}(E(\phi^{-1}z_2))^{\nu}(\overline{E(0)})^{\mu}E(0)$$

$$= \left\{\left(\frac{t - z_1^1}{1 - tz_1^1} - \frac{t - z_2^1}{1 - tz_2^1}\right)(t^2 - 1,0)\right.$$

$$\left.+ \sum_{j=2}^{d}\left(-(1 - t^2)^{\frac{1}{2}}\frac{z_1^j}{1 - tz_1^1} + (1 - t^2)^{\frac{1}{2}}\frac{z_2^j}{1 - tz_2^1}\right)(0, -(1 - t^2)^{\frac{1}{2}}\delta_i^j)\right\}$$

$$\times\frac{(1 - t^2)^{\nu}(1 - t^2)^{-(\nu+1)}}{(1 - tz_1^1)^{\nu}(1 - tz_2^1)^{\nu}}.$$

The two numerical factors in front of the vector terms can be written as

$$\frac{(t^2 - 1)(z_1^1 - z_2^1)}{(1 - tz_1^1)(1 - tz_2^1)} \text{ and } -(1 - t^2)^{\frac{1}{2}}\frac{z_2^j - z_2^j - t(z_2^1z_1^j - z_1^1z_2^j)}{(1 - tz_1^1)(1 - tz_2^1)}.$$

Therefore

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$$\begin{cases} A^1 = \dfrac{(z_1^1 - z_2^2)(1-t^2)}{(1-tz_1^1)^{\nu+1}(1-tz_2^1)^{\nu+1}}, \\ A^i = \dfrac{z_1^i - z_2^i - t(z_2^1 z_1^i - z_1^1 z_2^i)}{\text{same denominator}} \quad (i > 1). \end{cases}$$

If w is general, these formulae may be written as

$$A = \frac{P_w(z_1 - z_2)(1 - |w|^2) + Q_w(z_1 - z_2) - ((z_2,\bar{w})z_1 - (z,\bar{w})z_2)}{(1-(z_1,\bar{w}))^{\nu+1}(1-(z_2,\bar{w}))^{\nu+1}},$$

where P_w is orthogonal projection in the w direction and $Q_w = 1 - P_w$;

$$|w|^2 = \sum_{j=1}^{d} |w^j|^2, \quad (z,\bar{w}) = \sum_{j=1}^{d} z^j \bar{w}^j$$

is the standard Hermitean metric in $\mathbf{C}^d$. Explicitly:

$$P_w(\cdot) = \left(\cdot, \frac{w}{|w|}\right)\frac{w}{|w|},$$

so that we can as well write

$$A = \frac{(1-(z_2,\bar{w}))z_1 - (1-(z_1,\bar{w}))z_2 - (z_1 - z_2,\bar{w})w}{\text{previous denominator}}.$$

We can now make the Janson remark (see [P]) and pass to homogeneous notation. Writing $A = (A^1,\dots,A^d)$, $g = (g_1,\dots,g_d)$, $\langle A, g\rangle = \sum_{i=1}^d A^i g_i$ and putting

$$g_0 = -\sum_{i=1}^{d} g_i w^i$$

we get

$$\langle A,g\rangle = \frac{g_0(z_1 - z_2,\bar{w}) + \langle g, (1-(z_2,\bar{w}))z_1 - (1-(z_1,\bar{w}))z_2\rangle}{\text{previous denominator}}.$$

Write further

$$Z_k = (1, z_k^1,\dots,z_k^d) \ (k = 1,2), \quad W = (1, w^1,\dots,w^d), \quad G = (g_0,\dots,g_d)$$

and consider the standard pseudo-Hermitean metric in $\mathbf{C}^{d+1}$:

$$\|Z\|^2 = |Z^0|^2 - |Z^1|^2 - \cdots - |Z^d|^2,$$

$$(\!(Z, \bar{W})\!) = Z^0\bar{W}^0 - Z^1\bar{W}^1 - \cdots - Z^d\bar{W}^d.$$

Then

$$\langle g, A\rangle = \frac{\langle G, \langle\!\langle Z_2, \bar{W}\rangle\!\rangle Z_1 - \langle\!\langle Z_1, \bar{W}\rangle\!\rangle Z_2\rangle}{\langle\!\langle Z_1, \bar{W}\rangle\!\rangle^{\nu+1}\langle\!\langle Z_2, \bar{W}\rangle\!\rangle^{\nu+1}}.$$

If we view G as a (horisontal) form homogeneous of degree $-\mu$ (recall that $\mu = 2\nu + 1 = 2(\alpha + 1 + d) + 1$), the $\mathrm{SU}(1, d)$-invariant character of our formula is manifest.

It is now easy to make the transition to general $\boxed{s > 1}$.

Let $G = G(W)(\cdot)$ denote an s-linear form homogeneous of degree $-\mu$ in W (where now $\mu = 2\nu + s = 2(\alpha + 1 + d) + s$) and horisontal in the sense that

$$G(W)(\ldots, W, \ldots) = 0.$$

Then we may consider the kernel

$$\mathscr{A}(z_1, z_2) = \int_{\mathbf{B}} \frac{\langle G, (\langle\!\langle Z_2, \bar{W}\rangle\!\rangle Z_1 - \langle\!\langle Z_1, \bar{W}\rangle\!\rangle Z_2)^{\otimes s}\rangle}{\langle\!\langle Z_1, \bar{W}\rangle\!\rangle^{\nu+s}\langle\!\langle Z_2, \bar{W}\rangle\!\rangle^{\nu+s}} \| W\|^{2\mu} dI(w),$$

where thus $\nu = \alpha + 1 + d$, $\mu = 2\nu + 1$, and I is the $\mathrm{S}(1, d)$-invariant measure on $\mathbf{B}$. Note that the integrand is homogenous of degree 0 and so, projecting into $\mathbf{P}^d$, can be regarded as a function on $\mathbf{B}$. Therefore the integral makes sense.

3. Generalizing the transvectant

Again we start with $\boxed{s = 1}$.

Let us introduce bilinear operators J^i $(i = 1, \ldots, d)$, setting

$$J^i(f_1, f_2)(w) = \int_{\mathbf{B}\times\mathbf{B}} \overline{A^i(z_1, z_2, w)}\, f_1(z_1) f_2(z_2)\, d\mathscr{M}_\alpha(z_1)\, d\mathscr{M}_\alpha(z_2)$$

so that the bilinear form $H = H_g$ can be written

$$H(f_1, f_2) = \int_{\mathbf{B}} \sum_{i=1}^{d} J^i(f_1, f_2)(w)\, \overline{g_i(w)}\, d\mathscr{M}_\beta(w),$$

where as before $\beta + 1 + d = 2(\alpha + 1 + d)$ or $\mu = 2\nu + 1$.

Take first $w = 0$. Integrating by parts, as in the final remark in Section 1, we find

$$J^i(f_1, f_2)(0) = \sum_{i=1}^{d} \frac{\partial f_1(0)}{\partial w^i} f_2(0) - \frac{\partial f_2(0)}{\partial w^i} f_1(0). \tag{1}$$

On the other hand, it is easy to see that the operators J^i enjoy the following

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transformation property

$$\sum_{k=1}^{d} J^k(\tilde{f}_1, \tilde{f}_2)(z)\overline{\frac{\partial\phi^i}{\partial z^k}}(E(z))^{\mu}\bar{E}(z) = J^i(f_1, f_2)(\phi z), \tag{2}$$

where $\tilde{f}_1(z) = f_1(\phi z)E(z)^{-\nu}$, $\tilde{f}_2(z) = f_2(\phi z)E(z)^{-\nu}$.

Indeed, we find

$$\sum_{i=1}^{d} J^k(\tilde{f}_1, \tilde{f}_2)(w)\overline{\frac{\partial\phi^i}{\partial z^k}}(E(w))^{\mu}\overline{E(w)}$$

$$= \sum_{i=1}^{d}\int_{\mathbf{B}\times\mathbf{B}} \overline{A^k(z_1, z_2, w)}\, f_1(\phi z_1)(E(z_1))^{-\nu}f_2(\phi z_1)(E(z_1))^{-\nu}d\mathcal{M}_\alpha(z_1)d\mathcal{M}_\alpha(z_2)$$

$$\times \overline{\frac{\partial\phi^i}{\partial w^k}}(E(w))^{\mu}\overline{E(w)}$$

$$= \int_{\mathbf{B}\times\mathbf{B}} \overline{A^i(\phi z_1, \phi z_2, \phi w)}\,(\overline{E(z_1)})^{-\nu}(\overline{E(z_2)})^{-\nu}(E(w))^{-\nu}(\overline{E(w)})^{-1}f_1(\phi z_1)f_2(\phi z_1)$$

$$\times (E(z_1))^{-\nu}(E(z_2))^{-\nu}d\mathcal{M}_\alpha(z_1)d\mathcal{M}_\alpha(z_2)\,(E(w))^{\nu}\overline{E(w)}$$

$$= \int_{\mathbf{B}\times\mathbf{B}} \overline{A^i(\phi z_1, \phi z_2, \phi w)}\, f_1(\phi z_1)f_2(\phi z_1)d\mathcal{M}_\alpha(z_1)d\mathcal{M}_\alpha(z_2)$$

$$= J^i(f_1, f_2)(\phi w),$$

where we used that $d\mathcal{M}_\alpha(\phi z) = |E(z)|^{-2\nu}d\mathcal{M}_\alpha(z)$. This establishes (2).

Now take $z = 0$ in (2) and, as in Section 2, let ϕ be the symmetry interchanging 0 and w, where we first assume $w = (t, 0, \ldots, 0)$, $t > 0$. This gives

$$J^i(f_1, f_2)(w) = \sum_{k=1}^{d}\overline{\frac{\partial\phi^i(0)}{\partial z^k}}J^k(\tilde{f}_1, \tilde{f}_2)(0)(E(0))^{\mu}\overline{E(0)}.$$

Also by (1)

$$J^k(\tilde{f}_1, \tilde{f}_2)(0)$$

$$= \left\{\frac{\partial((f_1\circ\phi)E^{-\nu})}{\partial z^k}(f_2\circ\phi)E^{-\nu} - \frac{\partial((f_2\circ\phi)E^{-\nu})}{\partial z^k}(f_1\circ\phi)E^{-\nu}\right\}\bigg|_{z=0}$$

$$= \sum_{j=1}^{d}\left(\frac{\partial f_1(w)}{\partial w^j}\frac{\partial\phi^j(0)}{\partial z^k}f_2(w) - \frac{\partial f_2(w)}{\partial w^j}\frac{\partial\phi^j(0)}{\partial z^k}f_1(w)\right)(E(0))^{-2\nu}.$$

Thus

$$J^i(f_1, f_2)(w) = \sum_{j=1}^{d}\sum_{k=1}^{d}\overline{\frac{\partial\phi^i(0)}{\partial z^k}}\frac{\partial\phi^j(0)}{\partial z^k}\left(\frac{\partial f_1(w)}{\partial w^j}f_2(w) - \frac{\partial f_2(w)}{\partial w^j}f_1(w)\right).$$

By Section 2

$$\sum_{k=1}^{d} \overline{\frac{\partial \phi^i(0)}{\partial z^k}} \frac{\partial \phi^j(0)}{\partial z^k} = \begin{cases} (1-t^2)^2 & i=j=1 \\ 1-t^2 & i=j>1. \\ 0 & i \neq j \end{cases}$$

So (as $w = (t, 0, \ldots, 0)$)

$$\begin{cases} J^1(f_1, f_2) = (1-t^2)\left(\frac{\partial f_1(w)}{\partial w^1} f_2(w) - \frac{\partial f_2(w)}{\partial w^1} f_1(w)\right), \\ J^k(f_1, f_2) = \frac{\partial f_1(w)}{\partial w^k} f_2(w) - \frac{\partial f_2(w)}{\partial w^k} f_1(w) \quad (k > 1). \end{cases}$$

Passing to general w this finally gives

$$J^i = (\nabla^i f_1) f_2 - f_1(\nabla^i f_2)$$

where (cf. [R])

$$\nabla^i = \frac{\partial}{\partial z^i} - \bar{z}^i R, \quad R = \sum_{i=1}^{d} z^i \frac{\partial}{\partial z^i}.$$

Time for the Janson remark (see again [P])! We put $g_0 = -\sum_{i=1}^{d} z_i g_i$ and consider the form

$$G = (Z^0)^{-\mu} \sum_{i=0}^{d} g_i\left(\frac{Z^1}{Z^0}, \ldots, \frac{Z^d}{Z^0}\right) dZ^i$$
$$= (Z^0)^{-2\nu} \sum_{i=1}^{d} g_i\left(\frac{Z^1}{Z^0}, \ldots, \frac{Z^d}{Z^0}\right) d\left(\frac{Z^i}{Z^0}\right).$$

We further set

$$F_1 = (Z^0)^{-\nu} f_1\left(\frac{Z^1}{Z^0}, \ldots, \frac{Z^d}{Z^0}\right), \quad F_2 = (Z^0)^{-\nu} f_2\left(\frac{Z^1}{Z^0}, \ldots, \frac{Z^d}{Z^0}\right).$$

Then

$$\sum_{i=1}^{d} J_i \bar{g}_i = \sum_{i=0}^{d} \pm \left(\frac{\partial F_1}{\partial Z^i} F_2 - \frac{\partial F_2}{\partial Z^i} F_1\right) \bar{g}^i$$
$$= \langle\langle dF_1 \cdot F_2 - F_1 dF_2, G\rangle\rangle$$

so (1) can be written as

$$H(f_1, f_2) = \int_{\mathbf{B}} \langle\langle dF_1 \cdot F_2 - F_1 dF_2, G\rangle\rangle \| Z \|^{2\mu} dI(Z).$$

As in Section 2, we observe that the integrand in this formula is homogenous of degree 0.

It is now easy to write down the generalization to $\boxed{s > 1}$. For instance, if $s = 2$ we take

$$\left(\left(d^2 F \odot F_2 - 2\frac{\alpha + 1 + d + 1}{\alpha + 1 + d} dF_1 \odot dF_2 + F_1 \odot d^2 F_2, G\right)\right) \quad \text{etc.}$$

4. Big Hankel operators

So far we have only been concerned with Hankel forms, that is "small" Hankel operators. Now we turn to "big" Hankel operators. In other words, we are concerned with operators T of the type

$$Tf(z_1) = \int_{\mathbf{B}} \overline{\mathscr{A}(z_1, z_2)} K(z_1, z_2) f(z_2) d\mathscr{M}_\alpha(z_2),$$

mapping $A^{\alpha,2}(\mathbf{B})$ into its orthogonal complement $A^{\alpha,2}(\mathbf{B})^{\perp}$, where K is the reproducing kernel in $A^{\alpha,2}(\mathbf{B})$ and $\mathscr{A}$ a kernel which transforms as

$$\mathscr{A}(z_1, z_2) \mapsto \mathscr{A}(\phi z_1, \phi z_2).$$

This leads us to take

$$\mathscr{A}(z_1, z_2) = \int_{\mathbf{B}} \frac{\langle G, (\langle\langle Z_2, \bar{W}\rangle\rangle Z_1 - \langle\langle Z_1, \bar{W}\rangle\rangle Z_2)^{\otimes s}\rangle}{\langle\langle Z_1, \bar{W}\rangle\rangle^s \langle\langle Z_2, \bar{W}\rangle\rangle^s} \| W \|^{2s} dI(w).$$

5. Boundedness and trace ideal membership

So far all considerations have been purely formal. It is non time to ask the usual questions about boundedness (in the space $A^{\alpha,2}(\mathbf{B})$) and membership in trace ideal (Schatten-von Neumann) classes. However, we postpone this to a subsequent publication.

Note added May 1993. The results of this paper have been generalized to the case of general bounded symmetric domains in a forthcoming paper in the Rocky Mt. J. Math.

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