

## RAIKOV SYSTEMS AND ABSTRACT HARMONIC ANALYSIS

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This thesis investigates the structure of Raikov systems and the implications of this structure for the algebraic properties of  $M(G)$ , the convolution algebra of finite Borel measures on the locally compact abelian group  $G$ .

The techniques used are those of modern abstract harmonic analysis and new examples of constructions are obtained of both unexpected pathology and desirable regular phenomena.

A compact perfect subset of  $\Pi$  is a *Dirichlet set* if the constant function 1 can be uniformly approximated by continuous characters  $\chi \in \Pi \setminus \{1\}$  on  $A$ . The second chapter will show that given any Dirichlet set  $A$  on  $\Pi$  there exists a singly generated Raikov system containing  $A$  such that the Raikov system idempotent associated with this Raikov system is in the closure of the continuous characters  $\overline{\Pi} \subseteq \Delta M(\Pi)$ .

The third chapter shows that for each  $n \in \mathbb{Z}^+$  the Raikov systems generated by  $K_n$  subsets of  $D_n$  have associated Raikov system idempotent generalized characters lying in the closure of the continuous characters  $\overline{D}_n$  in the maximal ideal space  $\Delta M(D_n)$ .

The fourth chapter answers a question of Graham and McGehee [3], p. 411, in the negative by constructing a continuous tame probability measure which is supported on a proper Raikov system on  $\Pi$ . It also shows that there exists continuous tame probability measures in  $M_0(G)$  concentrated on proper Raikov systems on  $G$  where  $G$  is a countable

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product of cyclic groups.

The fifth chapter follows the work of Brown and Williamson [1] and define "churning" on  $\mathbb{D}_2 = \prod_{i=1}^{\infty} (\mathbb{Z}_2)_i$  and then studies some properties of measures under churning, and shows that  $M_0(\mathbb{D}_2)$  and  $\text{Rad } L^1(\mathbb{D}_2)$  are not closed under churning.

### References

- [1] G. Brown and J.H. Williamson, "Rearranging measures", *J. Austral. Math. Soc. Ser. A* **34** (1983), 16-30.
- [2] Charles F. Dunkl and Donald E. Ramirez, "Bounded projections on Fourier-Stieltjes transforms", *Proc. Amer. Math. Soc.* **31** (1972), 122-126.
- [3] Colin C. Graham, O. Carruth McGehee, *Essays in commutative harmonic analysis* (Grundlehren der mathematischen Wissenschaften, **238**. Springer-Verlag, Berlin, Heidelberg, New York, 1979).

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