Correspondence

DEAR EDITOR,

In a recent note entitled 'Digital patterns in perfect squares' [*Math. Gaz.* **84** (July 2000) pp. 289–291] Florian Luca made the following conjecture concerning perfect squares. For any given integer $K > 0$, there exist only finitely many perfect squares having at most K even digits when written in base 10.

This conjecture is false. We give examples showing that for any positive integer K, there are an infinite number of perfect squares with precisely K even digits when written in base 10. We do not offer explicit proofs, but let the examples speak for themselves.

Firstly, the following square numbers each have one even digit:

$$73^2 = 5329; \qquad 373^2 = 139129; \qquad 3373^2 = 11377129$$

$$33373^2 = 1113757129; \qquad 333373^2 = 111137557129;$$

$$3333373^2 = 11111375557129 \text{ etc.}$$

Note that the pattern continues with an extra digit '1' at the beginning, and an extra digit '5' in the middle.

Secondly, the following square numbers each have two even digits:

$$733^2 = 537289; \qquad 3733^2 = 13935289; \qquad 33733^2 = 1137915289;$$

$$333733^2 = 111377715289; \qquad 3333733^2 = 11113775715289 \text{ etc.}$$

As above, the pattern continues with an extra digit '1' at the beginning, and an extra digit '5' in the middle.

Thirdly, the following square numbers each have three even digits only:

$$7333^2 \qquad 37333^2 \qquad 337333^2 \qquad 3337333^2 \qquad 33337333^2 \text{ etc.}$$

By adding an additional '3' at the end, the number of even digits increases by one. Thus 7333^2 has four even digits (as do 373333^2, 3373333^2 etc), and 733333^2 has five even digits.

It should perhaps be pointed out that these are not the only examples. An alternative sequence with one even digit is provided by 7^2 37^2 337^2 3337^2 etc. It is also not the case that such series necessarily comprise the digits '3' and '7' exclusively. The numbers 17^2 137^2 1337^2 13337^2 etc. all have two even digits. However, this whole approach very much depends on the special relationship of the digits '3' and '7' with 10. Whether there are other radically different examples is a moot point.

Yours sincerely,

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DEAR EDITOR,

In my note 'An inductive proof of the arithmetic mean – geometric mean inequality', *Math. Gaz.* **84** (March 2000) p.101, are the words:

> 'On account of (2), the equality holds in the case $x = 0$, hence $G_n = a_{n+1}$. This implies that $a_1 = \dots = a_n = a_{n+1}$ in the case of equality.'

This is incorrect, because $a_{n+1} \notin \{a_1, \dots, a_n\}$.

It should have been:

> 'On account of (2), the equality holds in the case $x = 0$, hence $G_k = a_{k+1}$, $k = 1, 2, \dots, n - 1$ and $n \geqslant 2$. This implies that $a_1 = \dots = a_n$ in the case of equality.'

Yours sincerely,

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Currency corner: Pacioli's Summa

CHRIS PRITCHARD

In recent years the Italian mint has issued bimetallic L500 coins in a number of designs. One among them is inscribed:

1494 LUCA PACIOLI 1994

and shows the image of the Franciscan friar renowned for his invention of double entry bookkeeping. The coin commemorates the quincentenary of the publication of the book in which this contribution to accountancy first appeared, *Summa de arithmetica, geometrica, proportioni et proportionalita*. Historians of mathematics perhaps see it as offering little significant advance over the *Liber Abaci* of Fibonacci (Leonardo Pisano) except in terms of notation. The solution of quadratic equations, for example, is littered with abbreviations for the unknown quantity and its powers. However, it does contain a little treasure which sparked developments in probability theory. This was the statement and flawed solution of the 'division of stakes problem'. Pascal and Fermat offered correct solutions in the 1650s. More details of the life and work of Pacioli can be found in Nick MacKinnon's speculative article in the July 1993 issue of the *Gazette* (vol. 77, no. 479).

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