either $\alpha$, $\beta$ or $\nu$ must vanish, assumes that $\Delta$ is an invariant. There is, perhaps, no objection to this; yet it seems better in some ways to verify directly the expression of the original $\Delta$ in terms of the coefficients of $L$, $M$, $N$; and this leads almost at once to the result that the surface is a cylinder or paraboloid according as $\Delta$ is or is not zero. PERCY J. HEAWOOD.

## THE PILLORY.

IN the Indian Civil Service examination, Aug. 1911, the following problem was set:

*Prove that the series* $u_1+u_2+\ldots+u_n+\ldots,$

*in which all the terms are positive, cannot be convergent unless* $nu_n$ *tends to zero as a limit as* $n$ *increases indefinitely.*

This proposition is untrue, as the following series shows:

$$1+\tfrac{1}{2}+r^3+\tfrac{1}{4}+r^5+r^6+r^7+\tfrac{1}{8}+r^9+\ldots, \quad 0<r<1,$$

where $u_n=\dfrac{1}{2^s}$ if $n$ is of the form $2^s$ and $u_n=r^n$ otherwise.

This series converges to a value less than

$$(1+r+r^2+\ldots)+(\tfrac{1}{2}+\tfrac{1}{4}+\tfrac{1}{8}+\ldots)=\frac{1}{1-r}+1.$$

Furthermore, $nu_n$ has no limit as $n$ approaches infinity, but oscillates between 0 and 1. This disproves the proposition.

If it be required that the terms be *decreasing* as well as positive, or if it be required that $nu_n$ shall approach a limit, the proposition holds.

LESTER R. FORD.

Mathematical Department, Edinburgh University.

## CORRESPONDENCE.

*To the Editor of the Mathematical Gazette.*

DEAR SIR,—Will you allow me a few lines in explanation of my criticism of Mr. Hatton's proof of Desargues' Theorem (*Math. Gazette,* pp. 394 and 435)?

I was wrong in my implication that Mr. Hatton had argued in a circle by assuming the Fundamental Theorem in his proof of Desargues' Theorem, and therefore withdraw with apologies the phrase "logical unsoundness" with its implied accusation. But my objection is by no means wholly removed.

Mr. Hatton's proof, while not assuming the entire "Fundamental Theorem," is based on the Perspective Proposition: "If $OLAA'$ and $OMBB'$ are two sets of collinear points such that each is in perspective with a third set $ONCC'$ (*i.e.* such that $MN$, $BC$, $B'C'$ are concurrent in a point $X$, and $LN$, $AC$, $A'C'$ in a point $Y$), then $OLAA'$ and $OMBB'$ are in perspective (*i.e.* $LM$, $AB$, $A'B'$ are concurrent in a point $Z$)." Employing only the Axioms of Connection (or Projective Axioms), Desargues' Theorem can be deduced from this, and conversely. There would therefore be no logical objection to a proof of Desargues' Theorem, using the Perspective Proposition as an additional axiom; but Mr. Hatton proceeds to prove the Perspective Proposition by using ratios of segments. I contend that there is a logical objection to this in a system of Pure Projective Geometry, to which segments and ratios of segments are foreign. Mr. Hatton is, of course, free to reply that his field of discourse, like that of most English treatises on "Projective Geometry"—Mr. G. B. Mathews' recent book is the only exception that I know of—is not Pure Projective Geometry but Metrical Geometry

My objection then amounts to a protest against the use of extraneous subject matter in the proof of a theorem which is independent of such subject matter, just as I should object to the elementary proof of the existence of the common perpendicular to two skew lines on the ground that it employs the euclidean theory of parallels—always noting, however, that it is from the point of view of pure logic and not pedagogy that such objection is made.—Yours very truly,

D. M. Y. SOMMERVILLE.

13th January, 1915.

## REVIEWS.

**Elementary Applied Mechanics: Rules and Definitions.** By W G. HIBBINS, B.Sc. Pp. 30. Price 6d. (Longmans.)

An exceedingly useful little pamphlet. A good feature is the care with which the language of the definitions is chosen. For instance, the moment of a force is defined as the "*tendency* which a force has to turn," and is intentionally and correctly divorced from the definition of the "measure of a moment,"—to which the attention of authors of text-books may be directed. The use of "velocity" and "linear velocity" for "speed" and "velocity" is hardly so happy: nor is the definition of momentum as a "property," especially as this definition is given several pages after Newton's second Law, in which the words "rate of change of momentum" are used.

**A Senior Mental Arithmetic.** By S. GIBSON. Pp. 92. Price 1s. 6d. (Bell.)

Should prove useful to the boy or girl destined for trade. But from an educational point of view, the time would be much better spent in oral work in elementary algebraical substitutions and formula work in easy mensuration.

J. M. CHILD.

**Baumé and Specific Gravity Tables.** By N H. FREEMAN. Pp. 27. Price 2s. 6d. 1914. (London: E. & F Spon.)

Baumé's or Beaume's Hydrometer, extensively used in determining the alcoholic strength of spirit, is a variety of the common hydrometer, graduated by floating the instrument first in a ten per cent. solution of common salt, and then in pure distilled water at $54\frac{1}{2}°$ F The interval between the water-lines is divided into 10 degrees.

Mr. Freeman has computed specific gravities to seven places of decimals with first and second differences to seven places, corresponding to Baumé degrees.

A labour of zeal and of love, but not according to knowledge. A specific gravity computed to seven significant figures, of which possibly three are reliable in favourable circumstances! C. S. J

**Solutions of the Exercises in Godfrey and Siddons's Shorter Geometry.** By E. A. PRICE, B.A. Pp. viii + 160. 4s. 6d. net. 1914. (Cambridge University Press.)

Mr. Price's little volume will be found of considerable use to the private student, and even to teachers with such slender mathematical equipment that they are startled to find themselves in the presence of conics, conchoids, and the like (*e.g.* pp. 116-120). There are three pages of notes which will be useful to inexperienced teachers.

**A Treatise on Differential Equations.** By A. R. FORSYTH, F.R.S. Pp. xviii + 584. 14s. net. 1914. Fourth Edition. (Macmillan.)

The aim, scope, and signal merits of the above text-book are sufficiently well known to mathematicians in all countries to make a detailed notice of the fourth edition unnecessary. It will suffice to say that it is enlarged to the extent of some seventy additional pages. To quote from the preface, "Some portions of the book have been rewritten, particularly the early part of the chapter on the hypergeometric series, and parts of the chapter on