Correspondence

Dear Editor,

Diagonal Problem Conjecture

Is it possible to prove that for integer values $m \geqslant 0$ and $n \geqslant 0$, the following expression involving the Factorial and Gamma functions, viz.

$$\sum_{q=0}^{2m+1} \frac{(-1)^{1-q}}{q!} \frac{(q+n)!}{(q+2n+1)!} \frac{(2m+2n+q)!}{(2m+1-q)!} \frac{1}{\Gamma(n+q+\frac{1}{2})}$$

or equivalently

$$\frac{1}{\sqrt{n}} \sum_{q=0}^{2m+1} \frac{(-1)^{1-q}}{q!} \frac{((q+n)!)^2}{(q+2n+1)!} \frac{(2m+2n+q)!}{(2m+1-q)!} \frac{2^{2n+2q}}{(2n+2q)!}$$

is identically zero?

For initial values of m ($m = 0, 1, 2$) it is a straightforward but increasingly tedious exercise to show that the expression vanishes for arbitrary n-values, but it would be nice to see a general proof of the conjecture. For other randomly specified input values of m and n, it is also a straightforward matter to undertake a validation exercise using a spreadsheet to appreciate that the expression does appear to vanish, at least within the limits of accuracy associated with the evaluation of spreadsheet functions.

The above conjecture arose out of work carried out by the author many years ago, associated with the problem of determining the radiation pattern due to an electric field distribution that varied cosinusoidally across a circular aperture. It is also of relevance to the development of coefficients required in a Fourier-Bessel series expansion for the cosine function. The work involved matrix multiplication that resulted in a product matrix with various alternate diagonal terms vanishing, specifically those in cells where the $(2m+1+n)$th row intersected the nth column – hence the description "Diagonal Problem". In the event, only the above initial values for m were required by the author because of the rapid decay of other, associated multiplicative terms.

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Dear Editor,

Recently, while surfing the web, I came across the website [1] and became intrigued by the following problem, number 425, on page 161:

There was a diagram consisting of a pentagon labelled *ABCDEA* together with the five diagonals, and the problem was "A man started in a car from the town *A*, and wished to make a complete tour of these roads, going along every one of them once, and once only. How many different routes are there from which he can select? It is puzzling unless you can devise some ingenious method. Every route must end at the town *A*, from