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CONSTRUCTIONS OF THE MAXIMAL STRONGLY CHARACTER INVARIANT SEGAL ALGEBRAS AND THEIR APPLICATIONS

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Abstract

Let G denote any locally compact abelian group with the dual group Γ. We construct a new kind of subalgebra $L^1(G) \otimes_\Gamma S$ of $L^1(G)$ from given Banach ideals S in $L^1(G)$. We show that $L^1(G) \otimes_\Gamma S$ is the largest among all strongly character invariant homogeneous Banach algebras in S. When S contains a strongly character invariant Segal algebra on G, it is shown that $L^1(G) \otimes_\Gamma S$ is also the largest among all strongly character invariant Segal algebras in S. We give applications to characterizations of two kinds of subalgebras of $L^1(G)$-strongly character invariant Segal algebras on G and Banach ideals in $L^1(G)$ which contain a strongly character invariant Segal algebra on G.

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1. Notations and definitions

Throughout this article, T denotes the circle group. R denotes the additive group of real numbers. G denotes any locally compact abelian group with the dual group Γ. $P(L^1(G))$ denotes the space of all f in $L^1(G)$ whose Fourier transforms $\hat{f}$ have compact support.

For the convenience of the readers, we recall some definitions: An ideal S in $L^1(G)$ is called a *normed ideal* in $L^1(G)$ if S is also a normed linear space under some norm $\| \ \|_S$ such that $\| f * g \|_S \leq \| f \|_1 \| g \|_S$ for all $f \in L^1(G)$ and $g \in S$.

This definition is weaker than that of J. Cigler [2]. In addition, if $(S, \| \|_S)$ is also a Banach space, then S is called a *Banach ideal* in $L^1(G)$. A subalgebra S of $L^1(G)$ is called a *semi-homogeneous Banach algebra* on G if S is a Banach algebra under some norm $\| \|_S \geqslant \| \|_1$ and satisfies the property:

$(H\text{-}1)$ If $f \in S$ and $x \in G$, then $L_x f \in S$ and $\|L_x f\|_S = \|f\|_S$ (where $L_x f(y) = f(y - x)$).

If S satisfies the additional property:

$(H\text{-}2)$ For every $f \in S$, the map $x \to L_x f$ is continuous from G into $(S, \| \|_S)$, then S is called a *homogeneous Banach algebra* on G. The definition is equivalent to that of a homogeneous Banach space in Katznelson [6]. The proof can be found in [13, Theorem 3.2]. A semi-homogeneous Banach algebra S on G is called (*strongly*) *character invariant* if $\gamma \in \Gamma, f \in S$ imply $\gamma f \in S$ (and $\|\gamma f\|_S = \|f\|_S$), where $\gamma f(x) = (x, \gamma) f(x)$. In [13], H. C. Wang uses the word "character" instead of "strongly character invariant". A dense homogeneous Banach algebra in $L^1(G)$ is called a *Segal algebra* on G. For fundamental results on Segal algebras, see Reiter ([8, 9]) and Wang [13].

2. Construction of the maximal strongly character invariant homogeneous Banach algebras

Suppose that $(A, \| \|_A)$ and $(B, \| \|_B)$ are two normed linear spaces in $L^1(G)$ with $\| \|_A \geqslant \| \|_1$ and $\| \|_B \geqslant \| \|_1$. We introduce a new kind of linear subspaces of $L^1(G)$ as follows: The set $A \otimes^\Gamma B$ consists of all those elements $f \in L^1(G)$ such that

$$\text{(i)} \qquad f = \sum_n g_n * h_n$$

subject to the conditions:

$$\text{(ii)} \qquad \gamma g_n \in A \cap P(L^1(G)), \qquad \gamma h_n \in B \cap P(L^1(G)), \qquad \forall n \geqslant 1, \gamma \in \Gamma;$$

and

$$\text{(iii)} \qquad \sup_{\gamma \in \Gamma} \sum_n \|\gamma g_n\|_A \|\gamma h_n\|_B < \infty.$$

Clearly, (iii) implies that the series (i) converges in $L^1(G)$ and

$$\|f\|_1 \leqslant \text{the infimum of all possible values in (iii)}.$$

Denote the infimum by $\|f\|_\Gamma$. The above inequality means that

$$\|f\|_1 \leqslant \|f\|_\Gamma.$$

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We remark here that $A \otimes^\Gamma B$ may be zero. For example, take $A = L^1(T)$ and $B = C^1(T)$ = the space of all continuously differentiable functions on T. It follows from the above definitions that $A \otimes^\Gamma B = B \otimes^\Gamma A$ and $(A \otimes^\Gamma B, \| \|_\Gamma)$ is a normed linear space in $L^1(G)$. Moreover,

PROPOSITION 1. *Let A be a normed ideal in $L^1(G)$. If A or B satisfies the $(H\text{-}1)$ property, then $A \otimes^\Gamma B$ is not only a strongly character invariant semi-homogeneous Banach algebra on G but also a normed ideal in $L^1(G)$.*

PROOF. Let (f_m) be a sequence in $A \otimes^\Gamma B$ with $\sum_m \| f_m \|_\Gamma < \infty$. For each m there exists two sequences $(g_n^m) \subseteq A \cap P(L^1(G))$ and $(h_n^m) \subseteq B \cap P(L^1(G))$ such that

$$f_m = \sum_n g_n^m * h_n^m$$

and

$$\sup_{\gamma \in \Gamma} \sum_n \| \gamma g_n^m \|_A \| \gamma h_n^m \|_B \leqslant \| f_m \|_\Gamma + 2^{-m}.$$

This implies that

$$\begin{aligned} \sum_m \sum_n \| g_n^m \|_1 \| h_n^m \|_1 &\leqslant \sup_{\gamma \in \Gamma} \sum_m \sum_n \| \gamma g_n^m \|_A \| \gamma h_n^m \|_B \\ &\leqslant \sum_m \left(\sup_{\gamma \in \Gamma} \sum_n \| \gamma g_n^m \|_A \| \gamma h_n^m \|_B \right) \\ &\leqslant \sum_m (\| f_m \|_\Gamma + 2^{-m}) < \infty. \end{aligned}$$

Let $f = \sum_m \sum_n g_n^m * h_n^m$ in $(L^1(G), \| \|_1)$. From the above inequalities we find that

$$f \in A \otimes^\Gamma B,$$
$$f - \sum_{1 \leqslant k \leqslant m} f_k = \sum_{k > m} \sum_n g_n^k * h_n^k \quad \text{in } \left(L^1(G), \| \|_1\right)$$

and

$$\begin{aligned} \left\| f - \sum_{1 \leqslant k \leqslant m} f_k \right\|_\Gamma &\leqslant \sup_{\gamma \in \Gamma} \sum_{k > m} \sum_n \| \gamma g_n^k \|_A \| \gamma h_n^k \|_B \\ &\leqslant \sum_{k > m} \left(\sup_{\gamma \in \Gamma} \sum_n \| \gamma g_n^k \|_A \| \gamma h_n^k \|_B \right) \\ &\leqslant \sum_{k > m} \left(\| f_k \|_\Gamma + 2^{-k} \right) \\ &\to 0 \quad \text{as } m \to \infty, \end{aligned}$$

which shows that the series $\Sigma_m f_m$ converges to f in $(A \otimes^\Gamma B, \|\ \|_\Gamma)$, and so $(A \otimes^\Gamma B, \|\ \|_\Gamma)$ is complete. The remainder of the proof is based on the identities: $\gamma(f * g) = (\gamma f) * (\gamma g)$ and $\gamma(L_x f) = (x, \gamma) L_x(\gamma f)$. It is so straightforward as to be omitted.

PROPOSITION 2. *Suppose that A and B satisfy the hypotheses of Proposition* 1. *Let $A \otimes_\Gamma B$ denote the space of all $f \in A \otimes^\Gamma B$ such that $x \to L_x f$ is continuous from G into $(A \otimes^\Gamma B, \|\ \|_\Gamma)$; then $(A \otimes_\Gamma B, \|\ \|_\Gamma)$ is a strongly character invariant homogeneous Banach algebra on G.*

PROOF. In view of [13, Theorem 2.6], it suffices to show that $\gamma \in \Gamma, f \in A \otimes_\Gamma B$ imply $\gamma f \in A \otimes_\Gamma B$. We have

$$\begin{aligned} \|L_x(\gamma f) - L_y(\gamma f)\|_\Gamma &= \|(-x, \gamma)\gamma L_x f - (-y, \gamma)\gamma L_y f\|_\Gamma \\ &\leqslant |(-x, \gamma) - (-y, \gamma)|\, \|\gamma L_x f\|_\Gamma \\ &\quad + |(-y, \gamma)|\, \|\gamma(L_x f - L_y f)\|_\Gamma \\ &= |(-x, \gamma) - (-y, \gamma)|\, \|f\|_\Gamma + \|L_x f - L_y f\|_\Gamma. \end{aligned}$$

Since γ is continuous on G and $f \in A \otimes_\Gamma B$, it follows that $x \to L_x(\gamma f)$ is continuous from G into $(A \otimes^\Gamma B, \|\ \|_\Gamma)$ and so $\gamma f \in A \otimes_\Gamma B$. This completes the proof.

PROPOSITION 3. *If A and B are two Banach ideals in $L^1(G)$, then $A \otimes_\Gamma B \subseteq A \otimes^\Gamma B \subseteq A \cap B$ and* $\|\ \|_\Gamma \geqslant \max(\|\ \|_A, \|\ \|_B)$.

REMARK. This proposition does not hold in case of normed ideals, that is, it is necessary that $(A, \|\ \|_A)$ and $(B, \|\ \|_B)$ be complete. For example, consider $(A, \|\ \|_A) = (L^1(G), \|\ \|_1)$ and $(B, \|\ \|_B) = (P(L^1(G)), \|\ \|_1)$. Here $(B, \|\ \|_B)$ is not complete. It follows easily that $f \in A \otimes^\Gamma B$ and $\|f\|_\Gamma = \|f\|_1$ for all $f \in P(L^1(G))$, which implies $A \otimes_\Gamma B = A \otimes^\Gamma B = L^1(G) \not\subseteq A \cap B$.

PROOF OF PROPOSITION 3. For the sake of symmetry, it suffices to show that $A \otimes^\Gamma B \subseteq A$ and $\|\ \|_A \leqslant \|\ \|_\Gamma$. Let $f \in A \otimes^\Gamma B$. For any $\varepsilon > 0$, there exist $(g_n) \subseteq A \cap P(L^1(G))$ and $(h_n) \subseteq B \cap P(L^1(G))$ such that

$$f = \sum_n g_n * h_n$$

and

$$\sup_{\gamma \in \Gamma} \sum_n \|\gamma g_n\|_A \|\gamma h_n\|_B < \|f\|_\Gamma + \varepsilon.$$

We have

$$\begin{aligned}\sum_n \|g_n * h_n\|_A &\leqslant \sum_n \|g_n\|_A \|h_n\|_1 \leqslant \sum_n \|g_n\|_A \|h_n\|_B \\ &\leqslant \sup_{\gamma\in\Gamma} \sum_n \|\gamma g_n\|_A \|\gamma h_n\|_B \\ &> \|f\|_\Gamma + \varepsilon < \infty.\end{aligned}$$

This implies that there exists $\phi \in A$ such that $\phi = \Sigma_n g_n * h_n$ in $(A, \|\ \|_A)$ and consequently $\phi = \Sigma_n g_n * h_n$ in $(L^1(G), \|\ \|_1)$. Since $f = \Sigma_n g_n * h_n$ in $(L^1(G), \|\ \|_1)$, it follows that $\phi = f$. Therefore $A \otimes^\Gamma B \subseteq A$. On the other hand,

$$\|f\|_A = \|\phi\|_A \leqslant \sum_n \|g_n * h_n\|_A < \|f\|_\Gamma + \varepsilon.$$

It follows that $\|f\|_A \leqslant \|f\|_\Gamma$. This completes the proof.

THEOREM 4. *Let S be a Banach ideal in $L^1(G)$, then $L^1(G) \otimes_\Gamma S$ is the largest among all strongly character invariant homogeneous Banach algebras in S.*

REMARK. In general, $L^1(G) \otimes_\Gamma S$ is smaller than the maximal homogeneous Banach space S_c in S. (See [6] and [13] for the definition of B_c.)

PROOF. It suffices to show that if B is a strongly character invariant homogeneous Banach algebra in S, then $B \subseteq L^1(G) \otimes_\Gamma S$. We divide the proof into two steps. First, claim that $B \subseteq L^1(G) \otimes^\Gamma S$. For any $f \in B$, there exist $(g_n) \subseteq P(L^1(G))$ and $(h_n) \subseteq B \cap P(L^1(G))$ such that

$$f = \sum_n g_n * h_n \quad \text{in } (B, \|\ \|_B)$$

and

$$\sum_n \|g_n\|_1 \|h_n\|_B < \infty,$$

which follows immediately from [13, Theorem 3.7(i)] and [11, Theorem 2.6.8]. Since $B \subseteq S$, there exists a constant ρ such that $\|\ \|_S \leqslant \rho \|\ \|_B$, which implies

$$(*)\qquad \begin{aligned}\sup_{\gamma\in\Gamma} \sum_n \|\gamma g_n\|_1 \|\gamma h_n\|_S &\leqslant \rho \sup_{\gamma\in\Gamma} \sum_n \|g_n\|_1 \|\gamma h_n\|_B \\ &= \rho \sum_n \|g_n\|_1 \|h_n\|_B < \infty,\end{aligned}$$

and so $f \in L^1(G) \otimes^\Gamma S$. Therefore $B \subseteq L^1(G) \otimes^\Gamma S$. Next, claim that $B \subseteq L^1(G) \otimes_\Gamma S$. Since $B \subseteq L^1(G) \otimes^\Gamma S$, it follows that there exists a constant ρ' such

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that $\| \ \|_\Gamma \leqslant \rho' \| \ \|_B$. For any $f \in B$, we have

$$\begin{aligned} \| L_x f - L_y f \|_\Gamma &\leqslant \rho' \| L_x f - L_y f \|_B \\ &\to 0 \quad \text{as } y \to x \end{aligned}$$

which implies $f \in L^1(G) \otimes_\Gamma S$. Therefore $B \subseteq L^1(G) \otimes_\Gamma S$. This completes the proof.

3. A characterization of strongly character invariant Segal algebras

THEOREM 5. *Let S be a Segal algebra on G, then the following three properties are equivalent*:

(a) *There exists a norm under which S becomes a strongly character invariant Segal algebra on G.*

(b) $L^1(G) \otimes_\Gamma S = S$.

(c) $\sup\{\|\gamma f\|_S \colon \gamma \in \Gamma, f \in P(L^1(G))$ *and* $\|f\|_S = 1\} < \infty$.

PROOF. Applying Theorem 4 we see that (a) and (b) are equivalent. Now, claim that (b) implies (c). If $L^1(G) \otimes_\Gamma S = S$, then there exists a constant ρ such that $\|f\|_S \leqslant \|f\|_\Gamma \leqslant \rho \|f\|_S$ for all $f \in S$. This implies that for any $f \in P(L^1(G))$ we have

$$\sup_{\gamma \in \Gamma} \|\gamma f\|_S \leqslant \sup_{\gamma \in \Gamma} \|\gamma f\|_\Gamma = \|f\|_\Gamma \leqslant \rho \|f\|_S.$$

It follows that

$$\sup\left\{\|\gamma f\|_S \colon \gamma \in \Gamma, f \in P\left(L^1(G)\right) \text{ and } \|f\|_S = 1\right\} \leqslant \rho < \infty,$$

which shows that (b) implies (c). Next, claim that (c) implies (b). Assume that

$$\rho = \sup\left\{\|\gamma f\|_S \colon \gamma \in \Gamma, f \in P\left(L^1(G)\right) \text{ and } \|f\|_S = 1\right\} < \infty.$$

In view of Proposition 3, it suffices to show that $S \subseteq L^1(G) \otimes_\Gamma S$. The proof of Theorem 4 can be applied to this case if we use this ρ to play its role in Theorem 4 and replace $(*)$ in Theorem 4 by

$$(*)' \qquad \begin{aligned} \sup_{\gamma \in \Gamma} \sum_n \|\gamma g_n\|_1 \|\gamma h_n\|_S &= \sup_{\gamma \in \Gamma} \sum_n \|g_n\|_1 \|\gamma h_n\|_S \\ &\leqslant \rho \sum_n \|g_n\|_1 \|h_n\|_S < \infty. \end{aligned}$$

It is so easy as to be omitted.

EXAMPLE. Consider the following Segal algebras:

(a) $C^k(T)$ consists of all k-times continuously differentiable functions on T, with the norm $\|f\| = \sup_{0 \leqslant j \leqslant k} \|f^{(j)}\|_\infty$.

(b) $L^{(k)}(T)$ consists of all f in $L^1(T)$ such that for $j = 0, 1, \ldots, k-1$, $f^{(j)}$ are absolutely continuous on T and $f^{(j+1)} \in L^1(T)$, with the norm $\|f\| = \sup_{0 \leqslant j \leqslant k} \|f^{(j)}\|_1$.

(c) $L^{(k)}(R)$ consists of all f in $L^1(R)$ such that for $j = 0, 1, \ldots, k-1$, $f^{(j)}$ are absolutely continuous on R and $f^{(j+1)} \in L^1(R)$, with the norm $\|f\| = \sup_{0 \leqslant j \leqslant k} \|f^{(j)}\|_1$ (see [1], [6], [8], [12], [13]). Let S denote any one of $C^k(T)$, $L^{(k)}(T)$ and $L^{(k)}(R)$. It is well-known that S is character invariant. From Theorem 5 it is easy to show that there exists no norm under which S becomes a strongly character invariant Segal algebra.

4. A characterization of ideals in $L^1(G)$ which contain a strongly character invariant Segal algebra

THEOREM 6. *Let S be a Banach ideal in $L^1(G)$; then the following three properties are equivalent*:

(a) *There is the largest among all strongly character invariant Segal algebras in S.*

(b) *S contains a strongly character invariant Segal algebra on G.*

(c) *$P(L^1(G)) \subseteq S$ and $\sup_{\gamma \in \Gamma} \|\gamma f\|_S < \infty$ for all $f \in P(L^1(G))$.*

PROOF. Applying Theorem 4 we see that (a) and (b) are equivalent. Now, claim that (b) implies (c). Let B be a strongly character invariant Segal algebra in S. Then $P(L^1(G)) \subseteq S$ and there exists a constant ρ such that $\|\ \|_S \leqslant \rho \|\ \|_B$. This implies that for any $f \in P(L^1(G))$ we have

$$\sup_{\gamma \in \Gamma} \|\gamma f\|_S \leqslant \rho \sup_{\gamma \in \Gamma} \|\gamma f\|_B = \rho \|f\|_B < \infty.$$

This shows that (b) implies (c). Next, claim that (c) implies (a). In view of Theorem 4, it suffices to show that $P(L^1(G)) \subseteq L^1(G) \otimes_\Gamma S$. For any $f \in P(L^1(G))$ there exists $g \in P(L^1(G))$ such that $\hat{g} = 1$ on $\operatorname{supp} \hat{f}$. This implies that

$$f = g * f,$$

$$\sup_{\gamma \in \Gamma} \|\gamma g\|_1 \|\gamma f\|_S = \|g\|_1 \sup_{\gamma \in \Gamma} \|\gamma f\|_S < \infty$$

and

$$\begin{aligned} \|L_x f - L_y f\|_\Gamma = \|(L_x g - L_y g) * f\|_\Gamma &\leqslant \sup_{\gamma \in \Gamma} \|\gamma(L_x g - L_y g)\|_1 \|\gamma f\|_S \\ &= \|L_x g - L_y g\|_1 \sup_{\gamma \in \Gamma} \|\gamma f\|_S \\ &\to 0 \quad \text{as } y \to x. \end{aligned}$$

It follows that $f \in L^1(G) \otimes_\Gamma S$ and so $P(L^1(G)) \subseteq L^1(G) \otimes_\Gamma S$. This completes the proof.

EXAMPLE. Let α be a locally bounded function on Γ with $\alpha \geqslant 1$. Define $S(\alpha)$ as the space of all f in $L^1(G)$ such that $\lim \hat{f}(\gamma)\alpha(\gamma) = 0$. Under the norm $\| f \|_\alpha = \| f \|_1 + \sup_{\gamma \in \Gamma} |\hat{f}(\gamma)\alpha(\gamma)|$, $S(\alpha)$ forms a Segal algebra on G. (See [10].) We claim that $S(\alpha)$ contains no strongly character invariant Segal algebras on G if and only if α is unbounded on Γ. In this case, it follows that $L^1(G) \otimes_\Gamma S(\alpha)$ is not a Segal algebra on G. Now we give a detailed proof as follows: Take $f \in P(L^1(G))$ with $\hat{f}(0) = 1$. We have

$$\begin{aligned} \sup_{\chi \in \Gamma} |\alpha(\chi)| &= \sup_{\chi \in \Gamma} |\hat{f}(0)\alpha(\chi)| \\ &\leqslant \sup_{\chi \in \Gamma} \sup_{\gamma \in \Gamma} |\hat{f}(\gamma - \chi)\alpha(\gamma)| \\ &= \sup_{\chi \in \Gamma} \sup_{\gamma \in \Gamma} |(\chi f)\hat{}(\gamma)\alpha(\gamma)| \leqslant \sup_{\chi \in \Gamma} \| \chi f \|_\alpha . \end{aligned}$$

If α is unbounded on Γ, then $\sup_{\chi \in \Gamma} \| \chi f \|_\alpha = \infty$. From Theorem 6 we find that $S(\alpha)$ contains no strongly character invariant Segal algebras on G. If α is bounded on Γ, [10, Proposition 2] states that $S(\alpha) = L^1(G)$, which is a strongly character invariant Segal algebra on G. This completes the proof.

EXAMPLE. Define $F(R)$ as the space of all f in $L^1(R)$ such that $\lim \hat{f}(n) \log n = 0$. Under the norm $\| f \| = \| f \|_1 + \sup_n |\hat{f}(n)| \log n$, $F(R)$ forms a Segal algebra on R (see [4]). In this case, $G = R$, $\Gamma = R$ and so the largest strongly character invariant homogeneous Banach algebra in $F(R)$ is $L^1(R) \otimes_R F(R)$. By a similar argument as above we can show that $F(R)$ contains no strongly character invariant Segal algebras on R and $L^1(R) \otimes_R F(R)$ is not a Segal algebra on R.

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