

TOPICS IN THE THEORY OF LATIN SQUARES

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This thesis deals with various aspects of latin squares. The following is a summary of our results.

I. First, a new construction on latin squares is introduced and its properties analysed. This construction forms the basis of most of the results in the thesis.

II. The Evans Conjecture on partial latin squares is shown to be true.

III. Let L_n denote the number of distinct latin squares of order n . It is shown that $L_n \geq n!L_{n-1}$. As a consequence, it can be immediately deduced that L_n is strictly increasing (not previously known). Another consequence of the inequality is that we can write $L_n \geq n!(n-1)!(n-2)! \dots (n_0+1)!L_{n_0}$ for $n > n_0$ where n_0 is the largest order for which the number of latin squares is known. This is a slight improvement on Hall's lower bound for L_n and so establishes a new lower bound for L_n .

IV. The number of latin squares with the property that cell (i, i) contains i for all i in the range $1 \leq i \leq n-1$ is shown to be at least $L_n / (n!(n-1)!)$. This is shown to be relevant to two current unsolved

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problems on the structure of latin squares.

V. The problem of completing partial frequency squares (the frequency square is a generalization of the latin square) is considered. It is shown that the problem of completing a partial frequency square can be transformed into the problem of completing partial latin squares. This will allow the current knowledge on completing partial latin squares to be used to obtain results on completing partial frequency squares.

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