

# A note on using thermally driven solar wind models in MHD space weather simulations

Jens Pomoell<sup>1</sup> and Rami Vainio<sup>1</sup>

<sup>1</sup>Department of Physics, University of Helsinki  
P.O. Box 64, 00014 University of Helsinki, Finland  
email: jens.pomoell@helsinki.fi, rami.vainio@helsinki.fi

**Abstract.** One of the challenges in constructing global magnetohydrodynamic (MHD) models of the inner heliosphere for, e.g., space weather forecasting purposes, is to correctly capture the acceleration and expansion of the solar wind. In many current models, the solar wind is driven by varying the polytropic index so that a desired heating is obtained. While such schemes can yield solar wind properties consistent with observations, they are not problem-free. In this work, we demonstrate by performing MHD simulations that altering the polytropic index affects the properties of propagating shocks significantly, which in turn affect the predicted space weather conditions. Thus, driving the solar wind with such a mechanism should be used with care in simulations where correctly capturing the shock physics is essential. As a remedy, we present a simple heating function formulation by which the polytropic wind can be used while still modeling the shock physics correctly.

**Keywords.** shock waves, methods: numerical, solar wind, solar-terrestrial relations, Sun: coronal mass ejections

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## 1. Introduction

Although more than 50 years has passed since Parker theorized the existence of a supersonic solar wind (Parker 1958), the physical processes responsible for accelerating the fast solar wind and heating the solar corona remain unknown (see, e.g., Cranmer 2010 for a review). In spite of this, a number of magnetohydrodynamic (MHD) models have successfully been developed that are capable of reproducing reasonably accurately the physical conditions in the inner heliosphere under the steady state assumption (see, e.g., Cohen *et al.* 2008 for a short overview). Common to these models is that they all use some form of (more or less) ad-hoc heating mechanism to drive the solar wind. For instance, a popular choice used in many studies investigating coronal mass ejections (CMEs) is to use a polytropic index  $\gamma$  smaller than  $5/3$ , the value expected for a monoatomic plasma such as the solar corona.

Altering the polytropic index is not problem-free. For instance, the maximal compression ratio  $r$  of a MHD shock is given by  $r = (\gamma + 1)/(\gamma - 1)$ , which is equal to 4 for a monoatomic gas, and increases for smaller values of  $\gamma$ . However, the efficiency of shocks as particle accelerators is very sensitive to the magnitude of the compression ratio of the shock (e.g., Reames 1999). Thus, for applications where capturing the shock physics is essential, it is not clear if models using an altered polytropic index can be used. In this paper, we demonstrate the effects that altering the polytropic index has on the evolution of shocks in MHD simulations. We also show how it is possible to retain the correct shock physics and still use a solar wind solution generated by a model with a polytropic index not equal to five thirds.

## 2. Model

We solve numerically the equations of ideal MHD, given by

$$\begin{aligned}
 \partial_t \rho &= -\nabla \cdot (\rho \mathbf{v}) & \partial_t \mathbf{B} &= \nabla \times (\mathbf{v} \times \mathbf{B}) \\
 \rho \mathcal{D}\mathbf{v} &= -\nabla P + \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} + \rho \mathbf{g} & 0 &= \nabla \cdot \mathbf{B} \\
 S &= \mathcal{D}(P/\rho^\gamma) & \mathcal{D} &\equiv \partial_t + \mathbf{v} \cdot \nabla,
 \end{aligned}$$

using spherical coordinates in a two-dimensional azimuthally symmetric setting. Here  $\rho$  is the mass density,  $\mathbf{v}$  is the velocity field,  $\mathbf{B}$  is the magnetic field,  $P$  is the thermal pressure,  $\mathbf{g} = \frac{GM_\odot}{r^2} \hat{\mathbf{r}}$  is the gravitational acceleration,  $\gamma$  is the polytropic index and  $S$  is an energy source term. Note that we solve the equations not in the primitive form given above, but in conservative form using the variables  $\rho, \rho \mathbf{v}, \mathbf{B}$  and energy density  $u = \frac{P}{\gamma-1} + \frac{1}{2} \rho \mathbf{v}^2 + \frac{1}{2\mu_0} \mathbf{B}^2$ .

### 2.1. Solar wind model 1: $S = 0, \gamma = 1.05$

To obtain a steady state solar wind solution, we set  $S = 0$  and choose the adiabatic index to be  $\gamma = 1.05 \equiv \Gamma_1$ , a value commonly used in the literature. The initial magnetic field is set to a dipole field, and the density and radial velocity is initialized according to Parker’s hydrodynamical solar wind solution. We then integrate the MHD equations in time until a converged solution is reached. For the upcoming discussion, we denote the obtained steady state solution by  $\{\rho_1, P_1, \mathbf{v}_1, \mathbf{B}_1\}$ .

### 2.2. Solar wind model 2: $S \neq 0, \gamma = 5/3$

As the next step, we wish to obtain an identical solar wind solution as obtained in the previous case, but instead use  $\gamma = 5/3 \equiv \Gamma_2$ . To achieve this, an energy source  $S$  driving the wind is necessary. We derive  $S$  by requiring that in the steady state the two solutions are identical, i.e.  $\{\rho_1, P_1, \mathbf{v}_1, \mathbf{B}_1\} = \{\rho_2, P_2, \mathbf{v}_2, \mathbf{B}_2\}$  when  $\partial_t = 0$ . Plugging into the MHD equations gives

$$S = \mathbf{v}_1 \cdot \nabla \left( P_1 \rho_1^{-\Gamma_2} \right) \tag{2.1}$$

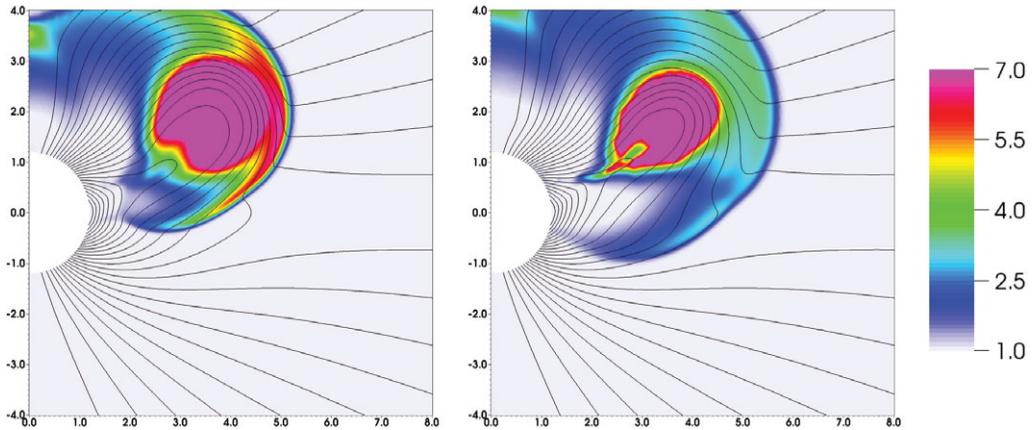
Applying this energy source term, we can retain  $\gamma = 5/3$  and still have the identical solar wind solution as obtained by using a lower polytropic index.

### 2.3. CME model

The final step in our simulation is to generate a CME. In this work, we simply superimpose on the solar wind solution a circular region with a higher density and an initial radial velocity. Additionally, we launch the CME 30° to the North from the equatorial plane.

## 3. Results and Discussion

The erupting CME launches a coronal shock wave evolving initially in a quasi-circular manner with the strongest regions near the nose of the shock. While the early evolution of the shock is morphologically similar for both models, there are important differences to note. Fig. 1 shows the compression ratio for the two simulation runs 48 minutes after the start of the eruption, with the left (right) figure corresponding to the simulation using solar wind model 1 (2). As can be seen, the compression ratio of the shock in the simulation using  $\gamma = 1.05$  exceeds 4 for the entire shock front except for the flank. On the other hand, with  $\gamma = 5/3$ , the compression ratio  $r > 3$  for a large part of the shock, but



**Figure 1.** Snapshots from the MHD simulations showing the compression ratio and magnetic field lines (black curves) 48 minutes after the start of the eruption. The image to the left corresponds to the case where  $\gamma = 1.05$  was used, and the image to the right to the case where  $\gamma = 5/3$  and an energy source term  $S$  was used. Note that except for the mechanism driving the background solar wind the two simulation setups were identical, using identical initial and boundary conditions as well as the same computational grid and numerical method. See on-line version for color figures.

does not exceed 4 anywhere. This indicates that our model including the energy source treats the shock physics consistently in terms of compression, which is important, e.g., for particle acceleration applications.

It is not only the compression ratio that is different for the two simulations. In fig. 1, it is evident that the shock with  $\gamma = 5/3$  has reached further out. Moreover, the shock structure at the southern flank is significantly different; not only has it reached further, but also the morphology near the skirt of the shock is different. This might be of importance to studies using MHD simulations that discuss the origin of so called EIT waves commonly observed in conjunction with CMEs.

#### 4. Summary and Conclusions

We have used MHD simulations to obtain two identical steady state solar wind solutions, but using different methods to drive the wind. In the first case a polytropic index  $\gamma = 1.05$  was used, while in the second case the polytropic index was set to  $\gamma = 5/3$ , but an appropriate extra source term in the energy equation was included. On top of these solutions, we launched a CME, and studied the shock launched by the eruption.

From the simulations we can conclude that using  $\gamma = 5/3$  is vital in order for the model to treat the shock physics consistently in terms of compression and dynamical evolution. This is especially important for studies where the shock obtained by the MHD simulation is used as input to particle acceleration simulations.

#### References

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