as they stand, take the digit in the ten's place of this sum and append it to the part of the quotient already found, thus extending the quotient by one digit, then repeat the process: with the proviso that when the above sum ends in one of the numbers *under* the same line which the divisor (in the right-hand column) itself is *under*, the ten's place is to be increased by one, viz. :—

								94
							85	95
					66	76	86	96
			47	57	67	77	87	97
		38	48	58	68	78	88	98
19	29	39	49	59	69	79	89	99

The process might be extended to such numbers as 996, etc.; and also to 101, 102, etc.

On the Decimalization of Money.

By John W. Butters, M.A., B.Sc.

As stated in the preceding paper, the method of expressing, at sight, shillings, pence, and farthings as a decimal of a pound to 3 places has long been known. It is sometimes referred to as the actuaries' rule. According to De Morgan, it occurs for the first time in Kersey's edition of *Wingate's Arithmetic*, 1673 (p. 191). It is also to be found in *Cocker's Decimal Arithmetic*, 1685 (although in a form which is not quite accurate). In some of the earlier books the method of conversion at sight *from* the decimal form is

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given, but not *vice versâ*. It is now found in most modern text-books in one form or another.

The method of extension beyond the third decimal place as given in Mr Hamblin Smith's paper is not quite new. The same method is used by De Morgan (*Companion to the British Almanac*, 1841) to find the *nearest* 4th place; a similar method is given by him in 1848 (*Companion to B. A.*) whereby the *actual* 4th and 5th places are obtained; the same method as Mr Hamblin Smith's, but with a different proviso, occurs in *Jackson's Commercial Arithmetic*, 1893. A much simpler method (given in a footnote to my former paper *) is to be found in the forty-ninth edition of a text-book on Arithmetic by Alexander Ingram and Alexander Trotter. This edition bears the date 1871, and I have reason for believing that the method was inserted by Trotter about that year. Mr F. C. Crawford, Edinburgh, informs me that he was taught the method by Trotter in January or February 1868. It occurs also in *Practical Arithmetic for Senior Classes*, by Henry G. C. Smith. To this there is no date.

So far as I know, no proof of the method has been given, and as this is also wanting in Mr Hamblin Smith's paper, it seems desirable to supply the deficiency.

Considered apart from its application to money, the rule may be stated as follows (a special case being taken for simplicity, although the method and proof are perfectly general):—In reducing $\frac{17}{96}$ to a decimal form, if at any stage we multiply the last two digits found (or their excess over 25, 50, or 75) by 4 and increase this product by 1 for each 24 contained in it, we obtain the next two digits.

We thus get successively the following pairs of digits:—

$$17+0=17$$
$$4\times 17+2=70$$
$$4(70-50)+3=83$$
$$4(83-75)+1=33$$
$$4(33-25)+1=33 \quad \text{and so on.}$$

We have now to show that $\cdot 17708333\ldots = 17/96$. From the method of formation it is easily seen that the series may be written in the following form:—

* See *Proceedings*, Vol. XX., p. 58, 1902.

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$$
\begin{array}{lcl}
\cdot 17 & = & 17/100 \\
\cdot 0070 & = & (4\times 17+2)/100^2 \\
\cdot 000083 & = & \{4(4\times 17+2-25\times 2)+3\}/100^3 \\
\cdot 00000033 & = & [4\{4(4\times 17+2-25\times 2)+3-25\times 3\}+1]/100^4 \\
\cdot 0000000033 & = & (4[4\{4(4\times 17+2-25\times 2)+3-25\times 3\}+1-25\times 1]+1)/100^5 \\
\&c. & = & \&c.
\end{array}
$$

Hence

$$
\begin{aligned}
\cdot 17708333\ldots = {} & 17/100 \\
& +4\,.\,17/100^2+2/100^2 \\
& +4^2.\,17/100^3+4\,.\,2/100^3-2/100^2+3/100^3 \\
& +4^3.\,17/100^4+4^2.\,2/100^4-4\,.\,2/100^3+4\,.\,3/100^4-3/100^3+1/100^4 \\
& +4^4.\,17/100^5+4^3.\,2/100^5-4^2.\,2/100^4+4^2.\,3/100^5-4\,.\,3/100^4+4\,.\,1/100^5-1/100^4+1/100^5 \\
& \qquad \&c. \qquad\qquad \&c. \\
= {} & 17/100+4\,.\,17/100^2+4^2.\,17/100^3+\ldots \\
= {} & (17/100)/(1-4/100) \\
= {} & 17/96
\end{aligned}
$$

It is here assumed that the same number occurs in the second and third columns (2 in this case); also that in the next two columns the same number (3 in this case) occurs; and so on. This is easily seen to be the case when we consider that each addition of 1 makes a 24 into 25; the following subtraction must therefore be of the *same* number of 25's as we have just previously added ones.

The method may be extended quite generally without the need of using a table such as in the preceding paper. To divide by $100 - a$, we multiply the last two digits found (or their excess over $100/a$, $2(100/a)$, $3(100/a)$, etc.) by a and add 1 for each $(100/a - 1)$ in the product; this gives us the next two digits; and so on.

With $1000 - a$ we use (and get) 3 digits at a time; and so on.

The treatment of such numbers as 104 will be readily seen from a particular case, *e.g.*, 17/104 we get the following pairs of numbers:

$$\begin{aligned} 17 - 1 &= 16 \\ 4(\ 25 - 16) - 2 &= 34 \\ 4(\ 50 - 34) - 3 &= 61 \\ 4(\ 75 - 61) - 3 &= 53 \\ 4(\ 75 - 53) - 4 &= 84 \\ 4(100 - 84) - 3 &= 61 \\ \therefore \quad 17/104 &= \cdot 163\dot{4}6153\dot{8}. \end{aligned}$$

In this case, instead of multiplying the *excess* over 25, 50, 75, we multiply the *defect* from 25, 50, 75, 100, and instead of *adding* 1 for each 25 that we are about to use in the following step, we *deduct* 1.

It is worthy of note that (as stated in the rule) the process may be applied at *any* stage, *e.g.*, having obtained, in the last example, 16 and then 34, we may continue with 63 instead of with 34.

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