Dear Editor,

Supercritical Galton–Watson branching processes: corrections to a paper of Foster and Goettge

Foster and Goettge (1976) define a rate of growth of a discrete branching process, $\{Z_n\}$ say, as a sequence $\{a_n\}$ of constants such that $\lim_{n\to\infty} a_n^{-1} Z_n$ exists almost surely and has positive probability of being positive and finite. They state that a classical (time homogeneous) Galton–Watson process has a rate of growth if and only if its mean family size m satisfies $1 < m < \infty$, whereas what is really meant is that $1 < m < \infty$ is a necessary and sufficient condition for $\{Z_n\}$ to have a single rate of growth which is 'achieved' (i.e. the limit is positive and finite) almost surely on the event of non-extinction. Subsequent work of Schuh and Barbour (1977) reveals that it is possible for a process with $m = \infty$ to have countably infinitely many rates of growth, namely any process which is *irregular* in the sense of their definition.

However, the main purpose of this note is to draw attention to a more fundamental error in the paper of Foster and Goettge (1976), whose aim is to exhibit an example of a Galton–Watson process in varying environment (GWPVE) which has countably infinitely many rates of growth.

The following is a necessary component of their argument. For a given sequence $\{n_i\}$ of positive integers, let A be the event that for all i, no individual in the ith generation has more than n_i offspring. Clearly if the sequence $\{n_i\}$ grows fast enough, A will have positive probability, in which case conditional on A, the process will behave as a GWPVE, in which the family size distribution at generation i will be obtained from the unconditional one by truncating at n_i and re-normalising.

I claim that the conditional process is indeed a GWPVE, as a few moments' thought will convince the reader; however, for $k \leqq n_i$ the probability p^*_{ik} that a member of the ith generation will have k children conditional on A will be proportional *not* to p_{ik} (the corresponding unconditional probability) as suggested *but* to $p_{ik}\pi^k_{i+1}$, where π_{i+1} is the unconditional probability that, among the descendants of a particular member of the $(i+1)$th generation, for all $j \geqq i+1$ no member of the jth generation has more than n_j offspring. In all but the most trivial cases $\pi_{i+1} < 1$ and this demonstrates the error.

It may be possible to patch up Foster and Goettge's construction, but the aforementioned essentially stronger result of Schuh and Barbour (1977) makes this unnecessary.

References

FOSTER, J. H. AND GOETTGE, R. T. (1976) The rates of growth of the Galton–Watson process in varying environment. *J. Appl. Prob.* **13**, 144–147.

SCHUH, H. J. AND BARBOUR, A. D. (1977) On the asymptotic behaviour of branching processes with infinite mean. *Adv. Appl. Prob.* **9**, 681–723.

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Yours sincerely,
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