REDUCTION OF PHASE NOISE IN INTERFEROMETRY WITH STATE-SPACE ANALYSIS

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1. INTRODUCTION

In radio mapping, one of the problems encountered is the random bias in the visibility estimate. The bias can be divided into two parts: (a) the positive bias due to the common sky background seen by all elements of the interferometer, and (b) the negative bias due to phase noise present in the system. The first kind of bias can be easily removed by subtracting the correlation between the signals at two interferometer sites when the source is not in the antenna beams from that measured with the source in the antenna beams. This bias will therefore not be considered here. In contrast, the second kind of bias is more difficult to remove. When the signal-to-noise ratio of the interferometer system is high, incoherent averaging techniques can be utilized in the fringe frequency or in the time domain (Clark et al., 1969; Moran, 1973).

Incoherent averaging in the time domain essentially consists of measuring the correlation between the two interferometer signals more than once with integration times much less than the coherence time of the system. By using these correlations and the probability density function of the measured correlation amplitudes, a maximum likelihood estimate of the correlation amplitude is found. The method over-estimates the correlation by a factor of $1 + 0.5\ \text{SNR}^{-2}$, and it loses the phase information about the source.

In removing phase noise effects from the interferometer data, the nature of the phase noise process at a given frequency must be determined. It is well-known that at low frequencies (e.g., 26 MHz), the ionosphere is the major source of the phase noise. Consequently, the stability of the ionosphere mainly determines the success or failure of an interferometer experiment at such low frequencies.

The system noise of the interferometer and imperfect local oscillators are also responsible for the phase fluctuations. At low frequencies, the phase noise contributed by the local oscillators is

C. van Schooneveld (ed.), Image Formation from Coherence Functions in Astronomy, 103-110.

usually a linear drift at a random rate. In the next section, a method will be presented for reducing different phase noise components from the interferometer phase.

2. METHOD

The method is based on empirical phase noise modeling combined with state-space formulation and discrete Kalman filtering. In the following discussion, it will be assumed that the interferometer data have been reduced by using standard reduction programs to remove local oscillator offsets, position errors for the source and interferometer baseline, and fringes. It is further assumed that the integration time used in this reduction process is sufficiently small in order to avoid degradation of the complex fringe function resulting from phase instabilities. Let there be N measurements of the complex fringe function for a point in the u-v plane. Let $\gamma(u,v,mT)$ be the complex visibility function measured for the m-th integration interval, where $m=1, 2, \ldots N$ is the time index; T is the lowest integration time in the first data reduction process (for example, the Mark I system uses a lowest integration time of 0.2 sec (Moran, 1976)). The total observation time, NT, must be small enough so that u and v do not change significantly. The function γ (u, v, mT) can be written as

$$\gamma(u,v,mT) = A(u,v,mT) \exp [j\phi(u,v,mT)] \tag{1}$$

where $A(u,v,mT)$ and $\phi(u,v,mT)$ denote the amplitude and phase, respectively. At the end of the first data reduction process, N pairs of $A(u,v,mT)$ and $\phi(u,v,mT)$ values are available, and it is desired to remove the noise in $\phi(u,v,mT)$. The measured phase function is decomposed as

$$\phi(mT) = \phi_{source}(mT) + \phi_{ion}(mT) + \phi_{osc}(mT) + \phi_{stat}(mT) \tag{2}$$

where we omitted u and v for brevity. The subscripts 'source', 'ion', 'ocs', and 'stat' are used to denote, respectively, the phase components due to the radio source observed; the differential phase noise component due to the propagation media; the noise term introduced by the local oscillators; and the phase noise term due to the system noise and finite integration time. (Let $\phi_{source}(t)$, $\phi_{ion}(t)$, $\phi_{osc}(t)$, and $\phi_{stat}(t)$ be the continuous forms of $\phi_{source}(mT)$, $\phi_{ion}(mT)$, $\phi_{osc}(mT)$, and $\phi_{stat}(mT)$, respectively.) Each term in (2) can be modeled in terms of its continuous form as a random process satisfying a certain stochastic differential equation. Each equation takes a form imposed by the physical process generating a particular phase noise. $\phi_{source}(t)$ is modeled as a random constant satisfying

$$\frac{d}{dt} \phi_{source}(t) = 0 \tag{3}$$

Since for a single point (u,v), the visibility phase is unique. $\phi_{ion}(t)$ is taken as a solution of the following equation

https://doi.org/10.1017/S0252921100074790 Published online by Cambridge University Press

$$\frac{d}{dt}\begin{bmatrix}\phi_{ion}(t)\\ \phi_{aux\ 1}(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\alpha^2 & -2\beta\end{bmatrix}\begin{bmatrix}\phi_{ion}(t)\\ \phi_{aux\ 1}(t)\end{bmatrix} + \begin{bmatrix}1\\ \alpha - 2\beta\end{bmatrix} w(t) \qquad (4)$$

where $\phi_{aux\ 1}(t)$ is an auxiliary random variable; α and β are the parameters describing the processes $\phi_{ion}(t)$ and $\phi_{aux\ 1}(t)$; and $w(t)$ is a white noise process. Equation (4) describes a random periodic process with auto-correlation function (Fitzgerald, 1966).

$$R(\tau) = \varphi_o^{\ 2} \exp\ (-\beta|\tau|) \cos \omega|\tau| \qquad (5)$$

where φ_o^2 is the variance of the process; ω is the angular fluctuation frequency; and τ is the time lag. This type of stochastic process for $\phi_{ion}(t)$ is suggested by the data shown in Figure 1, which is characteristic of almost all observations.

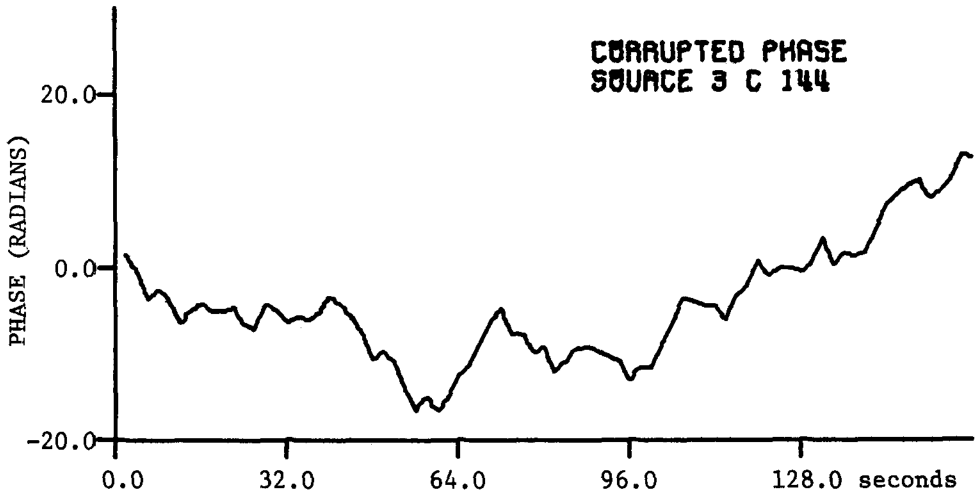

Figure 1. Phase data with oscillator drift and ionospheric perturbations. See also the note under "References".

$\phi_{osc}(T)$ was modeled as

$$\phi_{osc}(t) = ct \qquad (6)$$

where c is a random constant. This process satisfies the equation

$$\frac{d}{dt}\begin{bmatrix}\phi_{osc}(t)\\ \phi_{aux\ 2}(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\phi_{osc}(t)\\ \phi_{aux\ 2}(t)\end{bmatrix} \qquad (7)$$

where $\phi_{aux\ 2}(t)$ is the auxiliary variable whose initial value determines the value of the slope c of the process $\phi_{osc}(t)$. Equation (3), (4), and (7) can be combined into the following matrix equation

$$\frac{d}{dt}\begin{bmatrix}\phi_{source}(t)\\ \phi_{ion}(t)\\ \phi_{aux\ 1}(t)\\ \phi_{osc}(t)\\ \phi_{aux\ 2}(t)\end{bmatrix} = \begin{bmatrix}0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & -\alpha^2 & -2\beta & 0 & 0\\ 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\phi_{source}(t)\\ \phi_{ion}(t)\\ \phi_{aux\ 1}(t)\\ \phi_{osc}(t)\\ \phi_{aux\ 2}(t)\end{bmatrix} + \begin{bmatrix}0\\ 1\\ \alpha-2\beta\\ 0\\ 0\end{bmatrix} w(t) \qquad (8)$$

Equation (8) describes a 5-dimensional linear stochastic system whose state variables are $\phi_{source}(t)$, $\phi_{ion}(t)$, $\phi_{aux\ 1}(t)$, $\phi_{osc}(t)$, and $\phi_{aux\ 2}(t)$, the driving function being a vector white noise process. If α, β, and the initial values for the state variables are known, the state variables at any arbitrary time t can be generated by solving (8). In order to use these solutions for reducing phase noise, rewrite (2) in a continuous form:

$$\phi(t) = (1\ 1\ 0\ 1\ 0)\begin{bmatrix}\phi_{source}(t)\\ \phi_{ion}(t)\\ \phi_{aux\ 1}(t)\\ \phi_{osc}(t)\\ \phi_{aux\ 2}(t)\end{bmatrix} + \phi_{stat}(t) \qquad (9)$$

By using (8), (9), and the recursive Kalman filter algorithm, estimates of $\phi_{source}(mT)$, $\phi_{ion}(mT)$ and $\phi_{osc}(mT)$ are obtained (Okatan, 1977) at discrete times mT. To obtain residual interferometer phase, $\phi_{res}(mT)$, these estimates are used in the following equation:

$$\phi_{res}(mT) = \phi(mT) - [\hat{\phi}_{source}(mT) + \hat{\phi}_{ion}(mT) + \hat{\phi}_{osc}(mT)]\ ,\quad m=1, 2, \ldots N \qquad (10)$$

where the hat (^) stands for the predicted phase components. The quantity $\phi_{res}(mT)$ is now used with A(u,v,mT) to get a coherent average for the visibility by using the equation

$$\gamma_{coh}(u,v) = \frac{1}{N}\sum_{n=1}^{N} A(u,v,mT)\ \exp\ [j\phi_{res}(mT)]$$

In order to obtain α, β, and the initial values for the state variables, only a small portion of the phase data is used. The procedure is as follows:

(a) Remove the linear trend from the phase data by using the ordinary least squares method. The slope of the trend is the initial value for the state variable $\phi_{aux\ 2}(t)$. Set the initial value of $\phi_{osc}(t)$ to the value of the trend at the initial time.

(b) Fit the data from the preceding step to a second order autoregressive time series model having an autocorrelation function given by (5). From this process, β, φ_o^2, and ω are obtained. α is obtained from the relation

$$\alpha = (\beta^2 + \omega^2)^{1/2}$$

(c) Compute the variance of the residual process left over from the previous step to get an estimate for the variance of $\phi_{stat}(t)$. Set the initial value of $\phi_{ion}(t)$ to the initial value of the second order autoregressive process obtained at step (6) The initial value of $\phi_{aux\ 1}(t)$ is taken to be zero while the initial value of $\phi_{source}(t)$ is set to the best guess for a particular source.

In the Kalman filter algorithm, the uncertainties on the initial values of the state variables have to be supplied along with the variance of w(t). Once the initial parameters are obtained, they can be used for the other portions of the data. These parameters have to be updated as needed. Because the Kalman filter is recursive, the computer storage requirements are minimized. The computer time required for filtering phase noise is quite small and can be reduced further by decomposing the state matrix in (8). If we do not have a good guess for $\phi_{source}(t)$, it can be set to zero along with the uncertainty associated with it. Then, the Kalman filter assumes perfect information for this component and does not change it throughout the filtering process. Figure 2 shows a block diagram of the method.

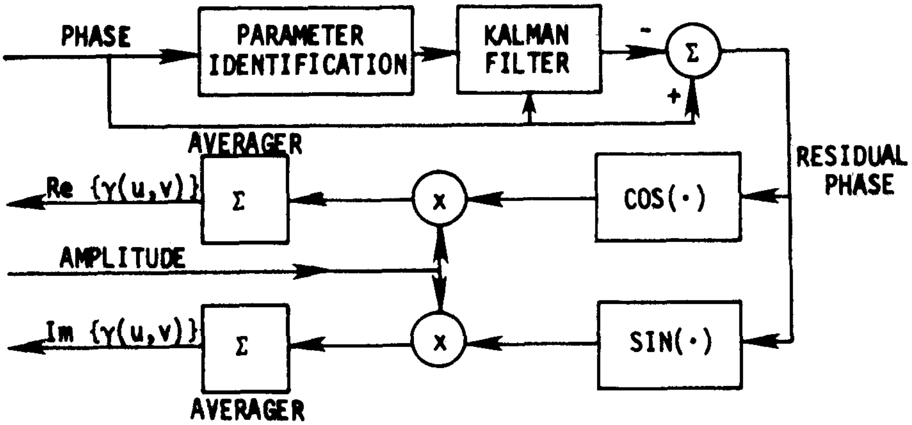

Fig. 2. Block diagram of the phase-noise reduction method.

3. RESULTS AND CONCLUSIONS

The method was applied to the data taken during VLBI experiments between Boulder, Colorado and Ames, Iowa in August, 1971. During these experiments, the ionospheric phase fluctuations were severe. The half-power Gaussian size of 3C 144 pulsar was determined as 2.20 $\mp$ 0.50 arc-sec before the technique was used. After using the method described in this paper, the size was determined as 1.34 $\mp$ 0.20 arc-sec, which is in close agreement with the recently published sizes of this source 1.3 (+0.23, -0.13) (Mutel et al., 1974). Figure 3 shows the residual interferometer phase obtained after the application of this method. The oscillation and linear trend are removed and the rms phase is small.

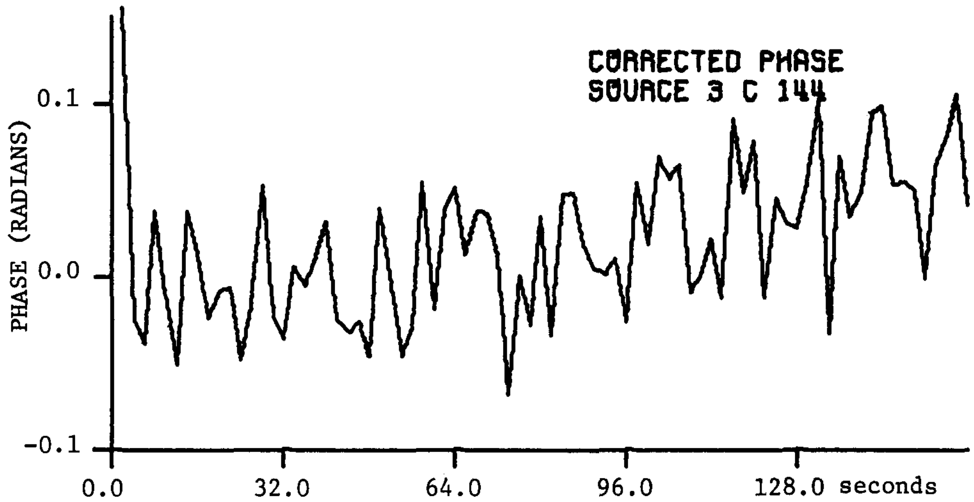

Figure 3. The phase data of fig. 1, corrected for oscillator drift and ionospheric perturbations.

4. RECOMMENDATIONS

A real time implementation of the method described here may be implemented in digital and analog interferometer systems. For the digital systems, high data rate can be a problem. Therefore, the rate has to be reduced before using the method. For continuous systems, the data rate does not introduce any problems; however, a solution of the nonlinear Riccati equation must be supplied during the filtering. For complicated phase noise models it may not be easy to obtain this solution.

ACKNOWLEDGEMENTS

We thank our colleague Professor S. C. Dutta Roy for his critical reading of the manuscript. We also thank the Engineering Research Institute for support during this investigation.

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Note.
The data of fig. 1 were collected on August 19, 1971, with a 26.3 MHz, 950 km baseline, interferometer operating between Boulder, Colorado and Ames, Iowa. The antennas at each site consisted of 160 pairs of crossed full-wave dipoles. The data were recorded using Mark I VLBI tape recording terminals on loan from the National Radio Astronomy Observatory.

DISCUSSION

Comment D.B. SHAFFER
1) As I understand it, the size you measured for 3C144 was too large because your visibility values were too low, and your technique removes ionosphere phase fluctuations which caused loss of coherence. Correct?
2) You can also go to shorter coherent integration intervals.
Reply A. OKATAN
1) Answer to the first question is yes. 2) If we want to keep uncertainties small, using shorter integration time is not useful, because the uncertainty on the estimate of the coherence function is proportional to the inverse square-root of the integration time.

Comment T.A. CLARK
As a collaborator in the acquisition of the 3C144 data presented, I would like to note that fringe amplitudes obtained by assuming partial coherence, e.g. ∿ 18 separate samples of 10 sec. each, gives visibility amplitudes quite similar to this approach and imply a 26 MHz apparent source size ∿ 1".5.

Comment P.N. WILKINSON
The closure phase is still a good observable at 26 MHz!
Reply A. OKATAN
Yes. But, if you don't have three or more stations operating at the same time, the closure phase cannot be observed.

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