

Transformation methods in nonlinear programming

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This thesis investigates a number of difficulties associated with the design and implementation of transformation methods for the solution of the continuous mathematical programming problem,

$$\begin{aligned} &\text{minimize} && f(x) && , && x \in E^n , \\ &\text{subject to} && g_i(x) \geq 0 && , && i = \overline{1, \ell} , \\ &\text{and} && h_i(x) = 0 && , && i = \overline{1, m} , \end{aligned}$$

where the problem functions $f(x)$, $g_i(x)$, $i = \overline{1, \ell}$, $h_i(x)$, $i = \overline{1, m}$, are at least continuous. The aim is to improve convergence properties of current transformation methods by designing modified methods which avoid the computational difficulties.

In Chapter 2, barrier function methods (interior-point methods) are investigated and a new family of barrier functions, exhibiting improved convergence properties, is proposed. The theoretical and computational aspects of two penalty function methods (exterior-point methods) are then considered in Chapter 3. Based on the properties of the barrier and penalty function approaches discussed in the second and third chapters, a new hybrid method is constructed in Chapter 4. The theoretical validity of the new approach is established and it is shown that under mild conditions on the problem (for example, the Kuhn-Tucker conditions) the computational difficulties of the parent methods are avoided and improved convergence properties are evident. For problems not satisfying such conditions, the hybrid method behaves as a barrier function method. In Chapter 5,

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unconstrained methods and projection methods for linearly constrained problems are considered and numerical evidence is presented to suggest that if linear constraints are active at the solution then improved convergence can be obtained by treating such constraints using projection instead of including them in barrier or penalty function methods.