

LETTER TO THE EDITOR

Dear Editor,

A simple comparison proof

In the paper [1] by R. E. Feldman the following is proved (we paraphrase it):

Result 1. Let ξ_1, ξ_2, \dots be independently ± 1 with probabilities $\frac{1}{2}, \frac{1}{2}$, let $(N_u)_{u \geq 0}$ be an independent Poisson process of rate $\mu > 0$, and let ζ be an independent random variable, exponentially distributed with parameter θ . Then, for each positive integer l ,

$$P\left(\sum_{i=1}^{N_u} \xi_i \geq l \text{ for some } u < \zeta\right) = A^l$$

where $A = 1 + \theta/\mu - \sqrt{(1 + \theta/\mu)^2 - 1}$.

As well as proving this by her own methods the author gives a complicated proof of the $l = 2$ case ‘for comparison’ using combinatorial methods of Feller. She also states that the $l > 2$ cases involve ‘even more complicated combinatorics and summations’. What we want to point out is that it is extremely easy to provide this comparison proof. First, by ladder considerations and the lack-of-memory property of ζ ’s exponential distribution it is immediate that the required probability is A^l for some A . Second, the probability that the first jump of the Poisson process occurs before ζ is $\mu/(\theta + \mu)$, and then by considering the first step of the randomized random walk one gets

$$A = \frac{\mu}{\theta + \mu} \left(\frac{1}{2} + \frac{1}{2}A^2\right).$$

The one root of this quadratic in the unit interval is the claimed evaluation of A .

Yours sincerely,

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Reference

[1] FELDMAN, R. E. (1991) Autoregressive processes and first-hit probabilities for randomized random walks. *J. Appl. Prob.* **28**, 347–359.