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Springer's Weyl Group Representation via Localization

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Abstract. Let G denote a reductive algebraic group over $\mathbb{C}$ and x a nilpotent element of its Lie algebra $\mathfrak{g}$. The Springer variety $\mathcal{B}_x$ is the closed subvariety of the flag variety $\mathcal{B}$ of G parameterizing the Borel subalgebras of $\mathfrak{g}$ containing x. It has the remarkable property that the Weyl group W of G admits a representation on the cohomology of $\mathcal{B}_x$ even though W rarely acts on $\mathcal{B}_x$ itself. Well-known constructions of this action due to Springer and others use technical machinery from algebraic geometry. The purpose of this note is to describe an elementary approach that gives this action when x is what we call parabolic-surjective. The idea is to use localization to construct an action of W on the equivariant cohomology algebra $H_S^*(\mathcal{B}_x)$, where S is a certain algebraic subtorus of G. This action descends to $H^*(\mathcal{B}_x)$ via the forgetful map and gives the desired representation. The parabolic-surjective case includes all nilpotents of type A and, more generally, all nilpotents for which it is known that W acts on $H_S^*(\mathcal{B}_x)$ for some torus S. Our result is deduced from a general theorem describing when a group action on the cohomology of the fixed point set of a torus action on a space lifts to the full cohomology algebra of the space.

1 Introduction

Let G be a reductive linear algebraic group over $\mathbb{C}$ with Lie algebra $\mathfrak{g}$, and fix a maximal torus T and a Borel subgroup B of G such that $T \subset B$. The flag variety $\mathcal{B} = G/B$ of G will be viewed as the variety of all Borel subgroups of G or, equivalently, as the variety of all Borel subalgebras of $\mathfrak{g}$. Let $W = N_G(T)/T$ be the Weyl group of the pair (G, T), and recall that W acts topologically on $\mathcal{B}$. Thus, the cohomology algebra $H^*(\mathcal{B})$ admits a representation as a graded W-module, which is well known to be isomorphic with the graded W-algebra $\mathbb{C}[\mathfrak{t}]/I_W^+$, the coinvariant algebra of W. Here $\mathfrak{t}$ is the Lie algebra of T and I_W^+ is the ideal in $\mathbb{C}[\mathfrak{t}]$ generated by the nonconstant homogeneous W-invariants. Note: throughout this paper, $H^*(Y)$ will denote the standard cohomology algebra of a space Y with complex coefficients.

A celebrated theorem of T. A. Springer [Spr1, Spr2] says that if x is a nilpotent element of $\mathfrak{g}$ and $\mathcal{B}_x$ is the Springer variety associated with x, namely, the closed subvariety of $\mathcal{B}$ consisting of all Borel subalgebras of $\mathfrak{g}$ containing x, then there is a graded $\mathbb{C}$-algebra representation of W on $H^*(\mathcal{B}_x)$ so that the cohomology restriction map $i_x^*: H^*(\mathcal{B}) \to H^*(\mathcal{B}_x)$ associated with the inclusion $i_x: \mathcal{B}_x \hookrightarrow \mathcal{B}$ is W-equivariant (see [H-S] for the proof of W-equivariance). As remarked often, the existence of this representation is quite surprising, because W itself does not usually act on $\mathcal{B}_x$, exceptions being when $x = 0$ or x is regular in $\mathfrak{g}$, and Springer's construction requires a lot of technical machinery. Subsequent definitions involve either replacing $H^*(\mathcal{B}_x)$ with

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an isomorphic algebra on which W is known to act (*cf.* [Kraft, D-P, JC1]) or replacing $\mathcal{B}_x$ with a space having both a W-action and isomorphic cohomology algebra (*cf.* [Ross, Slo, Treu]).

Our plan is to give a simple new construction of Springer's representation when x is a nilpotent element of $\mathfrak{g}$ that is what we call *parabolic-surjective*. We call a nilpotent $x \in \mathfrak{g}$ *parabolic* if it is regular in a Levi subalgebra $\mathfrak{l}$ of $\mathfrak{g}$, and we say x is *parabolic-surjective* if in addition the cohomology restriction map i_x^* is surjective. The key idea is to exploit the fact that in the parabolic-surjective case, there exists subtorus S of G acting on $\mathcal{B}_x$ so that $(\mathcal{B}_x)^S$ is stable under W so localization and the parabolic-surjective condition can be used to obtain an $\mathbb{C}[\mathfrak{s}]$-module action of W on the torus equivariant cohomology $H_S^*(\mathcal{B}_x)$ descending to an action on $H^*(\mathcal{B}_x)$, which turns out to coincide with Springer's action.

Recently, the existence of this W-action on $H_S^*(\mathcal{B}_x)$ was established by finding a geometric realization of $\operatorname{Spec}(H_S^*(\mathcal{B}_n))$: see [K-P] for the general parabolic-surjective setting and [G-McP] for x of type A. We note that although [G-McP] only treats the type A case, their argument is valid for all parabolic-surjective $x \in \mathfrak{g}$ for all $\mathfrak{g}$. The paper [A-H] establishes this action by employing Tanisaki's presentation of $H^*(\mathcal{B}_x)$ in type A.

We now state the main result that will give Springer's action for parabolic-surjective x by first obtaining it for $H_S^*(\mathcal{B}_x)$.

Theorem 1.1 *Let Y be a projective variety with vanishing odd cohomology. Also suppose that we have actions of an algebraic torus S and a finite group $\mathcal{W}$ on Y, and these two actions commute with each other. Let X be an S-stable subvariety of Y such that the cohomology restriction map $i^*: H^*(Y) \to H^*(X)$ is surjective, where $i: X \hookrightarrow Y$ is the inclusion. Then if $\mathcal{W}$ acts on $H^*(X^S)$ so that the cohomology restriction map $(i_S)^*: H^*(Y^S) \to H^*(X^S)$ induced by the inclusion $i_S: X^S \hookrightarrow Y^S$ is $\mathcal{W}$-equivariant, then $\mathcal{W}$ also acts on $H_S^*(X)$ by graded $\mathcal{W}$-algebra $\mathbb{C}[\mathfrak{s}]$-module isomorphisms. Moreover, the natural map $H_S^*(X) \to H^*(X)$ induces a representation of $\mathcal{W}$ on the graded algebra $H^*(X)$ such that all the maps in the following commutative diagram are $\mathcal{W}$-equivariant:*

(1.1)
$$\begin{array}{ccc} H_S^*(Y) & \xrightarrow{\iota^*} & H_S^*(X) \\ \downarrow & & \downarrow \\ H^*(Y) & \xrightarrow{i^*} & H^*(X). \end{array}$$

In the above diagram, ι^* is the map on equivariant cohomology induced by $i: X \hookrightarrow Y$. The proof, given in the next section, is an application of the localization theorem. The reader can easily reformulate this result as a statement involving topological torus actions. In the final section, we will verify the above assertions about parabolic-surjective Springer varieties.

2 Proof of the Main Theorem

We will begin by reviewing some facts about equivariant cohomology. Excellent references for the facts below are [A-B, Brion]. Recall that all cohomology is over $\mathbb{C}$. Let Y be a complex projective variety with trivial odd cohomology admitting a nontrivial action (S, Y) by an algebraic torus $S \cong (\mathbb{C}^*)^\ell$. In particular, the fixed point set Y^S is nontrivial. Recall that the S-equivariant cohomology algebra $H_S^*(Y)$ of Y is defined as the cohomology algebra $H^*(Y_S)$ of the Borel space $Y_S = (Y \times E)/S$, where E is a contractible space with a free S-action and S acts diagonally on the product. The projection $Y \times E \to E$ induces a map $Y_S = (Y \times E)/S \to E/S$, which in turn induces an $H^*(E/S)$-module structure on $H_S^*(Y)$. On the other hand, the inclusion $\nu_Y\colon Y \hookrightarrow Y_S$ along a fibre gives a map from $H_S^*(Y)$ to the ordinary cohomology $H^*(Y)$. Moreover, there is a natural identification $H^*(E/S) \cong \mathbb{C}[\mathfrak{s}]$, where $\mathfrak{s} = \mathrm{Lie}(S)$. Note that by the Künneth formula, $H_S^*(Y^S) = \mathbb{C}[\mathfrak{s}] \otimes H^*(Y^S)$. When $H_S^*(Y)$ is a free $\mathbb{C}[\mathfrak{s}]$-module, the action (S, Y) is said to be *equivariantly formal*. It is well known that equivariant formality is implied by the vanishing of odd cohomology of Y. The proof of our main result is based on the following well-known result, the first assertion of which is a special case of the localization theorem.

Theorem 2.1 *If (S, Y) is equivariantly formal, then the inclusion mapping $j_Y\colon Y^S \hookrightarrow Y$ induces an injection $(j_Y)_S^*\colon H_S^*(Y) \to H_S^*(Y^S)$. Moreover, the map $(\nu_Y)^*$ fits into an exact sequence*

$$0 \longrightarrow \mathbb{C}[\mathfrak{s}]^+ H_S^*(Y) \longrightarrow H_S^*(Y) \xrightarrow{(\nu_Y)^*} H^*(Y) \longrightarrow 0, \tag{2.1}$$

where $\mathbb{C}[\mathfrak{s}]^+$ is the augmentation ideal, generated by all the nonconstant homogeneous polynomials.

Proof Assume that $\mathcal{W}$ is a finite group acting on Y such that the action $(\mathcal{W}, Y)$ commutes with (S, Y). Then $\mathcal{W}$ acts linearly on both $H^*(Y)$ and $H^*(Y^S)$. Furthermore, it acts on $H_S^*(Y)$ and $H_S^*(Y^S)$ as $\mathbb{C}[\mathfrak{s}]$-module isomorphisms so that the map $j_Y^*\colon H_S^*(Y) \to H_S^*(Y^S)$ induced by the inclusion $j_Y\colon Y^S \hookrightarrow Y$ is a $\mathcal{W}$-equivariant $\mathbb{C}[\mathfrak{s}]$-module injection. Let X be an S-stable subvariety of Y such that the cohomology restriction map $i^*\colon H^*(Y) \to H^*(X)$ is surjective.

Since $\mathcal{W}$ acts on $H^*(X^S)$ and the restriction map $H^*(Y^S) \to H^*(X^S)$ is $\mathcal{W}$-equivariant, $\mathcal{W}$ also acts on $H_S^*(X^S)$ as a group of $\mathbb{C}[\mathfrak{s}]$-module isomorphisms so that the natural map $\mu\colon H_S^*(Y^S) \to H_S^*(X^S)$ is a $\mathcal{W}$-equivariant $\mathbb{C}[\mathfrak{s}]$-module homomorphism. Now consider the commutative diagram

$$\begin{array}{ccc} H_S^*(Y) & \xrightarrow{\iota^*} & H_S^*(X) \\ \Big\downarrow{\scriptstyle j_Y^*} & & \Big\downarrow{\scriptstyle j_X^*} \\ H_S^*(Y^S) & \xrightarrow{\mu} & H_S^*(X^S). \end{array}$$

We will define the action of $\mathcal{W}$ on $H_S^*(X)$ by imposing the requirement that ι^* be a $\mathcal{W}$-module homomorphism. To show this action is well defined, it suffices to show

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that the kernel of ι^* is a $\mathcal{W}$-submodule. Suppose then that $\iota^*(a) = 0$. By assumption, $H^*(Y) \to H^*(X)$ is surjective, and thus X also has vanishing odd cohomology. It follows that (S, X) is equivariantly formal and hence j_X^* is injective. Thus, to show $\iota^*(w \cdot a) = 0$ for any $w \in \mathcal{W}$, it suffices to show that $j_X^* \iota^*(w \cdot a) = 0$. But

$$j_X^* \iota^*(w \cdot a) = \mu j_Y^*(w \cdot a) = w \cdot \mu j_Y^*(a) = w j_X^* \iota^*(a) = 0,$$

since μj_Y^* is a $\mathcal{W}$-module homomorphism. Thus, $\mathcal{W}$ acts on $H_S^*(X)$, as claimed. It follows from this argument that j_X^* is $\mathcal{W}$-equivariant. To show that $\mathcal{W}$ acts on $H^*(X)$, consider the exact sequence (2.1) for X. As above, we can define the $\mathcal{W}$-action by requiring that $(\nu_X)^*$ be equivariant. It suffices to show its kernel is $\mathcal{W}$-stable. But if $(\nu_X)^*(a) = 0$, then $a = fb$ for some $f \in \mathbb{C}[\mathfrak{s}]^+$ and $b \in H_S^*(X)$. Thus,

$$w \cdot a = w \cdot fb = f(w \cdot b) \in \mathbb{C}[\mathfrak{s}]^+ H_S^*(X) = \ker(\nu_X)^*.$$

Finally, we remark that the above definitions make diagram (1.1) commutative. ∎

If one omits the assumption that cohomology restriction map $i^*: H^*(Y) \to H^*(X)$ is surjective, the best one can hope for is that $\mathcal{W}$ acts on the image $i^*(H^*(Y))$. The following result gives a sufficient condition for $\mathcal{W}$ to act in this case.

Theorem 2.2 *Assume that the setup in Theorem 1.1 holds except for the assumption that $i^*: H^*(Y) \to H^*(X)$ is surjective, and also assume that $H^*(X)$ has vanishing odd cohomology. Then there exists an action of $\mathcal{W}$ on $\iota^*(H_S^*(Y))$ by $\mathbb{C}[\mathfrak{s}]$-module isomorphisms. Moreover, if $\iota^*(H_S^*(Y))$ is free of rank $\dim i^*(H^*(Y))$, then the action of $\mathcal{W}$ on $\iota^*(H_S^*(Y))$ descends to $i^*(H^*(Y))$ so that the cohomology restriction map $i^*: H^*(Y) \to i^*(H^*(Y))$ is $\mathcal{W}$-equivariant.*

Proof For the first assertion, we have to show that the kernel of ι^* is $\mathcal{W}$-invariant. Since $H^*(X)$ has vanishing odd cohomology, j_X^* is injective, so this follows from the argument above. Next, note that if $\mathcal{N}$ denotes a free $\mathbb{C}[\mathfrak{s}]$-module of finite rank, then the $\mathbb{C}$-vector space dimension of $\mathcal{N}/\mathbb{C}[\mathfrak{s}]^+\mathcal{N}$ is equal to the rank of $\mathcal{N}$. Thus, it follows by assumption that the sequence

$$0 \longrightarrow \mathbb{C}[\mathfrak{s}]^+ \iota^*(H_S^*(Y)) \longrightarrow \iota^*(H_S^*(Y)) \xrightarrow{\nu} i^*(H^*(Y)) \longrightarrow 0$$

is exact, where ν is the restriction of $(\nu_X)^*$. Hence, as above, the kernel of $\iota^*(H_S^*(Y)) \to i^*(H^*(Y))$ is $\mathcal{W}$-stable, so $\mathcal{W}$ acts on $i^*(H^*(Y))$. Moreover, the map $H^*(Y) \to i^*(H^*(Y))$ is $\mathcal{W}$-equivariant. ∎

Remark 2.3 In the case when $S = \mathbb{C}^*$, since (S, X) is assumed to be equivariantly formal, the module $\iota^*(H_S^*(Y))$ is always free. This is because $\mathbb{C}[\mathfrak{s}]$ is a principal ideal domain and $H_S^*(X)$ is free.

3 The Weyl Group Action on $H_S^*(\mathcal{B}_x)$

We now return to the parabolic-surjective setting. First, recall that W acts as a group of homeomorphisms of $\mathcal{B}$ that commute with T. Let K be a maximal compact subgroup in G such that $H = K \cap T$ is a maximal torus in K. Then the natural mapping $K/H \to \mathcal{B}$ is a homeomorphism; but $W = N_K(H)/H$ acts on K/H (from the left) by

$w \cdot kH = k\dot{w}^{-1}H$, where $\dot{w} \in N_K(H)$ is a representative of w. Thus, W acts on $\mathcal{B}$ as asserted. Since this action commutes with the action of H on K/H, the group W acts on both $H_T^*(\mathcal{B}) = H_H^*(K/H)$ and $H^*(\mathcal{B})$ and the natural mapping $H_T^*(\mathcal{B}) \to H^*(\mathcal{B})$ is W-equivariant.

In order to apply Theorem 1.1, we need the following lemma from [JC1].

Lemma 3.1 *Let $x \in \mathfrak{g}$ be nilpotent, and suppose x is a regular element in the Lie algebra of the Levi $L = C_G(S)$ for a subtorus S of T. Then S acts on $\mathcal{B}_x$ with exactly $[W : W_L]$ fixed points. Moreover, every component of $\mathcal{B}^S$ contains exactly one point of $(\mathcal{B}_x)^S$, so W acts on $H^*((\mathcal{B}_x)^S)$ so that the cohomology restriction map $H^*(\mathcal{B}^S) \to H^*((\mathcal{B}_x)^S)$ is W-equivariant and surjective.*

Proof By assumption, $\mathcal{B}_x$ is the variety of Borel subalgebras of $\mathfrak{g}$ containing x, so $\mathcal{B}_x$ is stable under the action of S on $\mathfrak{g}$. Each irreducible component of $\mathcal{B}^S$ is isomorphic to the flag variety of L, so each component contains a unique fixed point of the one parameter group $\exp(tx)$, $t \in \mathbb{C}$, since x is regular in $\mathfrak{l}$. It follows that the cohomology restriction map $H^*(\mathcal{B}^S) \to H^*((\mathcal{B}_x)^S)$ is surjective. Moreover, since W permutes the components of $\mathcal{B}^S$, and each component contains a unique point of $\mathcal{B}_x$, we can uniquely define an action of W on $H^*((\mathcal{B}_x)^S) = H^0((\mathcal{B}_x)^S)$ by requiring that the cohomology restriction map be equivariant. Finally, it is well known that the number of components of $\mathcal{B}^S$ is the index $[W : W_L]$ of the Weyl group of L in W. ∎

Remark 3.2 In fact, W actually acts on $(\mathcal{B}_x)^S$ itself (see [JC1] or [JC2, Lemma 6.3 and p. 137]).

Let us now return to the problem considered in the introduction. As above, G is reductive linear algebraic group over $\mathbb{C}$ and $\mathcal{B}$ is its flag variety. By the main result (Theorem 1.1), we have the following corollary.

Corollary 3.3 *Let x be a parabolic-surjective nilpotent in $\mathfrak{g}$; say x is regular in the Lie algebra of the Levi subgroup $C_G(S)$. Then W acts on $H_S^*(\mathcal{B}_x)$, and this action descends to $H^*(\mathcal{B}_x)$ so that the diagram (1.1) is commutative for $Y = \mathcal{B}$ and $X = \mathcal{B}_x$. Consequently, this W-action is Springer's representation.*

Proof The only thing to show is that this action of W on $H^*(\mathcal{B}_x)$ coincides with Springer's representation. But this follows, since $i^* : H^*(\mathcal{B}) \to H^*(\mathcal{B}_x)$ is W-equivariant by [H-S]. ∎

This seems to give the most elementary construction of Springer's action in type A. It was originally conjectured in [Kraft] that for any $x \in \mathfrak{sl}(n, \mathbb{C})$, the action of $W = S_n$ on $H^*(\mathcal{B}_x)$ is equivalent to the action of W on the coordinate ring $A(\mathfrak{t} \cap C_y)$ of the schematic intersection of the diagonal matrices in $\mathfrak{sl}(n, \mathbb{C})$ and the closure C_y in $\mathfrak{sl}(n, \mathbb{C})$ of the conjugacy class of of the nilpotent y dual to x. This was immediately verified in [D-P] where it was shown that $H^*(\mathcal{B}_x) \cong A(\mathfrak{t} \cap C_y)$ as graded W-algebras. This isomorphism was extended in [JC1] to the case of parabolic-surjective nilpotents in an arbitrary $\mathfrak{g}$ that satisfy some additional conditions. Here the nilpotent y dual to x turns out to be a Richardson element in the nilradical of the parabolic subalgebra of $\mathfrak{g}$ associated with the Levi $\mathfrak{l}$ in which x is a regular nilpotent. We refer the reader to [JC1] for more details.

Finally, let us mention that by a well-known result of DeConcini, Lusztig, and Procesi [D-L-P], $\mathcal{B}_x$ has vanishing odd cohomology for any nilpotent $x \in \mathfrak{g}$. Moreover, the Jacobson-Morosov lemma guarantees that every Springer variety $\mathcal{B}_x$ has a torus action $(\mathcal{S}, \mathcal{B}_x)$. This suggests that $H_S^*(\mathcal{B}_x)$ should be studied in the general case. For example, when does W act on any of $H^*((\mathcal{B}_x)^S)$, $H_S^*(\mathcal{B}_x)$ or even $\iota^*(H_S^*(\mathcal{B}))$?

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