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ON THE SCATTERING OF WATER WAVES BY A SUBMERGED SLENDER BARRIER

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Abstract

An approximate analysis, based on the standard perturbation technique, is described in this paper to find the corrections, up to first order to the reflection and transmission coefficients for the scattering of water waves by a submerged slender barrier, of finite length, in deep water. Analytical expressions for these corrections for a submerged nearly vertical plate as well as for a submerged vertically symmetric slender barrier of finite length are also deduced, as special cases, and identified with the known results. It is verified, analytically, that there is no first order correction to the transmitted wave at any frequency for a submerged nearly vertical plate. Computations for the reflection and transmission coefficients up to $O(\varepsilon)$, where ε is a small dimensionless quantity, are also performed and presented in the form of both graphs and tables.

1. Introduction

In the linearised theory of water waves, scattering problems involving obstacles admit exact solutions only for a limited number of cases viz. when the obstacle is in the form of a thin plane barrier, which is either vertical [3, 4, 13] or a partially immersed barrier inclined at special angles [6], and is subjected to a normally incident wave train in deep water. The scattering problem involving a barrier which is not vertical but almost vertical can also be tackled. However, instead of an exact solution an approximate solution can be found.

The objective of the present investigation is to find the first order corrections to the reflection and transmission coefficients for the scattering of water waves by a fixed fully submerged slender barrier ($0 < a < y < b < \infty$), which is immersed from the depth a up to the depth b below the free surface, in deep water. To the authors' knowledge, the problem involving the diffraction of water waves by a just-immersed vertically

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symmetric slender barrier ($0 < y < a$), was first specified by Shaw [12], where he applied a method based essentially on an integral equation formulation, although the details have been omitted. In this study, a method based essentially on the standard perturbation technique is employed to find the solution (albeit approximately) of the problem under consideration.

Water wave scattering problems involving nearly vertical barriers have become increasingly important; despite this, little work has been done on the subject in recent years. The first problem in this area has been tackled by Shaw [12], where he considered the diffraction of water waves by a partially immersed curved but almost vertical barrier. Shaw [12] used Green's theorem to reduce the problem to the solution of a singular integral equation, wherein the associated singular integral equation has been solved up to first order by perturbation techniques, and corrections up to first order to the reflection and transmission coefficients have been obtained in terms of integrals involving the shape function describing the curved barrier. Later, Mandal and Chakrabarti [9] used a simplified perturbation technique, applied directly to the boundary value problem (BVP) along with the application of Green's Theorem (Evans [5]) to determine these corrections for the reflection and transmission coefficients, to the problem of diffraction of water waves by a fixed nearly vertical barrier. Subsequently, both the mathematical techniques, based on an integral equation formulation similar to [12], and a suitable exploitation of [5] along with an appropriately chosen perturbational technique [9], have been utilized successfully by Mandal and Kundu [11] in considering the problem of the scattering of water waves by a submerged nearly vertical plate. Since then, some attempts have been made to study this class of water wave problem, involving nearly vertical barriers, associated with Laplace's equation and few of its generalizations by employing different mathematical methods ([1, 2, 7, 8]).

The problem under consideration is tackled assuming linearity and the first order corrections to the reflection and transmission coefficients, which are the quantities of physical interest, are found here in terms of integrals involving the shape functions describing the two sides of the slender barrier, by (i) standard perturbation arguments, giving a sequence of BVPs, (ii) the known solution of the corresponding vertical plate problem, and (iii) Evans's trick [5], involving a judicious use of Green's Theorem, so that the complete solution of the first order BVP is not required. Corresponding results (i) for a submerged nearly vertical plate, and (ii) for a submerged vertically symmetric slender barrier of finite length follow as special cases of the general problem considered here. Numerical calculations have been done for the reflection and transmission coefficients for different values of the various parameters, up to first order, assuming particular shapes of the slender barrier, a vertically symmetric slender barrier, and a nearly vertical plate, each of finite length.

2. Formulation of the problem

Cartesian co-ordinates are chosen with y measured vertically downwards, and with $y = 0$ the undisturbed free surface. Linearized water wave theory is assumed so that there exists a velocity potential

$$\Phi(x, y, t) = \mathrm{Re}\{\phi(x, y)\exp(-i\sigma t)\},$$

where σ is the circular frequency. Thus, the problem under consideration is mathematically described by the following BVP:

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \quad \text{in the flow domain,} \tag{1}$$

$$K\phi + \frac{\partial\phi}{\partial y} = 0 \quad \text{on } y = 0, \tag{2}$$

$$\frac{\partial\phi}{\partial n} = 0 \quad \text{on } B, \tag{3}$$

$$r^{1/2}\nabla\phi \quad \text{is bounded as } r \to 0, \tag{4}$$

$$\phi, \nabla\phi \to 0 \quad \text{as } y \to \infty, \tag{5}$$

where K $(= \sigma^2/g$, a real constant) is the wave number, g is the acceleration due to gravity, B denotes the two sides of the slender barrier, n denotes the outward drawn normal to B, and r is the distance from the two sharp edges of the slender barrier.

Here the two sides of the slender barrier B are given by

$$B = B_1 \cup B_2, \qquad B_j : x = \varepsilon c_j(y), (j = 1, 2) \quad 0 < a < y < b < \infty,$$

where $\varepsilon > 0$ is a small dimensionless quantity, and the $c_j(y)$ are bounded and continuous functions of y for $a < y < b$ with

$$c_j(a) = 0, \quad c_j(b) = 0. \tag{6}$$

In this case, it is assumed that the y-axis passes through the upper and lower ends of the slender barrier. It is also assumed that the slender barrier possesses sharp edges at the two ends $(0, a)$ and $(0, b)$ respectively. The situation under consideration is sketched in Figure 1.

Finally we assume that

$$\phi(x, y) \sim \begin{cases} \phi^i(x, y) + R\phi^i(-x, y) & \text{as } x \to -\infty, \\ T\phi^i(x, y) & \text{as } x \to +\infty, \end{cases} \tag{7}$$

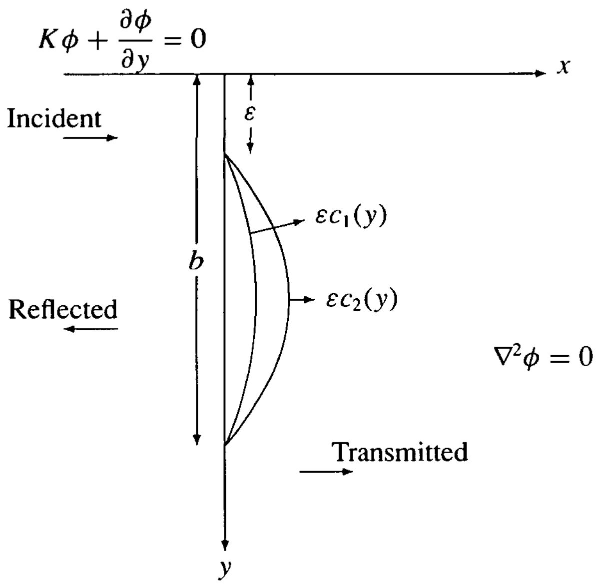

FIGURE 1. Slender barrier $a < y < b$.

where $\phi^i(x, y) = \exp(-Ky + iKx)$ is a train of surface water waves incident upon the barrier B from negative infinity, and R and T are the (complex) reflection and transmission coefficients, respectively, which are unknown.

Assuming that the parameter $\varepsilon > 0$ is very small, which gives a measure of the slenderness of the barrier, and neglecting $O(\varepsilon^2)$ terms, the boundary condition (3) can be approximated as (see [12])

$$\left.\begin{aligned} \frac{\partial \phi}{\partial x}(+0, y) &= \varepsilon \frac{d}{dy}\{c_1(y) \frac{\partial \phi}{\partial y}(+0, y)\} \\ \frac{\partial \phi}{\partial x}(-0, y) &= \varepsilon \frac{d}{dy}\{c_2(y) \frac{\partial \phi}{\partial y}(-0, y)\} \end{aligned}\right\}, \qquad a < y < b. \tag{8}$$

3. Solution by perturbation technique

The form of the approximate boundary conditions given by (8) suggest that we may expand the function $\phi(x, y)$, and the two unknown physical constants R and T, in terms of the small parameter ε, as follows (see [9]):

$$\phi(x, y) = \phi_0(x, y) + \varepsilon \phi_1(x, y) + 0(\varepsilon^2),$$
$$R = R_0 + \varepsilon R_1 + 0(\varepsilon^2),$$

$$T = T_0 + \varepsilon T_1 + 0(\varepsilon^2). \tag{9}$$

Here we restrict our attention to the determination of the constants R_0, T_0, R_1 and T_1 only, as we are interested in evaluating only the corrections to the reflection and transmission coefficients up to first order. Substituting the expansions given by (9) into the basic partial differential equation (1), the boundary conditions (2) and (8), the edge condition (4), and the requirements at infinity (5) and (7), we obtain after equating the coefficients of O(1) and O(ε) terms, that the functions ϕ_0 and ϕ_1 must be the solution of the following two independent boundary value problems.

BVP-0: The problem is to determine the function $\phi_0(x, y)$ satisfying

$$\nabla^2\phi_0 = 0 \text{ in the fluid region,}$$

$$K\phi_0 + \frac{\partial\phi_0}{\partial y} = 0 \text{ on } y = 0,$$

$$\frac{\partial\phi_0}{\partial x} = 0 \text{ on } x = 0, a < y < b,$$

$$r^{1/2}\nabla\phi_0 \text{ is bounded as } r \to 0,$$

where r is the distance from the sharp edges, $(0, a)$ and $(0, b)$, of the barrier,

$$\phi_0, \nabla\phi_0 \to 0 \text{ as } y \to \infty,$$

$$\phi_0 \sim \begin{cases} \phi^i(x, y) + R_0\phi^i(-x, y) & \text{as } x \to -\infty, \\ T\phi^i(x, y) & \text{as } x \to +\infty. \end{cases}$$

BVP-1: The problem is to determine the function $\phi_1(x, y)$ satisfying

$$\nabla^2\phi_1 = 0 \text{ in the region } y > 0,$$

$$K\phi_1 + \frac{\partial\phi_1}{\partial y} = 0 \text{ on } y = 0,$$

$$\left.\begin{aligned} \frac{\partial\phi_1}{\partial x}(+0, y) &= \frac{d}{dy}\{c_1(y)\frac{\partial\phi_0}{\partial y}(+0, y)\} \\ \frac{\partial\phi_1}{\partial x}(-0, y) &= \frac{d}{dy}\{c_2(y)\frac{\partial\phi}{\partial y}(-0, y)\} \end{aligned}\right\} \quad a < y < b, \tag{10}$$

$$r^{1/2}\nabla\phi_1 \text{ is bounded as } r \to 0,$$

$$\phi_1, \nabla\phi_1 \to 0 \text{ as } y \to \infty,$$

$$\phi_1 \sim \begin{cases} R_1\phi^i(-x, y) & \text{as } x \to -\infty, \\ T_1\phi^i(x, y) & \text{as } x \to +\infty. \end{cases}$$

The explicit solution $\phi_0(x, y)$ of the **BVP-0**, which corresponds to the vertical plate problem, is well known [4, 10, 11] and is given by

$$\begin{aligned}\phi_0(x, y) =&\phi^i(x, y) + R_0\phi^i(-x, y)\\ &+\frac{2i}{\pi\Delta}\int_0^\infty \frac{J(k)(k\cos ky - K\sin ky)}{k^2+K^2}\exp(kx)dk \text{ for } x<0\end{aligned}$$

and

$$\begin{aligned}\phi_0(x, y) =&T_0\phi^i(x, y)\\ &-\frac{2i}{\pi\Delta}\int_0^\infty \frac{J(k)(k\cos ky - K\sin ky)}{k^2+K^2}\exp(-kx)dk \text{ for } x<0,\end{aligned} \tag{11}$$

where

$$R_0 = -\frac{i\gamma}{\Delta},\quad T_0 = \frac{\alpha-\beta}{\Delta},\quad \Delta = \alpha-\beta-i\gamma,$$
$$J(k) = \int_a^b \frac{(d^2-u^2)\sin\, ku}{\{(u^2-a^2)(b^2-u^2)\}^{1/2}}du,$$

and

$$\alpha = \int_{-a}^{a} f(u)\ \exp(-Ku)\ du,$$
$$\beta = \int_{b}^{\infty} f(u)\ \exp(-Ku)\ du,$$
$$\gamma = \int_{a}^{b} f(u)\ \exp(-Ku)\ du,$$

with

$$f(u) = \frac{(d^2-u^2)}{\{(u^2-a^2)(b^2-u^2)\}^{1/2}},$$

and d^2 is given by $\int_a^b f(u)\exp(Ku)du = 0$.

Although the solution $\phi_0(x, y)$, corresponding to the **BVP-0** is known, it appears that it is not necessary to find the solution $\phi_1(x, y)$ to the **BVP-1**, for the determination of the first order corrections to the reflection and transmission coefficients. However, the physical quantities R_1 and T_1 can be found in the following manner using a technique similar to that used by Evans [5].

Determination of R_1: To find R_1, we apply Green's theorem to the harmonic functions $\phi_0(x, y)$ and $\phi_1(x, y)$ in the region bounded by the lines

$$y = 0,\ -X \le x \le X; \quad x = -X,\ 0 \le y \le Y; \quad y = Y,\ -X \le x \le X;$$
$$x = X,\ 0 \le y \le Y; \quad x = 0+,\ a < y < b; \quad x = 0-,\ a < y < b;$$

with $X, Y > 0$, and circles c_1 and c_2 of small radius δ with centres at $(0, a)$ and $(0, b)$, and we ultimately make $X, Y \to \infty$ and $\delta \to 0$. Using arguments similar to those in Evans [5], we find

$$\begin{aligned} iR_1 = \int_a^b \Big[& \phi_0(+0, y)\frac{\partial}{\partial y}\left\{c_1(y)\frac{\partial \phi_0}{\partial y}(+0, y)\right\} \\ & -\phi_0(-0, y)\frac{\partial}{\partial y}\left\{c_2(y)\frac{\partial \phi_0}{\partial y}(-0, y)\right\}\Big]\,dy, \end{aligned} \tag{12}$$

in which the boundary conditions given by (10) have been used. Utilizing (6), (12) can be reduced to the form

$$R_1 = i\int_a^b \left[c_1(y)\left\{\frac{\partial \phi_0}{\partial y}(+0, y)\right\}^2 - c_2(y)\left\{\frac{\partial \phi_0}{\partial y}(-0, y)\right\}^2\right] dy. \tag{13}$$

To derive the analytical expression for R_1, we use the results for $\phi_0(\pm 0, y)$, which can be deduced by using (11), and is given in [11] as follows

$$\phi_0(\pm 0, y) = \begin{cases} \exp(-Ky) & \text{for } y \le a, \\ \exp(-Ky) \pm \frac{i}{\Delta}\exp(-Ky)\int_b^y f(u)\exp(Ku)\,du, & \text{for } a < y < b, \\ \exp(-Ky) & \text{for } y \ge b. \end{cases} \tag{14}$$

Substituting the appropriate expression $\phi_0(\pm 0, y)$ into (13), the simplified form for the analytical expression of R_1 can be found, and is given by

$$\begin{aligned} R_1 = & i\int_a^b \Big[\{c_1(y) - c_2(y)\}\Big\{K^2\exp(-2Ky) - \frac{K^2}{\Delta^2}\exp(-2Ky)p(y) \\ & - \frac{1}{\Delta^2}q(y) + \frac{2K}{\Delta^2}\exp(-Ky)f(y)g(y)\Big\} + \frac{2iK}{\Delta}\{c_1(y) + c_2(y)\} \\ & \times \{K\exp(-2Ky)g(y) - \exp(-Ky)f(y)\}\Big]\,dy, \end{aligned} \tag{15}$$

where

$$g(y) = \int_b^y f(u)\exp(Ku)\,du, \quad p(y) = \{g(y)\}^2, \quad q(y) = \{f(y)\}^2.$$

Determination of T_1: Now, to determine T_1, the first order correction to the transmission coefficient T, we again utilize Evans' [5] idea involving the application of Green's theorem to the pair of harmonic functions $\phi_0(x, y)$ and $\phi_1(x, y)$ in the region mentioned earlier, and obtain finally,

$$iT_1 = \int_a^b \left[\Psi_0(+0, y)\frac{\partial}{\partial y}\left\{c_1(y)\frac{\partial \phi_0}{\partial y}(+0, y)\right\} - \Psi_0(-0, y)\frac{\partial}{\partial y}\left\{c_2(y)\frac{\partial \phi_0}{\partial y}(-0, y)\right\}\right] dy, \tag{16}$$

where $\Psi_0(\pm 0, y) = \phi_0(\mp 0, y)$, and the conditions on $x = \pm 0$ have been used. The expression for T_1, given by (16), can be simplified by using (6), and can be expressed in terms of $\phi_0(\pm 0, y)$ as

$$T_1 = i\int_a^b \{c_1(y) - c_2(y)\}\frac{\partial \phi_0}{\partial y}(+0, y)\frac{\partial \phi_0}{\partial y}(-0, y)dy. \tag{17}$$

Using the relation (14) in (17), an analytical expression for T_1 can be deduced:

$$T_1 = i\int_a^b \{c_1(y) - c_2(y)\}\left[K^2 \exp(-2Ky) - \frac{i}{\Delta}\{K \exp(-Ky)g(y) - f(y)\}^2\right]dy. \tag{18}$$

The expressions (15) and (18) are the general form of the analytical expressions for the first order corrections to the reflection and transmission coefficients for a surface water wave train incident upon a fixed fully submerged slender barrier. From (18), it is observed that T_1 in general does not vanish in contrast to the corresponding nearly vertical plate problem (cf. [11])

4. Special cases

(i) Submerged nearly vertical plate: When $c_1(y) = c_2(y)$, the slender barrier problem reduces to that involving a nearly vertical plate. The geometry is sketched in Figure 2. Thus, assuming $c_1(y) = c_2(y) = c(y)$, we find, from (15) and (18), that

$$R_1 = -\frac{4K}{\Delta}\int_a^b c(y)\{K \exp(-2Ky)g(y) - \exp(-Ky)f(y)\}\,dy \tag{19}$$

and

$$T_1 = 0. \tag{20}$$

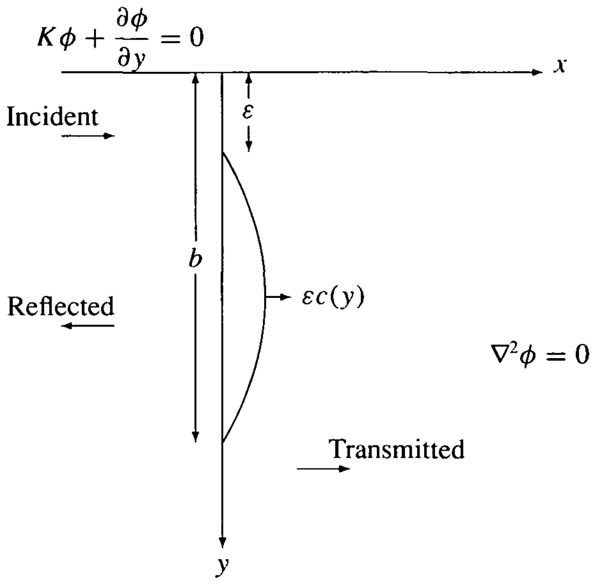

FIGURE 2. Nearly vertical plate $a < y < b$.

The expression (19) for R_1 can be written, after some elementary manipulation, as

$$R_1 = \frac{4K}{\Delta}\left[K\int_a^b c(y)\exp(-2Ky)\left\{\int_y^b \frac{(d^2-u^2)\exp(Ku)}{\{(u^2-a^2)(b^2-u^2)\}^{1/2}}du\right\}dy + \int_a^b \frac{(d^2-y^2)\exp(-Ky)c(y)}{\{(y^2-a^2)(b^2-y^2)\}^{1/2}}dy\right]. \tag{21}$$

Expressions (20) and (21) are in complete agreement with those obtained in [11] in connection with the diffraction of water waves by a submerged nearly vertical plate. It should be noted that in this case $T_1 = 0$, however, it does not vanish for the general problem considered here, that is, for the submerged slender barrier (see (18)).

(ii) Submerged symmetric slender barrier: If $c_1(y) = -c_2(y)$, the slender barrier becomes symmetric about the vertical (the situation is described in Figure 3), and thus by putting $c_1(y) = -c_2(y) = c(y)$ in (15) and (18), we find that

$$R_1 = 2i\int_a^b c(y)\left[K^2\exp(-2Ky) - \frac{1}{\Delta^2}\{K\exp(-Ky)g(y) - f(y)\}^2\right]dy, \tag{22}$$

$$T_1 = 2i\int_a^b c(y)\left[K^2\exp(-2Ky) + \frac{1}{\Delta^2}\{K\exp(-Ky)g(y) - f(y)\}^2\right]dy. \tag{23}$$

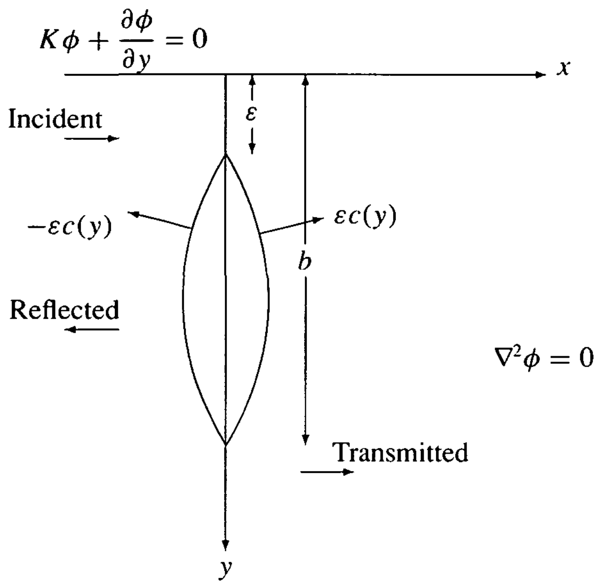

FIGURE 3. Symmetric slender barrier $a < y < b$.

Expressions (22) and (23) can be written in the simplified form

$$R_1 = P_1 - Q_1 \tag{24}$$

and

$$T_1 = P_1 + Q_1, \tag{25}$$

where

$$P_1 = 2iK^2 \int_a^b c(y)\exp(-2Ky)dy,$$
$$Q_1 = \frac{2i}{\Delta^2}\int_a^b c(y)\left[K\exp(-Ky)\int_b^y \frac{(d^2-u^2)\exp(Ku)}{\{(u^2-a^2)(b^2-u^2)\}^{1/2}}du - \frac{(d^2-y^2)}{\{(y^2-a^2)(b^2-y^2)\}^{1/2}}\right]^2 dy. \tag{26}$$

Analytical expressions for R_1 and T_1 given, respectively, by (24) and (25) are the first order corrections to the reflection and transmission coefficients for a surface water wave train incident upon a fixed submerged vertically symmetric slender barrier. It should be noted here that, in contrast with the submerged nearly vertical plate problem, in this case the expression for T_1 does not vanish.

5. Numerical results

The theory presented in this paper is also supported by numerical examples. For numerical computations of the reflection coefficient $|R|$ $(= |R_0 + \varepsilon R_1|)$ and transmission coefficient $|T|$ $(= |T_0 + \varepsilon T_1|)$ up to $O(\varepsilon)$, we have assumed a particular shape of the slender barrier by specifying $c_1(y) = (y - a)(b - y)/(b - a)$ and $c_2(y) = a \sin\{\pi(y - a)/(b - a)\}, a < y < b$. The various integrals are evaluated numerically by using a 16-point Gauss quadrature formula, and $|R|$ and $|T|$ are then obtained correct up to seven decimal places for different values of the various parameters.

In the Figure 4(i), curves are drawn for $|R|$ and $|T|$ as functions of u $(= a/b)$ for different values of $Kb = 0.2$ $(b = 0.1)$, 1.4 $(b = 0.7)$, 2.0 $(b = 1.0)$, and $\varepsilon = 0.001$. It is observed from each of the curves for $|R|$ that, for a fixed Kb, $|R|$ decreases as u increases (i.e. as the length of the barrier $b - a$ decreases), and $|R|$ decreases asymptotically to zero as $u \to 1$. Curves for $|T|$ show that, for a fixed Kb, $|T|$ increases with u, and increases asymptotically to 1 as $u \to 1$. All these observations are acceptable, since when $u \to 1$, the length of the barrier approaches zero so that the incident wave train does not feel the presence of the barrier, and hence almost all the wave energy is transmitted.

$|R|$ and $|T|$ are plotted graphically against u for different values of Kb in Figure 4(ii), assuming $\varepsilon = 0.005$. Figure 4(ii) shows similar qualitative behaviour to that already discussed in the case of Figure 4(i).

Curves for $|R|$ and $|T|$ as functions of Kb, for different values of u, and $\varepsilon = 0.001$, are shown in Figures 5(i) and 5(ii). It is observed from these figures that, for a fixed u, even for small u, that is, when there is a small gap between the upper edge of the barrier and the free-surface compared to the length of the barrier, $|R|$ first increases from zero, with Kb, attains its maximum, and then decreases to zero with further increase of Kb. It is also observed that, for a fixed u, $|T|$ decreases from 1, as Kb increases, until a minimum is reached, and then increases to 1 with Kb. These are plausible, since for small values of Kb corresponding to the incident wave train whose wave length is large compared to the length of the barrier, $|R|$ becomes small as the wave train does not feel the presence of the barrier, with the increase of Kb, the wave length of the incident wave train and the length of the barrier become comparable, and thus $|R|$ increases and $|T|$ decreases. On the other hand, for large values of Kb, the incident wave train becomes short and the wave energy is confined within a thin layer near the free surface, and almost all the wave energy is transmitted through the gap above the slender barrier submerged in water, without any reflection. Thus for each finite u, $|R| \to 0$ and $|T| \to 1$, as $Kb \to \infty$. Hence, $|R|$ and $|T|$ must have a maximum and minimum, respectively, for some intermediate values of Kb.

Assuming $\varepsilon = 0.005$, $|R|$ and $|T|$ are also calculated numerically for different

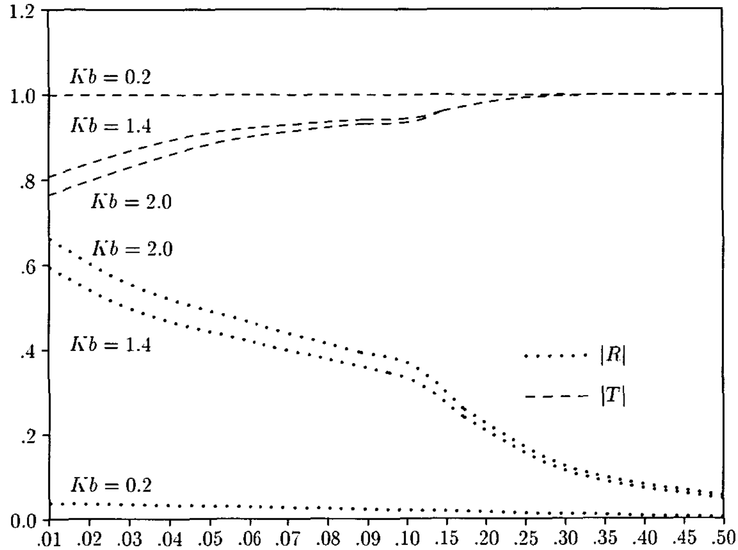

FIGURE 4(i). $|R|$ and $|T|$ versus u ($\varepsilon = 0.001$).

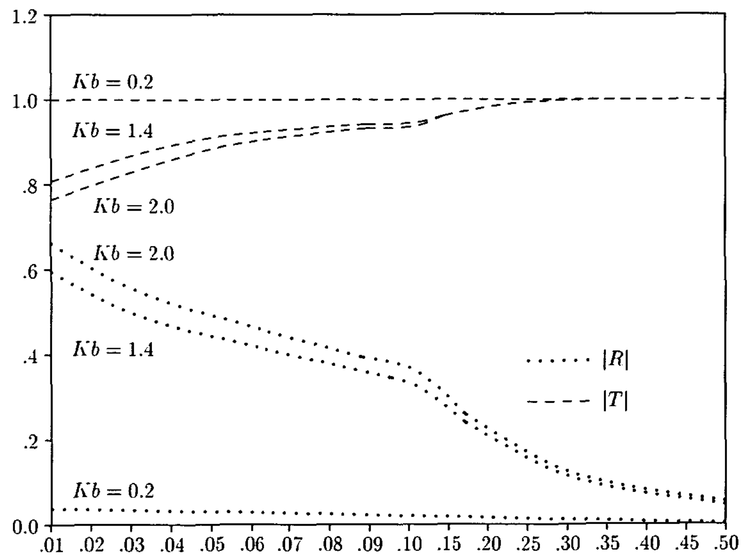

FIGURE 4(ii). $|R|$ and $|T|$ versus u ($\varepsilon = 0.005$).

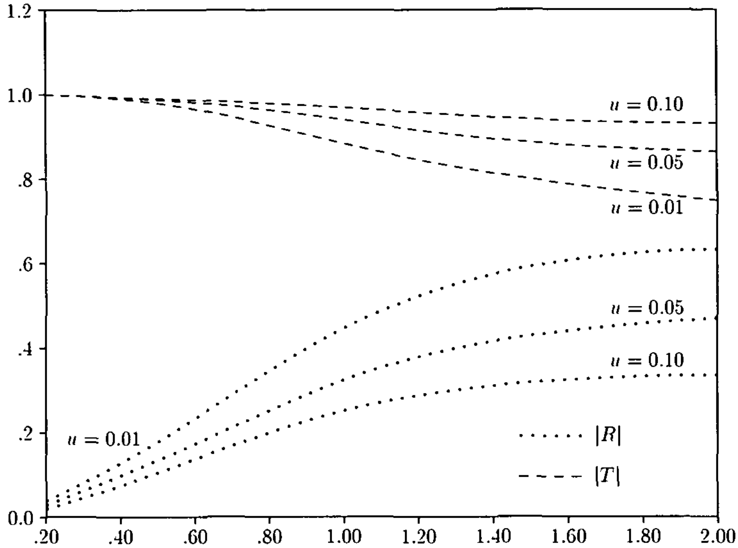

FIGURE 5(i). $|R|$ and $|T|$ versus Kb ($\varepsilon = 0.001$).

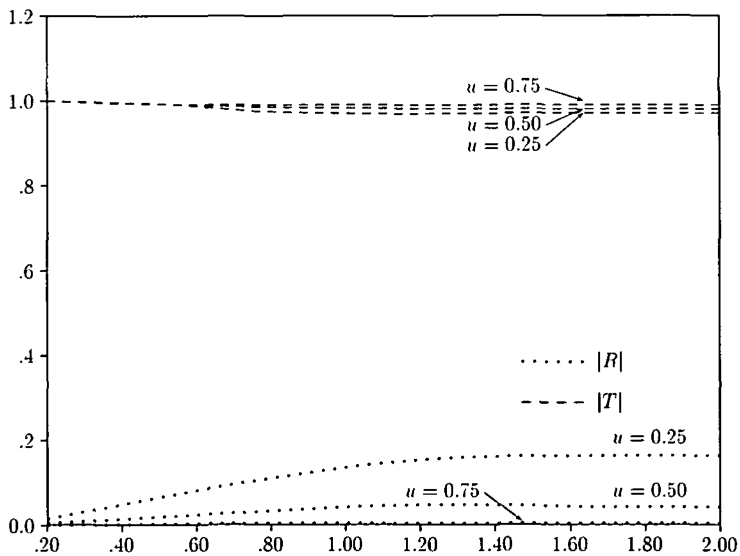

FIGURE 5(ii). $|R|$ and $|T|$ versus Kb ($\varepsilon = 0.005$).

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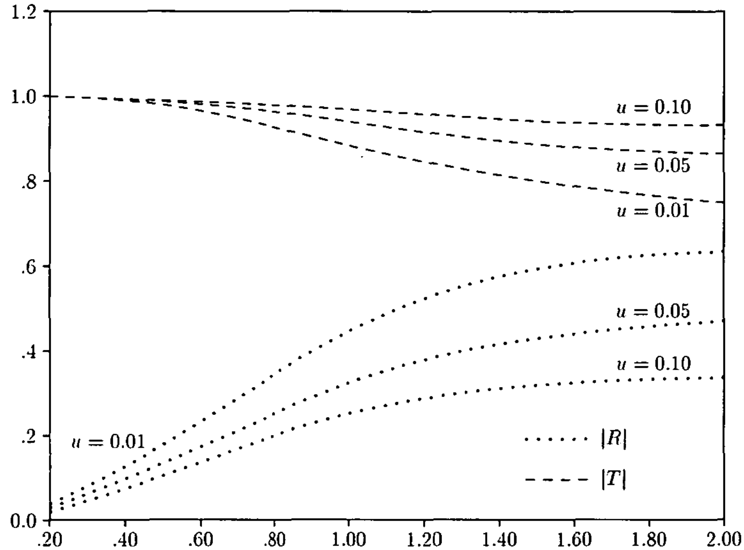

FIGURE 6(i). $|R|$ and $|T|$ versus Kb ($\varepsilon = 0.001$).

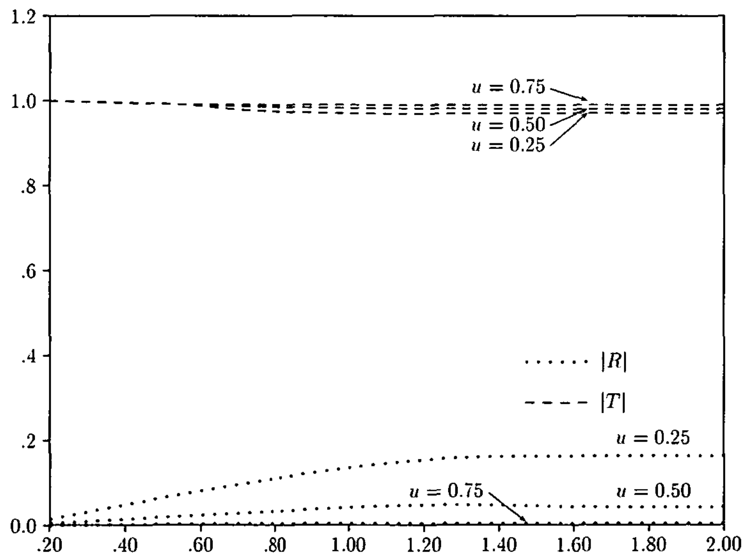

FIGURE 6(ii). $|R|$ and $|T|$ versus Kb ($\varepsilon = 0.005$).

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values of the various parameters, and are depicted in Figures 6(i) and 6(ii). From Figures 6(i) and 6(ii), the similar qualitative nature of $|R|$ and $|T|$, already described in the case of Figures 5(i) and 5(ii), is observed.

Curves for $\varepsilon = 0.005$ are drawn separately, since the values of $|R|$ and $|T|$ for $\varepsilon = 0.001$ and $\varepsilon = 0.005$ are so close, and distinction between them is not possible, if they were drawn on the same figure.

Table 1. $Kb = 0.2$ $(b = 0.1)$

	$\varepsilon = 0.001$		$\varepsilon = 0.005$	
u	$\|R\|$ (NVP)	$\|R\|$ (SSB)	$\|R\|$ (NVP)	$\|R\|$ (SSB)
0.01	0.0382909	0.0328509	0.0382909	0.0328565
0.05	0.0314056	0.0264243	0.0314056	0.0264199
0.10	0.0262840	0.0218080	0.0262840	0.0218002
0.25	0.0164733	0.0133065	0.0164733	0.0132982
0.50	0.0067540	0.0052904	0.0067540	0.0052861
0.75	0.0016064	0.0012264	0.0016064	0.0012254

Table 2. $Kb = 0.8$ $(b = 0.4)$

	$\varepsilon = 0.001$		$\varepsilon = 0.005$	
u	$\|R\|$ (NVP)	$\|R\|$ (SSB)	$\|R\|$ (NVP)	$\|R\|$ (SSB)
0.01	0.7264121	0.3556277	0.7264122	0.3558770
0.05	0.4606530	0.2618765	0.4606530	0.2619223
0.10	0.3268584	0.2017271	0.3268584	0,2017024
0.25	0.1538828	0.1065607	0.1538828	0.1065048
0.50	0.0467464	0.0352419	0.0467464	0.0352151
0.75	0.0087407	0.0069164	0.0087407	0.0069104

Table 3. $Kb = 1.4$ $(b = 0.7)$

	$\varepsilon = 0.001$		$\varepsilon = 0.005$	
u	$\|R\|$ (NVP)	$\|R\|$ (SSB)	$\|R\|$ (NVP)	$\|R\|$ (SSB)
0.01	0.7547233	0.5840841	0.7547235	0.5843242
0.05	0.5380489	0.4280253	0.5380490	0.4280099
0.10	0.3948318	0.3223990	0.3948318	0.3223010
0.25	0.1830797	0.1569966	0.1830797	0.1568968
0.50	0.0500798	0.0448438	0.0500798	0.0448092
0.75	0.0081969	0.0075399	0.0081969	0.0075336

Table 4. $Kb = 2.0$ $(b = 1.0)$

	$\varepsilon = 0.001$		$\varepsilon = 0.005$	
u	$\lvert R\rvert$ (NVP)	$\lvert R\rvert$ (SSB)	$\lvert R\rvert$ (NVP)	$\lvert R\rvert$ (SSB)
0.01	0.7095335	0.6540284	0.7095337	0.6540968
0.05	0.5171511	0.4779399	0.5171511	0.4777910
0.10	0.3798304	0.3532320	0.3798304	0.3530396
0.25	0.1681786	0.1589165	0.1681786	0.1587953
0.50	0.0403726	0.0388004	0.0403726	0.0387705
0.75	0.0056764	0.0055183	0.0056764	0.0055140

Assuming $c(y) = (y-a)(b-y)/(b-a)$, $a < y < b$, $|R|$ is tabulated for the nearly vertical plate as well as for the symmetric slender barrier for different values of the various parameters. Some representative numerical results are shown in Tables 1 to 4. It is observed from all the tables that, for a fixed Kb, $|R|$ decreases as u increases, and $|R| \to 0$ as $u \to 1$, both for the nearly vertical plate as well as that for the symmetric slender barrier. From these tables, it is also observed that, for a fixed u, as Kb increases $|R|$ first increases, and then decreases for both the nearly vertical plate as well as the symmetric slender barrier. It is also observed that the values of $|R|$ for $\varepsilon = 0.001$ in most cases differ from those for $\varepsilon = 0.005$ only in the fifth or sixth decimal places. As the terms of $O(\varepsilon^2)$ are neglected throughout the analysis, this indicates that the influence of ε is not of much significance for these type of curved barriers. From these tables it is further observed that more wave energy is reflected by the nearly vertical plate than the symmetric slender barrier. More specifically, it is found that among the slender barrier, nearly vertical plate and symmetric slender barrier, maximun wave energy is reflected by the nearly vertical plate and minimum wave energy is reflected by the slender barrier.

6. Discussion

Simple analytical expressions representing the first order corrections to the reflection and transmission coefficients, R_1 and T_1, respectively, are obtained here, assuming linear theory, for a surface water wave train incident upon a fixed submerged slender barrier, in deep water. The barrier is assumed to be of finite length whose upper and lower ends are at a depth a and b, respectively, below the free surface. The values of R_1 and T_1 are found by using a simple and straightforward perturbation technique somewhat similar to that employed in [9] in connection with the diffraction of water waves by nearly vertical barriers. Analytical expressions for R_1 and T_1 are obtained here, over the vertical barrier results, in terms of non-trivial integrals involving the shape functions describing the two sides of the slender barrier. These non-trivial integrals are also calculated, numerically, assuming the explicit form of $c_1(y)$ and

$c_2(y)$. As a special case, when two sides of the slender barrier become identical, the submerged slender barrier reduces to a submerged nearly vertical plate, and is observed to produce known results, involving corrections to the reflection and transmission coefficients, given by [11]. Another special case yields the corresponding analytical expressions for R_1 and T_1, for a submerged vertically symmetric slender barrier. Thus the present investigation provides with the generalisation to the problem of the scattering of water waves by a submerged nearly vertical plate [11] as well as that by a vertically symmetric slender barrier to that by a submerged slender barrier.

The problem of scattering of water waves by a submerged slender barrier ($0 < a < y < b < \infty$) is studied, in this paper, which to the best of the authors' knowledge has not been tackled before. However, it may be mentioned that Shaw [12] stated the problem of diffraction of water waves by a just immersed vertically symmetric slender barrier ($0 < y < a$) by employing a method based on an integral equation formulation, although he did not consider solutions for the reflection and transmission coefficients. Viewed in this light, the results obtained here may be looked on as the first order corrections to the reflection and transmission coefficients for the fully submerged slender barrier problem. To the authors, this is something new and has not been obtained before. Determination of R_1 and T_1 are, therefore, one of the many motives behind taking up the present study.

The reason for choosing a perturbation technique is that the perturbation technique is an extremely straightforward approach (although a bit pedestrian and old fashioned) which can be applied judiciously, to produce the desired results fairly easily and relatively quickly when compared to the integral equation technique, to handle scattering problems in water wave theory. It must be admitted that the approximations involved in the present paper are expressed in terms of non-trivial integrals whose evaluation must be done numerically. These non-trivial integrals can be evaluated numerically by using any one of the standard methods for numerical integration. Thus the numerical integration required here is much simpler than the procedures required in any direct numerical solution. It may be mentioned here that, while the numerical solution of the full problem is relatively straightforward for a barrier of substantial thickness, it is likely to be singular as the thickness tends to zero, that is, for a thin barrier. It may also be mentioned that, numerical computations are performed to support the theory presented here, although the theoretical development is the main contribution of the paper.

In fact, we have obtained here the corrections, up to first order, to the reflection and transmission coefficients for a submerged barrier which have not been derived so far. To check the accuracy of the approximations to the reflection and transmission coefficients,analytically, known results for a submerged nearly vertical plate given by [11], have been deduced from that for a submerged slender barrier (see Section 4(i)). It is interesting to note that, although, the correction to the transmission coefficient,

T_1, vanishes for a submerged nearly vertical plate, it does not vanish for a submerged slender barrier (see (18)) or for a submerged vertically symmetric slender barrier (see (25)). These interesting results have not been derived so far for the submerged slender barrier or for the submerged vertically symmetric slender barrier.

The approximate analytical approach described in this paper gives additional insight over numerical solutions and further analytical progress may be obtained

(i) by choosing a particular shape function for the curved barriers,

(ii) by determining the second order corrections to the reflection and transmission coefficients, R_2 and T_2, respectively. To determine R_2 and T_2, following the method described in this paper, it is required to find $\phi_1(x, y)$ for $x > 0$ as well as for $x < 0$. The explicit form of $\phi_1(x, y)$, the solution of the **BVP-1**, can be found by the procedure adopted by Vijaya-Bharathi *et al.* [2].

(iii) by considering the finite depth of water (which is important from the coastal engineering point of view).

7. Conclusion

From the analysis involved in the present investigation, the following conclusions can be drawn.

1. The boundary perturbation technique described in this paper is a simple and straightforward technique, to solve the scattering problem, as compared to the integral equation technique.

2. The final important results derived here are the first order corrections to the reflection and transmission coefficients R_1 and T_1, for the submerged slender barrier problem, which have not been derived previously.

3. Analytical expressions for R_1 and T_1 are derived here in terms of non-trivial integrals and the evaluation of these non-trivial integrals are much simpler than any direct numerical solution of the full problem.

4. To check the accuracy, the results obtained in this paper are compared with the known results for a submerged nearly vertical plate given by [11].

5. It is observed that T_1 vanishes for a submerged nearly vertical plate, however, it does not vanish for a submerged slender barrier or for a submerged vertically symmetric slender barrier.

6. Numerical results are also provided to describe the effect of finite barrier on the reflection and transmission coefficients.

7. Further analytical development of the problem considered here may be done.

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