Proof of the theorem that the mid points of the three diagonals of a complete quadrilateral are collinear.

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The following proof of this theorem assumes only *Euclid*, I. 43, and its converse, with the well-known deductions, "the line joining the mid points of two sides of a triangle is parallel to the third side," and "the mid point of one diagonal of a parallelogram is also the mid point of the other." The proof given by Dr Taylor in his *Conics* which suggested the method, makes use of ratios.

Let ABCD (Fig. 16) be a quadrilateral, AD, BC, produced meeting in E, and AB, DC, produced in F. Through each of the angular points of the figure draw parallels to AB, AD, giving two sets of four parallel lines,

	AGBF,	HCKL,	DMNP,	EQRS,	in the one set,
and	AHDE,	GCMQ,	BKNR,	FLPS,	in the other set.

By *Euclid*, I. 43,

$$\square^{m}AC = \square^{m}CR, \quad \text{and} \quad \square^{m}AC = \square^{m}CP.$$

$$\therefore \quad \square^{m}CP = \square^{m}CR, \quad \text{and} \quad \therefore \text{ CNS is a straight line.}$$

∴ the mid points of AC, AN, AS are collinear,

that is, the mid points of AC, BD, EF are collinear.

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