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Scope, motivation, and orientation

If one accepts gravitational forces on the Newtonian level of precision and ignores nuclear fission and fusion, then most physical phenomena on the scale of the Earth are accounted for by electrons, nuclei, and photons. Here photons play a double role: they mediate the interaction between charges, and appear freely propagating in the form of electromagnetic radiation. In their first role it often suffices to ignore all dynamical aspects and replace the photons by the effective electrostatic Coulomb interaction. Conversely, in the study of radiation phenomena, matter in the form of nuclei and electrons can mostly be replaced by prescribed macroscopic quantities like charge, current, and polarization densities. In our treatise we plan to dwell on the border area, where the interaction between photons and electrons, respectively nuclei, must be fully retained. Our goal is to discuss the dynamics of the coupled system, charges and their radiation field.

Although such a description might give the impression that we will deal with relativistic quantum electrodynamics (QED), in fact we will not even touch upon it. This theory has been devised for predicting a few very specific effects, like the anomalous g-factor of the electron, and it does so with astounding precision. Relativistic QED is, however, not well adapted to discuss, say, the fluorescence of the hydrogen atom. Thus the subject to be covered is what is commonly known as nonrelativistic quantum electrodynamics. In fact our enterprise also has a classical part. Just as in studying quantum mechanics a good grasp of classical mechanics is most useful, we believe that an understanding of classical electron theory, i.e. classical charges in interaction with the Maxwell field, serves as a solid basis for taking up the corresponding quantum theory. The classical models discussed will be semirelativistic with one exception, namely a fully relativistic theory of extended classical charges.

Classical electron theory was at the forefront of research in the early 1900s when the development of a dynamical theory of the then newly discovered electron was attempted. The basic prediction was an energy–momentum relation for

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the electron (compare with chapter 4), which, however, depended on the details of the particular electron model adopted. This enterprise came to a standstill because of the advent of the theory of special relativity, which, advancing with a totally different set of arguments, required a relativistically covariant link between energy and momentum for massive particles. Classical electron theory further deteriorated simply because it had become evident that for the investigation of radiation from atoms the newly born quantum mechanics had to be used. A brief revival occurred in the struggle to formulate a consistent relativistic quantum theory for the electron–positron field coupled to the photons. The hope was that a refined understanding of the classical theory should give a hint on how to quantize and how to handle correctly the ultraviolet infinities. But as the proper quantum field theory surfaced, classical considerations faded away. In fact the theory emerged in a worse state than before as summarized in the 1963 opinion of R. Feynman: "The classical theory of electromagnetism is an unsatisfactory theory all by itself. The electromagnetic theory predicts the existence of an electromagnetic mass, but it also falls on its face in doing so, because it does not produce a consistent theory."

Because of its peculiar history, classical electron theory never had any share in the good fortune of being rewritten, modernized, and rewritten again, as can be seen from a rapid sample of standard textbooks on electrodynamics. While the conventional chapters essentially follow the same intrinsic pattern, obviously with a lot of variations on details, once it comes to the chapter on radiation reaction, Pandora's box opens. As a student I was rather dissatisfied with such a state of affairs and promised myself to come back to it at some point. The first few chapters of this treatise are my own rewriting of the classical theory. It is based on two cornerstones:

- a well-defined dynamical theory of extended charges in interaction with the electromagnetic field;
- a study of the effective dynamics of charges under the condition that they are far apart and the external potentials vary slowly on the scale given by the size of the charge distribution. This is the *adiabatic limit*.

Our approach reflects the great progress which has taken place in the theory of dynamical systems. After all, charges coupled to their radiation field can be considered as one particular case, but with some rather special features. Perhaps the most unusual one is the appearance of a center manifold in the effective dynamics, in case friction through radiation is included.

For nonrelativistic QED the situation could hardly be more different. Through the efforts made in atomic physics and quantum optics a structured theory emerged which is well covered in textbooks and reviews. It would make little sense in trying to compete with them. However, almost exclusively this theory is based either on such drastic simplifications that an exact solution becomes possible or on second-order time-dependent perturbation theory. In recent years there has been substantial progress, mostly within the quarters of mathematical physicists, in gaining an understanding of *nonperturbative* properties of the full basic Hamiltonian, among others the structure of resonances, the relaxation to the ground state through emission of photons, the nonperturbative derivation of the g-factor of the electron, and the stability of matter when the quantized radiation field is included. These and other topics will be covered in the second half of the book. Readers less interested in the classical theory may jump ahead to chapter 12, where the conclusions of chapters 2–11 are summarized and the contents of the quantum part outlined.

A few words on the style are in order. First of all, I systematically develop the theory and discuss some of the most prominent applications. No review is intended. For a subject with a long history, such an attitude looks questionable. After all, what did the many physicists working in that area contribute? To compensate, I include one historical chapter, which as very often in physics is the history as viewed from our present understanding. Since there are excellent historical studies, I hope to be excused. Further, at the end of each chapter I add *Notes and References* intended as a guide to all the material which has been left out. The level of the book is perhaps best characterized as being an advanced textbook. I assume a basic knowledge of Maxwell's theory of electromagnetism and of nonrelativistic quantum mechanics. On the other hand, the central topics are explained in detail and, for the reader to follow the discussion, there is no need of further outside sources. This brings me to the issue of mathematical rigor. In the case of classical electron theory, many claims of uncertain status are in the literature, hardly any numerical work is available, and there are no quantitative experimental verifications, as yet, with the exception of the lifetime of an electron captured in a Penning trap. More than in other fields one has to rely on fixed points in the form of mathematical theorems, which seems to be the only way to disentangle hard facts from "truths" handed down by tradition. For the quantum theory we venture into the nonperturbative regime which by definition requires a certain mathematical sophistication. In a few cases I decided to provide the full proof of the mathematical theorem. Otherwise I usually indicate its basic idea to proceed then with the formal computation. To give always full details would overload the text on an unacceptable scale and, in addition, would be duplication, since mostly the complete argument can be found elsewhere in the literature. Of course, there are stretches, possibly even long stretches, where such a firm foundation is not available and one has to proceed on the basis of limited evidence.

Our introduction might give the impression that all basic problems are resolved, nonrelativistic quantum electrodynamics is in good shape, and one only has to turn to exciting applications. This would be a far too simplistic reading. What I hope is to bring the dynamics of charges and their radiation field properly into focus. Once this point is reached, there are many loose ends. On the theoretical side, to mention only a few of them: on the classical level, the comparison between the true microscopic and approximate particle dynamics could be more precise; a similar program for the relativistic theory of an extended charge is hardly tackled; in the quantum theory the removal of the ultraviolet cutoff at the expense of energy and mass renormalization is still not understood; and the dynamics of many charges remains largely unexplored. Also quantitative experimental confirmation of the effective dynamics of an electron, as given through the Lorentz–Dirac equation on its center manifold, remains on the agenda. The greatest reward would be if my notes encourage further research.