

**Tangents to a conic (or to confocal conics) at right angles meet on a circle.**

By Professor JACK.

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**Note on the Cooling of a Sphere in a mass of well-stirred Liquid.**

By Dr W. PEDDIE.

If the original distribution of temperature be symmetrical about the centre, the equation of conduction has the well-known form

$$\frac{\partial^2 u}{\partial r^2} = \frac{c\rho}{k} \frac{\partial u}{\partial t}, \quad \dots \dots \dots (1)$$

with the condition  $u = v$ ,  $\dots \dots \dots (2)$

$v$  being the temperature; and the initial condition gives

$$u = rf(r), \quad t = 0. \quad \dots \dots \dots (3)$$

If we assume that the rate of loss of heat from the liquid is very small, the condition that the gain of heat by the liquid is equal to the loss of heat by the sphere gives

$$-4\pi a^2 k \left( \frac{\partial v}{\partial r} \right)_{r=a} = M\sigma \left( \frac{\partial v}{\partial t} \right)_{r=a}, \quad \dots \dots \dots (4)$$

where  $a$  is the radius of the sphere,  $M$  is the mass of the liquid, and  $\sigma$  is its specific heat.

By the usual process we find that the solution

$$u = A_0 r + \sum A_m \sin a_m r \cdot e^{-\frac{k}{c\rho} a_m^2 t} \quad \dots \dots \dots (5)$$

suits (1) and (2); and (4) then gives

$$(1 + pa^2 a_m^2) \tan a_m a = a_m a, \quad \dots \dots \dots (6)$$

where  $p = M\sigma/4\pi a^2 c\rho$ .

From (3) we have

$$r[f(r) - A_0] = \sum A_m \sin a_m r. \quad (7)$$

Multiplying by  $a \sin a_m r$  and integrating from  $r=0$  to  $r=a$ , we find (by using (6)), on the right-hand side, the remainder

$$- \sum A_m p a^2 \sin a_m a \sin a_m a.$$

The form of this result at once suggests multiplication of (7) by  $r \sin a_m a$ . The remainder, on integration, is identical with the above. Hence we get

$$\int_0^a r[f(r) - A_0](a \sin a_m r - r \sin a_m a) dr = A_m \int_0^a \sin a_m r (a \sin a_m r - r \sin a_m a) dr.$$

The quantity  $A_0$  is the final temperature, and so we have

$$4\pi c \rho \int_0^a r^2 [f(r) - A_0] dr = M\sigma(A_0 - B),$$

where  $B$  is the initial temperature of the liquid.