BINARY ORBIT PERTURBATIONS AND THE ROTATIONAL ANGULAR MOMENTUM OF STELLAR INTERIORS

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Abstract. Calculations show that a significant variation in the minima of eclipsing binaries should arise for systems where axial precession exists. Several different angular velocity distributions are assumed in order to estimate the expected photometric variation as a function of the model parameters. It is found that the solid body rotation approximation is a reasonable representation unless interiors rotate more rapidly than present models predict.

Plavec (1960) has applied the detailed perturbation theory describing the interaction of close binary components as developed by Kopal (1959) to estimate the periodic variation of the orbital motion of twelve binaries typical of detached, semi-detached, and contact configurations. The perturbations are expressed in terms of apsidal motion, nodal regression, nutation and their cross products. He found that with the exception of the apsidal term, the perturbation periods are short, most less than one year, with amplitudes too small to be readily detected. The purpose of this work is to re-investigate the axial precession or nodal line regression by numerical evaluations of stellar models with the view of estimating the magnitude of observable photometric effects. The technique to be used was proposed originally by Luyten (1943).

For arbitrary axial orientations, the precession of the rotation axes of binary components must be accompanied by the precession of the orbital plane on the invariable plane of the system in order that the total angular momentum be conserved. In the case of an eclipsing binary, this perturbation causes the apparent inclination of the orbit to change with respect to the celestial sphere thereby changing the eclipse minima. The maximum differential inclination, Δi_{max}, of the orbital plane to the invariable plane occurs when the angular momentum vectors of the two components are coplanar and at the same azimuthal angle. In this case,

$$\tan(\Delta i_{\text{max}}) = \frac{J_1 \sin\theta_1 + J_2 \sin\theta_2}{L + J_1 \cos\theta_1 + J_2 \cos\theta_2}, \tag{1}$$

where L is the orbital angular momentum, J_1 and J_2 are the rotational angular momenta of the two components, and θ_1 and θ_2 are the inclination angles of the J_i vectors to the orbital normal. It has been shown by Kopal (1942) and by Hosokawa (1953) that the axial inclinations, θ_i, can be found from an analysis of the velocity curve during eclipse phases. During these phases the so-called rotational disturbance appears asymmetric in both phase and amplitude about conjunction if the eclipsed component rotates about an axis inclined to the orbital normal as projected on the plane of the sky. Since the effect is second order, high quality spectrographic data are required and the system must be free of 'complications'. Two such investigations have been reported. Kopal (1942) found a value of $\theta_1 = 15°$ for the B8 component of

A. Slettebak (ed.), Stellar Rotation, 187–192.

β Persei and Koch and Sobieski (1969) recently reported finding a θ_1 not less than 4° for the B2 component of 68 Herculis. A value of $\theta_i = 15°$ will be adopted for all calculations below.

The angular momentum, J, for a star of radius R in the case of radial symmetry is given by

$$J = \tfrac{8}{3}\pi\Omega_0 \int_0^R \varrho(r)\, w(r)\, r^4\, \mathrm{d}r\,, \tag{2}$$

where Ω_0 is the surface value of the angular velocity, $w(r)$ is the angular velocity distribution, and $\varrho(r)$ is the density distribution with distance from the stellar center. Equation (2) was integrated numerically using the density distributions for the 10 $M_\odot$ and 2.5 $M_\odot$ models of Schwarzschild (1958) combined with the angular velocity distributions published by Roxburgh (1964) and by Clement (1969). Their $w(r)$ distributions based on an electron scattering opacity were coupled with the 10 $M_\odot$ model density distribution while the $w(r)$ distribution for the models involving the Kramer opacity were used with the 2.5 $M_\odot$ density distribution. Since Clement's $w(r)$ distributions are neither axisymmetric nor radially symmetric, a distribution appropriate to an astrographic latitude of 30° was adopted to simplify the calculations. His radiative braked (R.B) and viscous braked (V.B.) cases are considered separately. Calculations were also made for the case of solid body rotation for reference purposes. A surface angular velocity for each mass model and a representative orbital angular momentum for each system investigated were read from a mean curve passed through the basic data given in Table I. For these tabulated systems, solid body rotation and a radius of gyration equal to 0.2753, appropriate to the Eddington standard model, were assumed to calculate the angular momenta listed in column 8. Absolute dimensions are from the Kopal and Shapley Catalogue (1956) and equatorial velocities are those determined by Koch *et al.* (1965). With the boundary condition thus derived the angular momenta for the different angular velocity distributions, calculated by using Equation (2), are listed in Table II. These results were applied, in turn, to the solution of Equation (1). The values of $\Delta i_{\max}$ thus found are tabulated in Table III.

Several conclusions apropos of these calculations can be drawn.

(1) For even moderate axial inclinations, the estimated values for the differential inclination are surprisingly large. A characteristic value and a maximum value for $\Delta i_{\max}$ can be taken as 0.°4 and 1.°4, respectively. These values will be used below to estimate characteristic photometric variations.

(2) Since the orbital angular momentum is significantly greater than the rotational angular momentum, the contributions by the two components are approximately additive. The differential inclination is not a strong function of mass. Although the rotational angular momentum is larger for the more massive stars, this increase is counterbalanced by the larger orbital angular momentum expected in close systems.

(3) The most significant result, however, is that with the exception of the radiative braked model the differential inclination is sensibly model independent. The result

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TABLE I

Basic data for close binary systems

	Sp. C.	Mass ($M_\odot$)	V_{eq} (km/sec)	Radius ($R_\odot$)	$\Omega_0 \times 10^5$ (sec^{-1})	Period (days)	log (J/M)	log $(L/\Sigma M)$
σ Agl	B3	6.8	123	4.2	4.21	1.95	17.435	19.24
	B4	5.4	142	3.3	6.18		17.393	
WW Aur	A3	1.81	41	1.9	3.10	2.53	16.614	18.83
	A3	1.75	39	1.9	3.10		16.614	
U Cep	B8	2.9	310	2.4	18.56	2.49	17.594	18. 70
	G8	1.4	–	2.9			–	
AH Cep	B0.5	16.5	210	6.1	4.95	1.78	17.830	19.21
	B1	14.2	195	5.5	5.09		17.752	
Y Cyg	B0	17.4	146	5.9	3.56	3.00	17.658	19.25
	B0	17.2	148	5.9	3.56		17.658	
68 Her	B2	7.9	119	4.5	3.80	2.05	17.441	18.80
	B5	2.8	90	4.3	3.01		17.310	
RX Her	B9.5	2.75	78	2.1	5.34	1.78	16.937	18.67
	A0.5	2.33	68	1.8	5.43		16.810	
δ Lib	A0	2.6	85	3.5	3.49	2.33	17.196	18.64
	G2	1.1	–	3.5			–	
U Oph	B4	5.30	107	3.4	4.52	1.68	17.283	18.94
	B6	4.65	87	3.1	4.03		17.153	
β Per	B8	5.2	60	3.6	2.39	2.87	17.056	18.66
	K0	1.0	–	3.8			–	
ξ Phe	B6	6.1	100	3.4	4.23	1.67	17.254	18.84
	A0	3.0	–	2.0			–	
V Pup	B1	16.6	180	6.0	4.31	1.45	17.756	19.17
	B3.5	9.8	–	5.3			–	
U Sge	B7	6.7	76	4.1	2.66	3.38	17.215	18.78
	G2	2.0	–	5.4			–	
μ Sco	B1.5	18.0	225	4.8	6.73	1.45	17.755	19.12
	B3	9.3	–	5.3			–	
BH Vir	F9	0.87	90	1.1	11.76	0.82	16.718	17.92
	G2	0.90	90	1.2	10.78		16.756	
RS Vul	B4	4.6	80	3.9	2.95	4.48	17.217	18.78
	F9	1.4	–	5.3			–	

TABLE II

Rotational angular momenta

Mass ($M_\odot$)	log J (cgs units)			
	Solid	Rox.	V.B.	R.B.
2.5	50.60	50.66	50.68	51.36
10.0	51.73	51.92	51.79	52.30

TABLE III

Computed differential inclinations

M_1	M_2/M_1	θ_1	θ_2	log L (cgs)	Δi_{max}			
					Solid	Rox.	V.B.	R.B.
2.5	0.25	15°	0°	52.04	0°.27	0°.31	0°.32	1°.42
2.5	1.00	15°	0°	52.39	0°.12	0°.14	0°.14	0°.63
2.5	1.00	15°	15°	52.39	0°.24	0°.28	0°.29	1°.27
10.0	0.25	0°	15°	53.10	0°.02	0°.03	0.°03	0°.12
10.0	0.25	15°	0°	53.10	0°.32	0.°48	0°.36	1°.10
10.0	0.25	15°	15°	53.10	0°.34	0.°51	0°.39	1°.22
10.0	1.00	15°	0°	53.46	0°.14	0°.21	0°.16	0°.48
10.0	1.00	15°	15°	53.46	0°.27	0°.42	0°.31	0°.96

found for solid body rotation becomes a fair general approximation and one must expect that only for rapid core rotation will model delineation through observation be possible.

The photometric variation of the eclipse minima can be related directly to the differential inclination through the geometric eclipse depth, p_0. In the notation of Russell and Merrill (1952),

$$\cos i = r_g(1 + kp_0)$$

while the brightness at minimum is

$$l_0 = (1 - \alpha_0 L_s)$$

for occultations and

$$l_0 = (1 - \tau\alpha_0^{tr} L_g)$$

for transits. Since $\alpha_0 = \alpha_0(k; p_0)$, the variation of the eclipse minima, expressed in magnitudes is given by

$$\Delta m = -2.5 \log\left\{\frac{l_0(k, p_0 \pm \Delta p_0)}{l_0(k, p_0)}\right\}, \tag{3}$$

where

$$\Delta p_0 = -\frac{\sin i}{r_s}\Delta i.$$

Table IV lists line results found by applying Equation (3) to several representative model solutions as well as to the two 'observed' systems β Per and 68 Her. The last column in Table IV lists estimates of the nodal regression period calculated using the method and relevant constants given by Plavec.

TABLE IV

Photometric variations

Model	i	r_s/r_g	r_s	L_s	Δi	Δm^{oc}	Δm^{tr}	P_1/P_0
Assumed	84°	0.85	0.30	0.85	$+1°.4$	0.181		103
$M_1 = 4\ M_{\odot}$					$+1°.4$	0.052		
$M_2 = 1\ M_{\odot}$					$-0°.4$	-0.048		
					$-1°.4$	-0.171		
Assumed	84°	0.85	0.30	0.15	1°.4		0.087	12
$M_1 = 10\ M_{\odot}$					0°.4		0.020	
$M_2 = 2.5\ M_{\odot}$					$-0°.4$		-0.019	
					$-1°.4$		-0.082	
β Per	80°.5	0.74	0.20	0.72	1°.4	0.133		64
					0°.4	0.039		
					$-0°.4$	-0.039		
					$-1.°4$	-0.142		
68 Her	78°.6	0.80	0.28	0.19	1°.4		0.058	52
					0°.4		0.017	
					$-0°.4$		-0.017	
					$-1°.4$		-0.058	

Notes: Limb-darkening of 0.6 assumed in all cases.
$L_s + L_g \equiv 1$.
P_1/P_0 = nodal regression period expressed in units of the orbital period for assumed synchronism of rotation.

One may conclude that the magnitude of the photometric variation is adequately large to be detected by photoelectric means. Since the expected precessional period can be quite short for semi-detached and contact systems, each series of eclipse observations must be treated separately. This applies both to the photometric data necessary for determining the depth of the eclipse minimum and to the velocity data required to define the instantaneous axial inclination.

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