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# Some lacunary and random Fourier series

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A subset $\Delta$ of the dual of a compact abelian Hausdorff topological group $G$ is called Sidon iff every (complex-valued) continuous function on $G$ whose Fourier coefficients vanish off $\Delta$ has absolutely convergent Fourier series. In the first part of this thesis a weighted analogue is studied: if $W$ is a complex-valued function on $\Delta$, $\Delta$ is called a $W$-Sidon set iff for every continuous function $f$ whose Fourier coefficients vanish off $\Delta$, $W\hat{f}$ is (absolutely) summable.

In Chapter 2, $W$-Sidon sets are characterised in many different ways and some variants of these characterisations are shown to lead back to Sidon sets. Also $W$-Sidon sets $\Delta$ for which $W \in l^2(\Delta)$, are shown to behave similarly to finite sets in the Sidon theory.

In Chapter 3, $W$-Sidon sets $\Delta$ with $W \notin l^2(\Delta)$ and which are not Sidon are constructed for the circle group. These sets provide new counterexamples to a multiplier problem and also show that $W$-Sidon sets need not be $\Lambda(p)$ for any $p \geq 1$.

In Chapter 4 the algebra of all $W$'s making a set $W$-Sidon is investigated and necessary and sufficient conditions for the set to be Sidon, cast in terms of it. Also $W$-Sidon sets are characterised using $p$-Sidon sets (another recent generalisation of Sidon sets).

In Chapter 5, further analytic properties of $W$-Sidon sets (including some which come from multiplier theory) are pursued and a necessary condition on the growth of $W^2$ obtained.

The second part of this thesis, Chapter 6, strengthens existing

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results on random Fourier series. For the circle group $\mathbb{T}$ we prove the existence of a (complex-valued) continuous function $f$ on $\mathbb{T}$ and a sequence $\omega$ of plus or minus ones on $\mathbb{Z}$ (the dual of $\mathbb{T}$) for which $\omega\hat{f}$ is not the Fourier transform of a function in any Lebesgue space $L^p(\mathbb{T})$ for $p > 2$, but which nevertheless satisfies

$$(*) \qquad \sum_{n\neq 0} \left|n^{-\alpha}\hat{f}(n)^{\beta}\right| < \infty \text{ for each } \alpha, \beta > 0 .$$

A related result is also proved: there is a function $f$, integrable over $\mathbb{T}$, and a sequence $\omega$ as above for which $\omega\hat{f}$ is not the Fourier transform of a measure, although $f$ again satisfies (*).

An appendix and list of references relate the results in this thesis to the literature.

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