SOME LIE ADMISSIBLE ALGEBRAS

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Several studies have been made to obtain larger classes of non-associative algebras from classes of algebras with a known structure. Thus, we have right alternative algebras (**2**)* and non-commutative Jordan algebras (**6**), (**7**), (**8**), and (**9**). These algebras are defined by a subset of the set of identities of the algebras from which they derive their names. Also, Albert (**1**), among others has studied Jordan admissible algebras. This paper is concerned with algebras which are related to Lie algebras in that they satisfy some of the identities of a Lie algebra and are Lie admissible. Theorem 2 answers a question raised by Albert in (**1**).

1. For an algebra $\mathfrak{A}$, the algebra $\mathfrak{A}^{(-)}$ is defined as the same vector space as $\mathfrak{A}$, but with a multiplication given by $[x, y] = xy - yx$ where juxtaposition denotes multiplication in $\mathfrak{A}$. The algebra $\mathfrak{A}^{(-)}$ is clearly an anticommutative algebra. $\mathfrak{A}$ is said to be Lie admissible if $\mathfrak{A}^{(-)}$ is a Lie algebra, that is,

$$[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$$

and

$$[x, y] + [y, x] = 0.$$

An algebra $\mathfrak{A}$ is Lie admissible (**1**, p. 573) if and only if

$$R_{[x,y]} - L_{[x,y]} = [R_x - L_x, R_y - L_y] \tag{1}$$

where R_a and L_a are right and left multiplications by a in the algebra $\mathfrak{A}$.

Let Φ be a field of characteristic $\neq 2, 3$. One way to generalize Lie algebras would be to study an algebra $\mathfrak{A}$ which satisfies the identity

$$x(yz) + y(zx) + z(xy) = 0 \tag{2}$$

or equivalently

$$L_yL_x + R_xL_y + R_{xy} = 0. \tag{3}$$

The identity (2) does not seem restrictive enough to allow a satisfactory structure theory. If we take the algebra over Φ with basis s, t, u, and v and define $su = tu = u^2 = vu = s$, $sv = tv = uv = v^2 = t$, and all other products

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*See Kleinfeld (**5**) for an extensive bibliography of works on right alternative algebras.

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equal 0 we have an algebra which satisfies (2) but will not satisfy either of the equivalent identities

$$(4) \qquad (xy)z + (yz)x + (zx)y = 0,$$

$$(5) \qquad L_{xy} + L_y R_x + R_x R_y = 0.$$

For (4) implies $3x^2x = 0$ or $x^2x = 0$, but $v^2v = tv = t \neq 0$.

Thus (2) is not sufficient to make an algebra power-associative. However, if (2) and (4) are assumed, then $x^3 = x^2x = xx^2 = 0$ and (2) implies $x^2(xx) + x(xx^2) + x(x^2x) = x^2x^2 = 0 = (x^2x)x$. To complete a proof that an algebra satisfying (2) and (4) is power-associative, that is, $x^\lambda x^\mu = x^{\lambda+\mu}$, observe that $x^n = x^{n-1}x = 0$ for $n \geqslant 3$.

The example given above also shows that an algebra may satisfy (2), but not be Lie admissible. For $[t, [u, v]] + [u, [v, t]] + [v, [t, u]] = [t, t - s] + [u, -t] + [v, s] = 0 + s - t \neq 0$. An algebra that satisfies (3) and (5) is Lie admissible. This is easily seen by rewriting (3) with x and y interchanged to obtain $R_{[x,y]} = [L_x, L_y] + R_y L_x - R_x L_y$. Likewise, $L_{[x,y]} = -[R_x, R_y] + L_x R_y - L_y R_x$. Subtraction yields (1).

An anticommutative algebra is flexible, that is $(xy)x = x(yx)$. As a first step towards generalizing Lie algebras it appears natural to study flexible algebras which satisfy (2) and (4). Such an algebra need not be a Lie algebra as shown by an algebra with basis u, v over Φ and with multiplication given by $v^2 = u$ and $u^2 = uv = vu = 0$.

A flexible algebra satisfying (3) and (5) is a Jordan admissible algebra and by Schafer **(7)** is a non-commutative Jordan algebra. To see that such an algebra is Jordan admissible, write the condition for Jordan admissibility in the form $[R_x + L_x, R(x^2) + L(x^2)] = 0$, **(1**, p. 574**)**. Let $x = y$ in (3) and (5), then $R(x^2) = -L_x^2 - R_x L_x$ and $L(x^2) = -L_x R_x - R_x^2$. In a flexible algebra R_x and L_x commute, so the commutator above is equal to 0.

In what follows, the full strength of (2), (4), and flexibility is not needed. The algebras we shall be concerned with are power-associative, flexible and Lie admissible. That these conditions are, in general, less restrictive may be seen by examining an associative algebra.

2. In this section extensive use will be made of the known structure of semi-simple Lie algebras of characteristic 0. The material that will be used may be found in **(3)**, **(4)**, or **(11)**.

Albert has shown **(1**, p. 576**)** that an algebra is a flexible Lie admissible algebra if and only if

$$(6) \qquad R_{[x,y]} = [R_x, R_y - L_y].$$

Let $D_y = D(y)$ represent the linear transformation $R_y - L_y$, then (6) may be written as $R(xD_y) = [R_x, D_y]$ or in a flexible Lie admissible algebra D_y is a derivation.

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As in the study of Lie algebras we shall say that an element x of an algebra $\mathfrak{A}$ belongs to the characteristic root α of the linear transformation D_y, if for some integer $h \geqslant 1$, $x(D_y - \alpha I)^h = 0$.

LEMMA. *Let $\mathfrak{A}$ be a flexible Lie admissible algebra over an arbitrary algebraically closed field Ω of characteristic 0. For $x, y, z \in \mathfrak{A}$, and $\alpha, \beta \in \Omega$ roots of D_z, if x, y belong to α, β respectively, then xy and yx belong to $\alpha + \beta$ whenever $\alpha + \beta$ is a root of D_z and equal 0 otherwise.*

Since D_z is a derivation, $xy(D_z - (\alpha + \beta)I) = x(D_z - \alpha I)y + x(y(Dz - \beta I))$ and the lemma may be proved by induction just as for Lie algebras.

THEOREM 1. *Let $\mathfrak{A}$ be a flexible, Lie admissible algebra over an arbitrary algebraically closed field Ω of characteristic 0. If $\mathfrak{A}^{(-)}$ is a semi-simple Lie algebra, then $\mathfrak{A}$ is a direct sum of simple, flexible, Lie admissible algebras.*

Assuming that $\mathfrak{A}$ is in addition power-associative, Weiner **(10)** obtained the same conclusion. Since very few results have been obtained for non-power-associative algebras we include a proof of Theorem 1.

Since $\mathfrak{A}^{(-)}$ is semi-simple we may write $\mathfrak{A}^{(-)} = \mathfrak{B}_1 \oplus \ldots \oplus \mathfrak{B}_t$, where the $\mathfrak{B}_i$, $i = 1, 2, \ldots, t$, are simple Lie algebras. This decomposition of $\mathfrak{A}^{(-)}$ will decompose the underlying vector space of $\mathfrak{A}$ into a vector space direct sum of subspaces $\mathfrak{A}_i$, $i = 1, 2, \ldots, t$, where the subspace $\mathfrak{A}_i$ is the same vector space as $\mathfrak{B}_i$. We shall show that the $\mathfrak{A}_i$ are ideals of $\mathfrak{A}$ and that $\mathfrak{A}$ is a direct sum of the $\mathfrak{A}_i$.

In each $\mathfrak{B}_i$ fix a Cartan subalgebra $\mathfrak{H}_i$ and relative to $\mathfrak{H}_i$ a classical basis of $\mathfrak{B}_i$. Let x, y be arbitrary elements of $\mathfrak{A}_k$. Since $xy \in \mathfrak{A}$, we may write $xy = \sum b_i$, $b_i \in \mathfrak{A}_i$ and $\mathfrak{B}_i$. If $b_j \neq 0$, then there is an $x_j \in \mathfrak{B}_j$ such that $b_j x_j - x_j b_j \neq 0$, for otherwise Ωb_j would be an ideal of the simple algebra $\mathfrak{B}_j$. For any $x_j \in \mathfrak{B}_j$, $(xy)D(x_j) = \sum b_i D(x_j) = (xD(x_j))y + x(yD(x_j)) = b_j D(x_j)$ since $\mathfrak{A}^{(-)}$ is a direct sum of the $\mathfrak{B}_i$. For the same reason $(xy)D(x_j) = 0$, if $j \neq k$. Thus for all $j \neq k$, $b_j D(x_j) = [b_j, x_j] = 0$ and by the remark above $b_j = 0$. Hence $xy = b_k$ and we have shown that $\mathfrak{A}_k$ is a subalgebra of $\mathfrak{A}$.

It should be noted that the notion of belonging to a root in $\mathfrak{A}$, as defined, is the same as belonging to a root in $\mathfrak{A}^{(-)}$. Furthermore, since the non-zero root spaces of $\mathfrak{B}_i$ are one dimensional, the non-zero root spaces in the algebra $\mathfrak{A}_i$ are also of dimension one.

Let $x \in \mathfrak{A}_j$, $y \in \mathfrak{A}_k$, $j \neq k$ and in addition suppose that x belongs to a non-zero root of $\mathfrak{H}_j \subset \mathfrak{A}_j$. Thus, there is an $h \in \mathfrak{H}_j$ such that $xD_h = \alpha x$ where $\alpha \neq 0$ and $\alpha \in \Omega$. Write $xy = \sum b_i$. Apply D_h to obtain $(xy)D_h = (xD_h)y + x(yD_h) = \alpha xy = [b_j, h]$. Since $\alpha \neq 0$, $xy \in \mathfrak{A}_j$. Let $h \in \mathfrak{H}_j$ and let x_β be the basis element belonging to the root β. Also, let $y \in \mathfrak{A}_k$, $k \neq j$. The quantity $x_\beta y \in \mathfrak{A}_j$. For $x_{-\beta}$ belonging to the root $-\beta$, $(x_\beta y)D(x_{-\beta}) = [x_\beta, x_{-\beta}]y = h_\beta y$ which also is an element of $\mathfrak{A}_j$. From the distributive laws, we have that each $\mathfrak{A}_j$ is an ideal of $\mathfrak{A}$. It follows that $\mathfrak{A} = \mathfrak{A}_1 \oplus \ldots \oplus \mathfrak{A}_t$. Each $\mathfrak{A}_i$ is simple, for each (two-sided) ideal of $\mathfrak{A}$ is an ideal of $\mathfrak{A}^{(-)}$.

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COROLLARY. *Let $\mathfrak{A}$ be a flexible algebra over a field Φ of characteristic* 0. *If $\mathfrak{A}^{(-)}$ is the direct sum of central simple Lie algebras, then $\mathfrak{A}$ is a direct sum of simple, flexible, Lie admissible algebras.*

The decomposition of $\mathfrak{A}^{(-)}$ decomposes $\mathfrak{A}$ into a vector space direct sum $\mathfrak{A} = \mathfrak{A}_1 + \ldots + \mathfrak{A}_t$. Let Ω be the algebraic closure of Φ. Apply the theorem to $\mathfrak{A}_\Omega = \mathfrak{A}_{1\Omega} + \ldots + \mathfrak{A}_{t\Omega}$. Each $\mathfrak{A}_{i\Omega}$ is a simple, flexible, Lie admissible algebra and $\mathfrak{A}_{i\Omega}\mathfrak{A}_{j\Omega} = 0$, $i \neq j$. If $u \in \mathfrak{A}$ and $x \in \mathfrak{A}_i$, then $ux, xu \in \mathfrak{A}_{i\Omega}$. Also, ux, $xu \in \mathfrak{A}$ and therefore $ux, xu \in \mathfrak{A}_i$.

THEOREM 2. *Let $\mathfrak{A}$ be a flexible, power-associative algebra, over an arbitrary, algebraically closed field Ω of characteristic* 0. *If $\mathfrak{A}^{(-)}$ is a simple Lie algebra, then $\mathfrak{A}$ is a simple Lie algebra isomorphic to $\mathfrak{A}^{(-)}$.*

Since $\mathfrak{A}^{(-)}$ is a simple Lie algebra over an algebraically closed field of characteristic 0, we may fix a classical basis of $\mathfrak{A}^{(-)}$. Let $\bar{\mathfrak{H}}$ be the fixed Cartan subalgebra of $\mathfrak{A}^{(-)}$ and $\mathfrak{H}$ the corresponding subspace of $\mathfrak{A}$. If x, y belong to the roots $\alpha(\mathfrak{H})$ and $\beta(\mathfrak{H})$, then xy and yx belong to the root $(\alpha + \beta)(\mathfrak{H})$ (provided $\alpha(\mathfrak{H}) + \beta(\mathfrak{H})$ is a root). Thus $[x, y] \in \Omega z$, where z belongs to the root $(\alpha + \beta)(\mathfrak{H})$. In a simple Lie algebra the non-zero root spaces are one dimensional and it follows that $xy, yx \in \Omega z$.

The elements of $\mathfrak{H}$ belong to the root zero and hence $\mathfrak{H}$ is a power-associative subalgebra of $\mathfrak{A}$. Also, $\mathfrak{H}$ is commutative, for $[h, h'] = 0$, $h, h' \in \bar{\mathfrak{H}}$. The proof of the theorem will follow readily once we further determine the structure of $\mathfrak{H}$. We shall do this in several steps.

(i) The algebra $\mathfrak{H}$ is a nil algebra. For suppose $h \in \mathfrak{H}$ is not nilpotent, then the subalgebra $\mathfrak{I}$ of $\mathfrak{H}$ generated by h is an associative non-nilpotent algebra and has an idempotent. Let e be this idempotent, $e \in \bar{\mathfrak{H}}$. For an element x of the fixed basis of $\mathfrak{A}^{(-)}$ belonging to a non-zero root, $\alpha(\mathfrak{H})$, there exists $\alpha, \beta \in \Omega$ such that $ex - xe = \alpha x$ and $ex = \beta x$. This implies that $xe = (\beta - \alpha)x$. We use the identity $(R_e + L_e - I)(R_e - L_e) = 0$ (**1**, (11)) which is valid in a power-associative algebra of characteristic 0. For $x \neq 0$, this identity applied to x yields $2\alpha\beta = \alpha^2 + \alpha$. Thus $\alpha = 0$ or $\alpha = 2\beta - 1$. Since $\mathfrak{A}^{(-)}$ is simple, x may be further restricted so that $\alpha \neq 0$. Let $y \neq 0$ belong to the root $-\alpha(\mathfrak{H})$. Then $[x, y] \neq 0$ and $[x, y] \in \mathfrak{H}$. Not both xy and yx can equal 0. Set $ey = \beta' y$, $[e, y] = \alpha' y = -\alpha y$ and $ye = (\beta' - \alpha')y = (\beta' + \alpha)y$. From the lemma xy belongs to the root $\beta + \beta'$, relative to e, if $\beta + \beta'$ is a root. Also, $xy \in \mathfrak{H}$, that is, xy is either 0 or xy belongs to the root zero. Likewise yx is either 0 or belongs to the root zero. Thus, either $\beta + \beta' = 0$ or $(\beta' + \alpha) + (\beta - \alpha) = 0$. In either case $\beta' = -\beta$. As for α and β we may obtain the result, $\alpha' = -\alpha = 2\beta' - 1$. Combining $-\alpha = -2\beta - 1$ and $\alpha = 2\beta - 1$ we reach a contradiction.

(ii) The algebra $\mathfrak{H}$ is a nil algebra of bounded index; that is, there is a $t > 1$ such that $h^t = 0$ for all $h \in \mathfrak{H}$. For $h \in \mathfrak{H}$ the algebra $\mathfrak{I}$ generated by h is an associative nil algebra. Thus $\mathfrak{I}^{k+1} = 0$ for $k \leqslant \dim \mathfrak{I} \leqslant \dim \mathfrak{H}$. In particular $h^t = 0$ for $t = \dim \mathfrak{H} + 1$.

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(iii) The algebra $\mathfrak{H}$ is such that $\mathfrak{H}^2 = 0$. Let $x^t = 0$ for all $x \in \mathfrak{H}$, $t \geqslant 3$. Let n be the least positive integer such that $3n \geqslant t$. Since $\mathfrak{A}$ is power associative, $x^{3n} = 0$. Let $h \in \mathfrak{H}$ and set $g = h^n$. Also, let x be a basal element of $\mathfrak{A}^{(-)}$ belonging to a non-zero root relative to $\overline{\mathfrak{H}}$. From the lemma and the known structure of $\mathfrak{A}^{(-)}$, there exist $\alpha, \beta, \gamma, \delta \in \Omega$ such that $gx - xg = \alpha x$, $gx = \beta x$, $g^2x - xg^2 = \gamma x$, and $g^2x = \delta x$. From this we may write $xg = (\beta - \alpha)x$ and $xg^2 = (\delta - \gamma)x$.

The flexible law implies

$$(g^2g^2)x - g^2(g^2x) + (xg^2)g^2 - x(g^2g^2) = 0.$$

Since $g^3 = 0$, $g^4 = 0$; and it follows that $-\delta^2x + (\delta - \gamma)^2x = 0$. Thus $\gamma = 0$ or $\gamma = 2\delta$, since $x \neq 0$. The flexible law also states $(gg^2)x - g(g^2x) + (xg^2)g - x(g^2g) = 0$. However, $g^3 = 0$, so $-\beta\delta x + (\beta - \alpha)(\delta - \gamma)x = 0$. If $\gamma \neq 0$, then $\alpha = 2\beta$. We may also write, $(gg)x - g(gx) + (xg)g - x(gg) = 0$. This implies that $\gamma x - \beta^2 x + (\beta - \alpha)^2 x = 0$. If $\gamma \neq 0$, this is a contradiction. Thus $\gamma = 0$ and $[g^2, x] = 0$. By linearity $[g^2, u] = 0$, for all $u \in \mathfrak{A}^{(-)}$. Since $\mathfrak{A}^{(-)}$ is simple, $g^2 = 0$, that is, $h^{2n} = 0$.

By repeated application of the above, we have $h^t = h^{2n} = h^{2m} = \ldots = 0$ with $t \geqslant 2n \geqslant 2m \geqslant \ldots \geqslant 2$. Equality between two components will occur only when the left-hand exponent is 2 or 4. We will now examine the situation when $h^4 = 0$. Set $hx - xh = \lambda x$, $hx = \mu x$, $h^2x - xh^2 = \nu x$, and $h^2x = \xi x$. If we apply the flexible law to the triple h^2, h^2, x, we have $(-2\nu\xi + \nu^2)x = 0$ and $\nu = 0$ or $\nu = 2\xi$.

Since $h^5 = 0$, if $\nu \neq 0$, $(h^3h^2)x - h^3(h^2x) + (xh^2)h^3 - x(h^2h^3) = 0$ yields $xh^3 + h^3x = 0$. Substitute h^3x for $-xh^3$ after using the flexible law with the triple h^2, h, x. This gives $4h^3x - \nu(2\mu - \lambda)x = 0$. This result may be used to simplify $(hh^3)x - h(h^3x) + (xh^3)h - x(h^3h) = 0$ and we find $2\mu = \lambda$. This implies $h^3x - xh^3 = 0$.

If $\nu = 0$, $h^2x = xh^2$. The first equation in the preceding paragraph may be rewritten as $[h^3, h^2x] = 0$ or $[h^3, \xi x] = 0$. Also $h^3x - h(h^2x) + (xh^2)h - xh^3 = 0$ or $[h^3, x] - \lambda\xi x = 0$. It follows that $[h^3, x] = 0$.

By linearity, $[h^3, u] = 0$ for all $u \in \mathfrak{A}$. The simplicity of $\mathfrak{A}^{(-)}$ implies $h^3 = 0$. We may now conclude $h^2 = 0$ for all $h \in \mathfrak{H}$. To complete the proof that $\mathfrak{H}^2 = 0$, we observe that $(h + h')^2 = 0$ and $[h, h'] = 0$.

We may now determine the remainder of the structure of $\mathfrak{A}$. Let $x \neq 0$ be a basis vector of $\mathfrak{A}$ and $(\mathfrak{A}^{(-)})$ belonging to a non-zero root of $\mathfrak{H}$. For $h \in \mathfrak{H}$, there exist $\alpha, \beta \in \Omega$ such that $hx - xh = \alpha x$, $hx = \beta x$, and $xh = (\beta - \alpha)x$. From $h^2 = 0$ and $(hh)x - h(hx) + (xh)h - x(hh) = 0$ it follows that $\alpha = 0$ or $2\beta = \alpha$. More precisely, it is always true that $2\beta = \alpha$. For there is an $h' \in \mathfrak{H}$ such that $[h', x] \neq 0$, since x belongs to a non-zero root. In case $\alpha = 0$ we may write $(h'h)x - h'(hx) + (xh)h' - x(hh') = 0 = [xh, h'] = [x, h']$ or $\beta = 0$.

Next let x, y be basis vectors of $\mathfrak{A}$ belonging to non-zero roots $\alpha(\mathfrak{H})$ and $\rho(\mathfrak{H})$ respectively. If $\alpha(\mathfrak{H}) + \rho(\mathfrak{H})$ is not a root, then by the lemma $xy = yx = 0$.

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Otherwise let $z \neq 0$ belong to the root $\alpha(\mathfrak{H}) + \rho(\mathfrak{H})$ and then there exist $\sigma, \tau \in \Omega$ such that $xy - yx = \sigma z$, $xy = \tau z$, and $yx = (\tau - \sigma)z$. From the above, if $[h, x] = \alpha x$ and $[h, z] = \omega z$, then $2hx = -2xh = \alpha x$ and $2hz = -2zh = \omega z$. We use the flexible law once again to write $(hx)y - h(xy) + (yx)h - y(xh) = 0$. Thus $\frac{1}{2}\alpha xy - \tau hz + (\tau - \sigma)zh + \frac{1}{2}\alpha yx = \frac{1}{2}[\alpha\tau - \omega\tau - (\tau - \sigma)\omega + \alpha(\tau - \sigma)]z = 0$. Since $z \neq 0$, $\alpha = \omega$ or $2\tau - \sigma = 0$. The quantity $2\tau - \sigma$ is independent of which $h \in \mathfrak{H}$ is used. If $2\tau - \sigma \neq 0$ then $\alpha(h) = \omega(h)$ and the root forms $\alpha(\mathfrak{H})$ and $\omega(\mathfrak{H})$ are equal. This implies y belongs to the root zero which is a contradiction, thus $2\tau - \sigma = 0$.

This completes the multiplication table of $\mathfrak{A}$ and $\mathfrak{A}$ is seen to be a Lie algebra. The mapping $u \to \frac{1}{2}u$ gives the isomorphism between $\mathfrak{A}$ and $\mathfrak{A}^{(-)}$.

COROLLARY. *Let $\mathfrak{A}$ be a flexible, strictly power-associative algebra over a field Φ of characteristic* 0. *If $\mathfrak{A}^{(-)}$ is a central simple Lie algebra, then $\mathfrak{A}$ is a simple Lie algebra isomorphic to $\mathfrak{A}^{(-)}$.*

Let Ω be the algebraic closure of Φ. By the theorem, $\mathfrak{A}_\Omega$ is a Lie algebra. Since $\mathfrak{A}$ is an algebra contained in $\mathfrak{A}_\Omega$, $\mathfrak{A}$ is anticommutative and satisfies the Jacobi identity. Thus $\mathfrak{A}$ is a Lie algebra.

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