

## IS THERE A COUNTABLE OMEGA-UNIVERSAL LOGIC?

ALEXANDER C. PASEAU 

Faculty of Philosophy, University of Oxford  
and

FELIX WEITKÄMPER 

Institut für Informatik, Ludwig-Maximilians-Universität München

**Abstract.** Some informal arguments are valid, others are invalid. A core application of logic is to tell us which is which by capturing these validity facts. Philosophers and logicians have explored how well a host of logics carry out this role, familiar examples being propositional, first-order and second-order logic. Since natural language and standard logics are countable, a natural question arises: is there a countable logic guaranteed to capture the validity patterns of any language fragment? That is, is there a countable omega-universal logic? Our article philosophically motivates this question, makes it precise, and then answers it. It is a self-contained, concise sequel to ‘Capturing Consequence’ by A.C. Paseau (RSL vol. 12, 2019).

**§1. Introduction.** Some informal arguments are valid, others invalid. One of logic’s roles is to tell us which is which. A logic succeeds in this role to the extent that it underwrites the validity of valid informal arguments and the invalidity of invalid ones. These informal arguments may be drawn from mathematics or elsewhere.

Consider a fragment of natural language consisting of sentences  $s_1, \dots, s_n$ . These sentences stand in logical relations to one another: perhaps, say,  $s_1$  and  $s_2$  imply  $s_3$  but not  $s_4$ ;  $s_5$  and  $s_6$  imply one another; and so on. The pattern of these implications and non-implications might be called the sentences’ *implicational structure* or *implicational network*.

In its core application, the job of a logic is to capture implicational structure. To do so, the logic associates to each sentence  $s_i$  in the target domain a formal sentence  $\sigma_i$  of its language.<sup>1</sup> It discharges its function of capturing implicational structure perfectly just when: a subset  $S$  of  $\{s_1, \dots, s_n\}$  implies a sentence  $s$  if and only if the corresponding subset  $\Sigma$  of formal sentences implies, according to the logic, the sentence  $\sigma$  corresponding to  $s$ . For example,  $\{s_1, s_2\}$  implies  $s_3$  just when  $\{\sigma_1, \sigma_2\}$  implies  $\sigma_3$ .

Some logics are better suited to this role than others. One reason for the success of first-order logic is its ability to capture the implicational structure of large swathes of

---

Received: August 23, 2024.

2020 *Mathematics Subject Classification*: Primary 03A05.

*Key words and phrases*: applications of logic, formalization, universal logic, abstract logic, conservative translation, consequence isomorphism.

<sup>1</sup> As should be clear, in this informal introduction we are using the word ‘domain’ in the everyday sense, not the model-theoretic one.



informal language, especially mathematical language. An obvious failing of a logic, as far as this role is concerned, is if it is too small to capture the relevant implicational structure. As a limiting case, take a logic with just one sentence. A one-sentence logic is incapable of capturing the implicational structure of any target domain consisting of two logically inequivalent sentences. By the same token, a logic consisting of finitely many sentences fails to capture the implicational structure of any target domain consisting of infinitely many logically inequivalent sentences. In each case, we can tell from the outset—merely on cardinality grounds—that no logic of that kind is up to the job.

In certain cases, we may be able to guarantee that a logic can capture the implicational structure of target domains of a certain kind. Suppose we are interested in target domains consisting of fewer than 10 sentences, and that some logic  $\mathcal{L}$  provably captures the implicational structure of any domain consisting of fewer than 10 sentences. Then  $\mathcal{L}$  is guaranteed to capture the implicational structure of any of the target domains of interest.

A one-sentence logic, we observed, cannot capture the implicational structure of a domain of two logically inequivalent sentences; nor can a finite-sentence logic do the same vis-à-vis a domain of infinitely many logically inequivalent sentences. There is a metaphorical but very useful way of describing this common failing. In each case, the logic is not *capacious* enough to model the respective implicational structures. It simply does not contain not enough ‘room’ to do so. Conversely, a logic that captures the implicational structure of any domain consisting of fewer than 10 sentences has enough ‘room’ to capture any of these. More generally, one logic may be more capacious or roomy than another by dint of its ability to capture more implicational structures.

A natural question is whether a single logic can do the job of capturing *any* informal target domain. Philosophers and linguists have made considerable progress on this question, closely examining parts of technical and less technical language, for example the domain of arithmetic (think of Peano Arithmetic) or temporal locutions (think of tense logic). This progress notwithstanding, we are still far from the final or even near-final answer. A closely related question is whether there is a logic so capacious that, whatever implicational structure a target domain might have, the logic can accommodate it. That is, is there a logic guaranteed to be capacious enough so far as informal arguments go? And if so, how large must such a logic be? Although a logic may have other failings, if it is maximally capacious then it can capture *any* implicational structure. Conversely, whatever its other virtues, if a logic is not maximally capacious in this sense, it may fail to capture *some* implicational structures.

The next section will make these motivating remarks and our question more precise. The remaining sections will then establish some results that answer it.

**§2. The set-up.** Informal arguments are expressed in natural language, including its technical parts. For example, algebraic geometry, although highly technical, is an informal practice: algebraic geometers give arguments and express claims in mathematical English, French, etc. The same goes for topologists, cosmologists, biologists, lawyers, and so on.

We may call the target domain (algebraic geometry, cosmology, etc.)  $T$ . The set of natural-language sentences is countably infinite, since the lexicon of any natural

language is finite and its formation rules allow for sentences of arbitrarily large finite length. This assumption is completely standard in both philosophy and linguistics. It also applies to natural language augmented with technical vocabulary. It follows that  $T$ , the target domain, consists of a countable (i.e., finite or countably infinite) set of sentences. We may also assume that  $T$  has a determinate implicational structure that logic tries to capture. This structure may be opaque and part of the role of the logic may be to elucidate it. Moreover,  $T$ 's implicational structure may only crystallise if we theorise about  $T$  in a certain way.

Suppose we use logic  $\mathcal{L}$ , with set of sentences  $Sen(\mathcal{L})$  and consequence relation  $\models_{\mathcal{L}}$ , to try to capture  $T$ 's implicational structure. If  $\mathcal{L}$  is to do this, we must formalise the sentences of  $T$  in  $\mathcal{L}$ , that is, use a formalisation *Form* which we can think of as a function mapping the elements of  $Sen(T)$  to  $Sen(\mathcal{L})$ . Logic  $\mathcal{L}$  perfectly captures the required implicational structure just when, for any subset  $S$  of  $Sen(T)$  and element  $s$  of  $Sen(T)$ ,  $S$  implies  $s$  if and only if  $Form(S) \models_{\mathcal{L}} Form(s)$ .

Now let's think about logics, each logic being characterised by a set of sentences and consequence relation on that set. When comparing logics, the informal idea we would like to capture is that one logic is more capacious than another if any implicational structure found in the latter can be found in the former. To that end, suppose  $\mathcal{L}_1$  and  $\mathcal{L}_2$  are two logics with respective sentence sets  $Sen(\mathcal{L}_1)$  and  $Sen(\mathcal{L}_2)$  and consequence relations  $\models_{\mathcal{L}_1}$  and  $\models_{\mathcal{L}_2}$ . The map  $j : Sen(\mathcal{L}_1) \rightarrow Sen(\mathcal{L}_2)$  is a *conservative translation* from  $\mathcal{L}_1$  to  $\mathcal{L}_2$  just when, for all  $\Gamma \subseteq Sen(\mathcal{L}_1)$  and  $\phi \in Sen(\mathcal{L}_1)$ ,

$$\Gamma \models_{\mathcal{L}_1} \phi \text{ if and only if } j(\Gamma) \models_{\mathcal{L}_2} j(\phi).$$

We write  $\mathcal{L}_1 \leq \mathcal{L}_2$  when there is a conservative translation from  $\mathcal{L}_1$  to  $\mathcal{L}_2$ , thereby defining a partial order on logics, and  $\mathcal{L}_1 < \mathcal{L}_2$  when there is a conservative translation from  $\mathcal{L}_1$  to  $\mathcal{L}_2$  but none from  $\mathcal{L}_2$  to  $\mathcal{L}_1$ .<sup>2</sup>

As an illustration, consider  $PL_n$ , propositional logic with  $n \geq 1$  sentence letters and  $PL_{\omega}$ , propositional logic with a countable infinity of sentence letters. The logics  $PL_n$  and  $PL_{\omega}$  have a countable and expressively adequate set of connectives and are equipped with their usual consequence relation. It is easy to check that for natural numbers  $m$  and  $n$ ,  $PL_m \leq PL_n$  if and only if  $m \leq n$ , and that  $PL_n < PL_{\omega}$  for all natural numbers  $n$ .

Predicate logics provide another illustration. Let  $FOL_{\omega}$  be first-order logic with a countable infinity of variables, constants, predicate and function symbols of all arities, a countable and expressively adequate set of truth-functional connectives, standard formation rules and its standard consequence relation. Perhaps surprisingly, it may be shown that there is a map  $j : Sen(PL_{\omega}) \rightarrow Sen(FOL_{\omega})$  that is (a) a bijection, (b) a conservative translation and (c) has an inverse that is also a conservative translation.<sup>3</sup> Next, let  $SOL_{\omega}$  be second-order logic with a countable infinity of first-order variables and for each arity a countable infinity of second-order predicate and function variables of that arity, a countable infinity of constants, predicate and function symbols of all arities, a countable and expressively adequate set of truth-functional connectives, standard formation rules and its standard consequence relation. In contrast to the result relating  $PL_{\omega}$  and  $FOL_{\omega}$ , we note the strict inequality  $FOL_{\omega} < SOL_{\omega}$ . The fact

<sup>2</sup> The notion of conservative translation is usually applied to deductive systems. Here, it is adapted to semantic consequence relations.

<sup>3</sup> See Paseau [2] for the proof and Paseau [3] for follow-up philosophical discussion.

that  $\text{FOL}_\omega \leq \text{SOL}_\omega$  follows trivially from the fact that  $\text{FOL}_\omega$  is a sublogic of  $\text{SOL}_\omega$ , so that the inclusion map from  $\text{Sen}(\text{FOL}_\omega)$  to  $\text{Sen}(\text{SOL}_\omega)$  is a conservative translation. The fact that there is no conservative translation from  $\text{SOL}_\omega$  to  $\text{FOL}_\omega$  follows from the fact that  $\text{FOL}_\omega$  is compact, whereas  $\text{SOL}_\omega$  is not.<sup>4</sup>

The natural question prompted by these examples, raised at the end of Paseau [2], is whether there is a logic that is as capacious as possible, as far as countable domains go. If so, what is it? Could it be any of the well-known and well-studied logics, such as  $\text{SOL}_\omega$  for example? And what is its minimum size—can it be countable or must it be uncountable? To make these questions more precise, let us say that logic  $\mathcal{L}$  is countable just when  $\text{Sen}(\mathcal{L})$  is countable (i.e., countably infinite or finite). And say that logic  $\mathcal{UL}_\omega$  is  $\omega$ -universal just when, for any other countable logic  $\mathcal{L}$ ,  $\mathcal{L} \leq \mathcal{UL}_\omega$ . So, our question is this: is there a countable  $\mathcal{UL}_\omega$ ?

In the remainder of this paper, we show that there is *no* countable  $\mathcal{UL}_\omega$ . In fact, we show that the cardinality of any  $\omega$ -universal logic must exceed  $2^\omega$ . Under the assumption that there is no cardinal properly between  $2^\omega$  and  $2^{2^\omega}$ , a consequence of the Generalised Continuum Hypothesis, we can even conclude that the smallest  $\omega$ -universal logic has a sentence set of precisely size  $\aleph_2$ .

**§3. Definitions.** To address these questions, we begin by noting three standard features of a consequence relation for a logic. Here,  $A$  and  $B$  are sets of sentences of the logic in question, and  $\phi$  a sentence.

1. If  $\phi \in A$  then  $A \models \phi$ . (Any set of sentences implies any sentence of that set.)
2. If  $A \subseteq B$  and  $A \models \phi$  then  $B \models \phi$ . (If a set of sentences implies a sentence then so does any of its supersets.)
3. If  $B = \{\phi : A \models \phi\}$  then  $B = \{\phi : B \models \phi\}$ . (The closure of the closure of a set of sentences is its closure.)

These features noted, it will be useful to change perspective slightly to facilitate the formal discussion. We now take as our central notion the closure of the set of sentences. We write  $\langle A \rangle = \{\phi : A \models \phi\}$  and call  $\langle \rangle$  the *hull function*.<sup>5</sup> The hull function is a function from sets of sentences of a logic to other sets of sentences (specifically: the former's closure). Hull functions and consequence relations are interdefinable:  $\langle A \rangle = \{\phi : A \models \phi\}$  and  $A \models \phi$  if and only if  $\phi \in \langle A \rangle$ . The hull function may therefore be referred to as the *consequence structure* of the logic.

For our general results, we take a logic to consist of a set of sentences, and a hull function, which represents its consequence relation. The three properties noted above are then captured by the following.

**DEFINITION 1.** We take a logic or logical system to be a pair  $(L, \langle \rangle)$  of a set  $L$  and a hull function  $\langle \rangle : \mathcal{P}(L) \rightarrow \mathcal{P}(L)$  satisfying the following conditions:

1. For all  $A \subseteq L$ ,  $A \subseteq \langle A \rangle$ .
2. For all  $A \subseteq B \subseteq L$ ,  $\langle A \rangle \subseteq \langle B \rangle$ .
3. For all  $A \subseteq L$ ,  $\langle \langle A \rangle \rangle = \langle A \rangle$ .

<sup>4</sup> See Paseau [2] for details.

<sup>5</sup> This should not be confused with the notion of a Skolem hull.

These three conditions are minimal abstract conditions to impose on any logic of the sort we are interested in. The size of the logic  $(L, \langle \rangle)$  is the size of  $L$ , i.e., the size of the logic's set of sentences. Our abstract perspective allows these sentences to be whatever we would like them to be.<sup>6</sup>

As we would like to investigate relations between logical systems, we will need an appropriate notion of embedding between them. In the present context, this is given naturally as follows.

**DEFINITION 2.** *If  $(X, \langle \rangle)$  and  $(Y, \langle \rangle)$  are logical systems, an embedding (of logical systems) of  $(X, \langle \rangle)$  in  $(Y, \langle \rangle)$  is an injective map  $f$  from  $X$  to  $Y$  such that for all  $A \subseteq X$ ,  $\langle f(A) \rangle \cap f(X) = f(\langle A \rangle)$ . A bijective embedding is called an isomorphism of logical systems.*

It is easy to check that an embedding is a conservative translation as defined in the previous section.

**§4. The cardinality of  $\omega$ -universal logics.** We begin by noting that, as already demonstrated by Paseau [2],  $\omega$ -universal logics do exist. In fact, we easily obtain an  $\omega$ -universal logic by taking the disjoint union of all possible countable consequence structures. Since a consequence structure is given as a map from the power set of a countable set to itself, there can only be at most  $(2^\omega)^{2^\omega} = 2^{2^\omega}$  consequence structures on a countable set. Consider a logic with  $2^{2^\omega}$  copies of  $\omega$  such that the consequence structure on the  $\xi$ th copy, for  $\xi < 2^{2^\omega}$ , is given by the  $\xi$ th of the  $2^{2^\omega}$  such possible structures. This logic, the disjoint union of all the countable consequence structures, is an  $\omega$ -universal logic of cardinality  $2^{2^\omega}$ .

We proceed to show that this construction is best possible if there is no cardinal properly between  $2^\omega$  and  $2^{2^\omega}$ , a consequence of the Generalised Continuum Hypothesis, and that unconditionally there is no  $\omega$ -universal logic of cardinality less than or equal to  $2^\omega$ .

We proceed in two steps. First we investigate how many mutually non-isomorphic countable logics there actually are. We will see that the naïve upper bound of the number of maps from the power set of  $\omega$  to itself is indeed sharp: there are in fact  $2^{2^\omega}$  mutually non-isomorphic countable logics. Then, the main result follows by simple cardinal arithmetic.

We start with a lemma taken from Harzheim [1], where it is a special case of Theorem 9.1.25. Let  $(\mathfrak{D}, \leq)$  be a partially ordered set. Then an *antichain* of  $\mathfrak{D}$  is a subset  $\mathfrak{A}$  of  $\mathfrak{D}$  such that for no  $a, b \in \mathfrak{A}$ ,  $a \leq b$  or  $b \leq a$ .

**LEMMA 1.** *The power set  $(\mathcal{P}(\omega), \subseteq)$  of  $\omega$  has an antichain of cardinality  $2^\omega$ .*

Since the proof is rather elementary, we include it here.

*Proof.* Partition  $\omega$  into two disjoint infinite sets  $M$  and  $N$ , and enumerate their elements as  $(m_1, m_2, \dots)$  and  $(n_1, n_2, \dots)$  respectively. Now for any subset  $S \subseteq \omega$ , consider the set

$$\omega_S := \{m_i \mid i \in S\} \cup \{n_i \mid i \notin S\}.$$

<sup>6</sup> For instance, despite the name, the set of sentences can include open formulas.

Then for any two distinct subsets  $S$  and  $T$  of  $\omega$ , neither  $\omega_S \subseteq \omega_T$  nor  $\omega_T \subseteq \omega_S$ . Thus,  $\{\omega_S \mid S \subseteq \omega\}$  is an antichain of cardinality  $2^\omega$ .  $\square$

We note that this result does not require the use of any high-powered principles such as the Axiom of Choice, since the partition of  $\omega$  into  $M$  and  $N$  is definable (e.g., take  $M$  to be the even numbers and  $N$  the odd ones). In any case, we shall make use of any standard set-theoretic principles we need freely in the metatheory, as usual.

We can use this antichain to pin down the number of mutually non-isomorphic countable logics.

**PROPOSITION 1.** *There are  $2^{2^\omega}$  mutually non-isomorphic countable logics.*

*Proof.* Let  $\mathfrak{A}$  be an antichain of  $(\mathcal{P}(\omega), \subseteq)$  of cardinality  $2^\omega$ . Then  $\mathfrak{A}$  has itself  $2^{2^\omega}$  distinct subsets  $\mathfrak{X} \subseteq \mathfrak{A}$ .

Consider a logic whose set of sentences  $L$  is defined as  $\omega \cup \{\phi\}$ , with  $\phi \notin \omega$ . Thus the sentences in  $L$  are the natural numbers (think of these as representing a countably infinite collection of sentences) plus some other sentence, which for suggestiveness we have called  $\phi$ .

For every  $\mathfrak{X} \subseteq \mathfrak{A}$  and set of sentences  $A \subseteq L$ , let

$$\langle A \rangle_{\mathfrak{X}} := \begin{cases} A \cup \{\phi\} & \exists X \in \mathfrak{X} : X \subseteq A \\ A & \text{otherwise.} \end{cases}$$

Because  $\mathfrak{A}$  is an antichain, the logics  $(L, \langle \rangle_{\mathfrak{X}})$  are distinct for distinct  $\mathfrak{X}$ . For suppose that  $\mathfrak{X}_1$  and  $\mathfrak{X}_2$  are distinct subsets of the antichain  $\mathfrak{A}$ , such that without loss of generality  $A$  is an element of  $\mathfrak{X}_1$  but not  $\mathfrak{X}_2$ . Then  $\langle A \rangle_{\mathfrak{X}_1} = A \cup \{\phi\}$ . However, because  $\mathfrak{A}$  is an antichain, there is no  $X \in \mathfrak{X}_2 : X \subseteq A$ , and so  $\langle A \rangle_{\mathfrak{X}_2} = A$ . Thus the  $(L, \langle \rangle_{\mathfrak{X}})$  are distinct for distinct  $\mathfrak{X}$ .

For the rest of the argument, note that every  $(L, \langle \rangle_{\mathfrak{X}})$  is a countable logic. We have shown that because  $\mathfrak{A}$  is an antichain, the  $(L, \langle \rangle_{\mathfrak{X}})$  are distinct for distinct  $\mathfrak{X}$ . Since, however, there are only  $2^\omega$  mappings from  $L$  to itself, each isomorphism class can only have at most  $2^\omega$  distinct members. Finally, since  $2^\omega < 2^{2^\omega}$ , the pigeonhole principle implies that there are indeed  $2^{2^\omega}$  isomorphism types.  $\square$

We can now conclude the argument.

**THEOREM 1.** *There is no  $\omega$ -universal logic of cardinality not exceeding  $2^\omega$ . If there is no cardinal properly between  $2^\omega$  and  $2^{2^\omega}$ , the smallest  $\omega$ -universal logic has cardinality  $2^{2^\omega}$ .*

*Proof.* Let  $\mathcal{L}$  be a logic of cardinality not exceeding  $2^\omega$ . Then  $\mathcal{L}$  has at most  $(2^\omega)^\omega = 2^\omega$  countable subsets. Therefore, no more than  $2^\omega$  mutually non-isomorphic countable logics can be embedded in  $\mathcal{L}$ . It follows that  $\mathcal{L}$  is not  $\omega$ -universal as there are  $2^{2^\omega}$  mutually non-isomorphic countable logics.

As we have shown there to exist an  $\omega$ -universal logic of cardinality  $2^{2^\omega}$ , this implies that when there is no cardinal properly between  $2^\omega$  and  $2^{2^\omega}$ , the smallest  $\omega$ -universal logic has cardinality  $2^{2^\omega}$ .  $\square$

By way of conclusion, we note that if the Generalised Continuum Hypothesis holds, the smallest  $\omega$ -universal logic has cardinality  $\aleph_2 = 2^{2^\omega}$ . This is because under it, the first two uncountable cardinals are  $\aleph_1 = 2^\omega$  and  $\aleph_2 = 2^{2^\omega}$ .

**§5. Conclusion.** We have answered, in the negative, the question of whether a countable  $\omega$ -universal logic exists. No countable logic, including well-known and well-studied ones such as  $\text{PL}_\omega$ ,  $\text{FOL}_\omega$ ,  $\text{SOL}_\omega$ , can be  $\omega$ -universal. This is a limitative result on the power of countable logics.

We end with two sets of philosophical comments on this result. The first consists of three brief comparisons with other well-known limitative results. The comparisons will help clarify the nature of our result.

- (i) Unlike Gödelian incompleteness, the present result has nothing to do with deductive systems. We have shown there is no logic of countable size into which all countable logics embed. Logics are here thought of as sets of sentences with a consequence relation, which may or may not be capturable by a deductive system. The notion of an embedding is the corresponding one; it too is not deductively constrained. Unlike Gödelian incompleteness, our result is therefore purely on the semantic side of things.
- (ii) Our result belongs to the abstract study of logical systems. It is therefore more general than, say, an independence result such as that the Continuum Hypothesis is independent of the axioms of ZFC. It is also more general than a generalisation of that same independence result to a class of set theories. This is because our result is about any countable logical system, not just set theories. It is also, of course, not an independence result. It reveals that no countable logic can be ‘as capacious is possible’, where capaciousness is understood as the ability to embed a countable logic.
- (iii) Another famous limitative result is the undecidability of first-order logic. Our result differs from this one in two main ways. First, it is not about decidability or computable notions. In our treatment, logical systems are individuated by their sets of sets of sentences and their consequence structure. These are not constrained to be computable, or semi-computable, etc. Second, our result is about *all* countable logical systems. So it is much more general than a fact about a particular one such as first-order logic, or any restricted class of countable systems.

The second comment concerns the result’s import. We have shown that no countable logic can capture the implicational structure of *any* countable target domain. It would be a mistake—a quantifier-shift fallacy—to think this means that no countable logic can capture the implicational structure of *some* given target domain. Given a target domain (e.g., a mathematical or scientific theory ‘in the wild’), it might be that a well-known countable logic such as first-order logic can completely capture its implicational structure. More strongly, such a logic may even be able to capture the implicational structure of any target domain of interest, perhaps because the target domains found in mathematics, the sciences, etc. exhibit a limited range of implicational structures. But what this article proves is that we cannot identify a single countable logic  $\mathcal{L}$  capacious enough to capture the implicational structure of *any* countable target domain, whatever it might be. Any sufficiently capacious  $\mathcal{L}$  must have more sentences than the cardinality of the continuum.

**Acknowledgements.** We would like thank an RSL referee for comments.

## BIBLIOGRAPHY

- [1] Harzheim, E. (2005). *Ordered Sets*. New York: Springer.
- [2] Paseau, A. C. (2019). Capturing consequence. *The Review of Symbolic Logic*, **12**, 271–295.
- [3] ———. (2021), Propositionalism. *The Journal of Philosophy*, **118**, 430–449.

FACULTY OF PHILOSOPHY

UNIVERSITY OF OXFORD

OXFORD, UK

*E-mail:* [alexander.paseau@philosophy.ox.ac.uk](mailto:alexander.paseau@philosophy.ox.ac.uk)

INSTITUT FÜR INFORMATIK

LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MUNICH, GERMANY

*E-mail:* [felix.weitkaemper@lmu.de](mailto:felix.weitkaemper@lmu.de)