Put shortly, the proof is this :—

FIGURE 18 (*b*).

Let ABCD be the position of maximum area. Then the flat triangle AOD = BOC and their angles at O are equal,

$$\therefore \; OA . OD = OB . OC.$$

∴ A, B, C, D are concyclic.

It obviously applies to any quadrilateral.

The following is not open to the same objection :—

Let ABC′D′, ABC″D″ be two equal areas on opposite sides of the maximum position.

Bisect D′D″ and C′C″ and let AD, BC meet in O. Then AD′OD″, BC′OC″ are kites having OD′ = OD″, OC′ = OC″, and D′C′ = D″C″.

∴ triangles OD′C′, OD″C″ are congruent.

∴ after taking away the common ∠ D″OC′, ∠ D′OD″ = C′OC″

∴ their halves ∠ AOD″ and ∠ BOC′ are equal.

Also, since triangles OD′C′, OD″C″ are congruent and the quadrilaterals are equal,

$$\therefore \; OD'ABC' = OD''ABC''.$$

Take away OD″ABC′ and the kites are proved equal in area and so are their halves, AOD″ and BOC′.

It follows that $\quad OA . OD'' = OB . OC'.$

This is true for every such pair of equal quadrilaterals, and therefore for the coincident pair, when D′D″ coincide on OA and C′C″ coincide on OB.

∴ for the maximum position

$$OA . OD = OB . OC,$$

i.e., ABCD is cyclic.

An application of Sturm's Functions.

By J. D. HÖPPNER.

https://doi.org/10.1017/S0013091500032065 Published online by Cambridge University Press