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ANALYSIS OF GLACIER RUN-OFF AND METEOROLOGICAL OBSERVATIONS*

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ABSTRACT. Linear models of the relationships between meteorological observations and the flow of river Tungnaá at the western margin of glacier Vatnajökull were investigated by means of spectral analysis and estimation of the impulse response. Most of the variation of Tungnaá is confined to the lowest frequencies and the diurnal variations. The temperature has most effect on the rapid variations around 1 cycle/day whereas the largest coherences with the precipitation are in the lowest frequencies. The wind explains over 20% of the variations in the frequency range from 0–1 cycle/day, but this is partly due to its coherence with the precipitation. The time lag between changes in the temperature and the river is about 2 h, but the time lag between precipitation and the river is longer. Analysis of longer records of daily observations from Þjórsá shows that the coherence of the run-off and temperature increases at frequencies too low to be estimated from these data. At frequencies over 1 cycle/day most of the observed variations of the river cannot be explained by means of a linear relationship with the meteorological series.

RÉSUMÉ. *L'analyse des débits glaciaires et des observations météorologiques.* Des relations linéaires entre les observations météorologiques et le débit de la rivière Tungnaá sur la bordure Ouest du glacier Vatnajökull furent recherchées par le moyen d'analyse spectrale et par estimation de la réponse. Le plupart des variations de la Tungnaá se limitent aux très basses fréquenes et aux variations diurnes. La température a le maximum d'effet sur les variations rapides d'environ un cycle par jour, tandis que les plus grandes cohérences avec les précipitations se trouvent dans les plus basses fréquences. Le vent explique plus de 20% des variations dans la gamme de fréquence entre 0 et 1 cycle par jour, mais c'est en partie due à sa liaison avec les précipitations. Le retard entre les changements de température et ceux de la rivière est d'environ deux heures, mais le temps de réponse entre les précipitations et le débit de la rivière est plus long. L'analyse d'enregistrement ancien d'observations quotidiennes venant de Þjórsá montre que le lien entre les débits et la température augmente à des fréquences trop faibles pour être estimées à partir de ces données. A des fréquences supérieures à un cycle par jour, la plupart des variations observées de la rivière ne peuvent pas être expliquées par une relation linéaire avec les séries météorologiques.

ZUSAMMENFASSUNG. *Analyse von Gletscher-Abfluss und meteorologischen Beobachtungen.* Die linearen Beziehungen zwischen meteorologischen Beobachtungen und der Wasserführung des Flusses Tungnaá am Westrand der Vatnajökull wurden durch spektrale Analyse und durch Schätzung der Impulsreaktion untersucht. Der grösste Teil der Schwankungen im Tungnaá beschränkt sich auf den Bereich der niedrigsten Frequenzen und der Tagesvariationen. Die Temperatur beeinflusst von allem die kurzdauernden Schwankungen im Bereich von 1 Zyklus pro Tag. Die Niederschläge hingegen äussern sich grösstenteils nur mit niedrigsten Frequenzen. Über 20% der Variationen im Frequenzbereich 0–1 Zyklus pro Tag kann durch Wind erklärt werden, jedoch liegt dies zum Teil daran, dass Wind und Niederschläge miteinander gekoppelt sind. Temperaturveränderungen machen sich mit etwa 2 Stunden Verzögerung im Fluss bemerkbar, Niederschläge aber erst nach einem längeren Zeitraum. Eine Analyse längerer Aufzeichnungen von täglichen Beobachtungen am Fluss Þjórsá zeigt, dass sich der Zusammenhang zwischen Wasserführung und Temperatur bei Frequenzen verstärkt, die zu niedrig sind, als dass sie aus unseren Daten abgeschätzt werden könnten. Bei Frequenzen über 1 Zyklus pro Tag können die meisten beobachteten Schwankungen des Flusses nicht durch lineare Beziehung zu den meteorologischen Daten erklärt werden.

INTRODUCTION

The climate at Tungnaárjökull and Jökulheimar, as in the whole of southern Iceland, is markedly maritime with considerable cloudiness, heavy precipitation, mild winters and cool summers. In such conditions the heat balance of glaciers can be expected to be more dependent upon meteorological factors than in continental climates where the radiation is most important as shown by Lang (1968). On Hoffellsjökull in the most maritime part of Vatnajökull at its south-eastern margin the global radiation only accounts for 10–40% of the ablation, increasing with the altitude, as discussed by Ahlmann and Thorarinsson (1938), but on Bægisárjökull in the central northern Iceland at a higher altitude the global radiation accounts for about 54% of the ablation according to Björnsson (in press). No direct observations of global radiation were carried out at Jökulheimar.

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The impact of any particular meteorological factor may vary rapidly with time, but during a time interval long enough to enable direct observations of the change in the volume of the glacier, several meteorological variables will have contributed significantly to the ablation. It is then difficult to obtain a sufficient number of observations to separate empirically the effect of each component. A large proportion of the melt water is carried away by rivers. In favourable conditions river flow can be measured with considerable accuracy over long periods of time. By the use of numerical spectral analysis we can estimate how much of the variation of the river can be explained as linear functions of meteorological observations. We have applied this procedure to observations of the river Tungnaá at the western margin of Vatnajökull and meteorological observations from the station at Jökulheimar which is close by the outlet (Fig. 1). A previous description of observations from Tungnaá and meteorological conditions was given by Sumarlidason (1965).

Location

Bárðarbunga (2 000 m) and Háabunga (1 700 m) are the two main ice centres of the western part of Vatnajökull from which several broad but short outlet glaciers descend down to the south-west and west to the central Iceland plateau (Fig. 1). The snouts of those glaciers

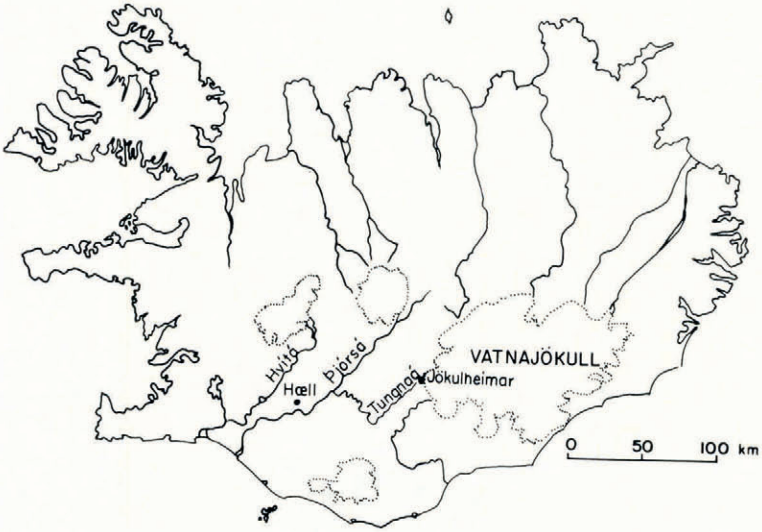

Fig. 1. Iceland. The location of Jökulheimar at the western margin of Vatnajökull and the main glacial melt-water streams in south-west Iceland.

reach down to an elevation of 700–1 000 m, increasing gradually in height from south to north. No marginal creep has been recorded in those glaciers but all of them seem to move by surges at intervals of some decades as discussed by Thorarinsson (1964, 1969). Figure 2 shows the snout of Tungnaárjökull, which is 190 km² in area according to a map of 1946, but the area of each outlet glacier is changeable depending on their interplay and their stage in the surge cycle. The last surge of Tungnaárjökull reached the ice margin in the year 1945 when it advanced 1.5–2 km (Freysteinsson, 1969). Since then Tungnaárjökull has been continuously receding. Figure 2 shows the position of the ice margin in 1946 and 1960 according to aerial photographs of those years. It also shows a rough estimation of the ice margin in 1970. Since 1955 the average retreat of Tungnaárjökull has been about 75 m/year

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and the average thinning of Vatnajökull about 60–80 cm/year in water equivalent (Sigbjarnarson, 1971).

The Jökulheimar meteorological station (Fig. 2) is located at lat. 64° 18′ N., long. 18° 15′ W., at an elevation of 675 m on the edge of a lava plateau with some separated hills and N.E.–S.W. trending ridges, built up of pillow lava and volcanic tuff, reaching a maximum elevation of 750 m within a distance of 4 km. To the south from Jökulheimar there is a 4 km wide outwash plain in the front of the glacier snout. The lava plateau and the outwash plain are surrounded by 800–1 000 m high N.E.–S.W. trending mountain massifs. A narrow ridge, Jökulgrindur, trends to the north-east from Jökulheimar. The surge of Tungnaárjökull in 1945

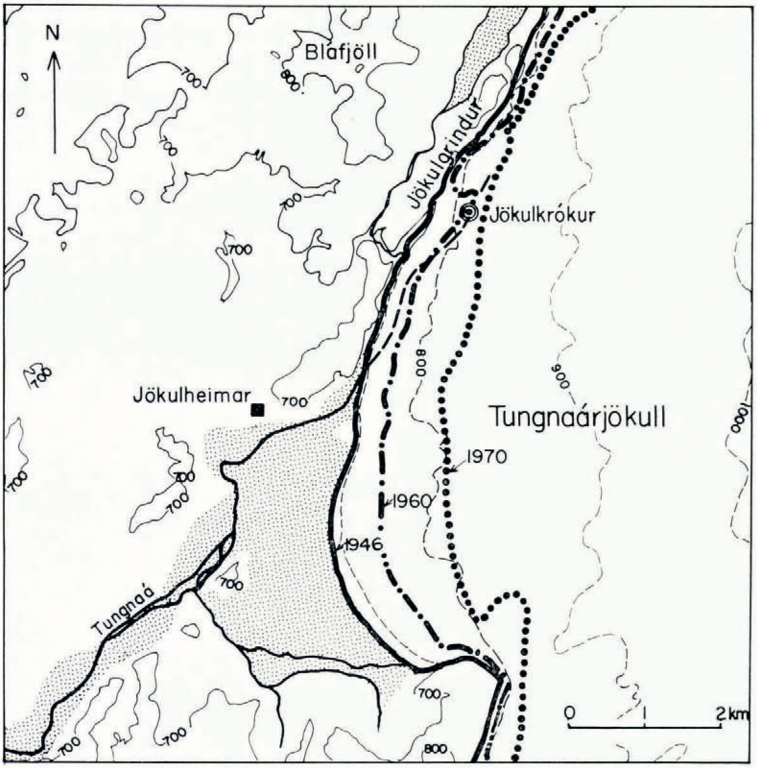

Fig. 2. The snout of Tungnaárjökull, showing the location of Jökulheimar meteorological station and Tungnaárkrókur water-level recorder.

ceased there but reached somewhat further down on the outwash plain. In 1946 the trunk of Tungnaá river issued from the glacier at the south-east corner of Jökulgrindur. The retreat of Tungnaárjökull after 1945 began on the outwash plain, but the glacier has been gradually leaving the Jökulgrindur ridge. Thus the open channel of Tungnaá river has every year been extended farther towards the north-east along the south-east slopes of Jökulgrindur. In 1970 there was only a 200–300 m wide ice bridge back across Tungnaá river along Jökulgrindur, about 5 km to the north-east from Jökulheimar. Tungnaá river used to flow in braided channels on the outwash plain providing no cross-section suitable for measurements. But in 1965 a single river channel appeared in Jökulkrókur (Fig. 2), where the bedrock consists of rather loose pillow lava, providing an opportunity for continuous discharge measurements.

The measurements

Since 1963 a meteorological station has been operated at Jökulheimar each summer. Observations of wind, cloud cover and dew point are carried out 6 times each day at 9, 12, 15, 18, 21 and 24 o'clock. The precipitation is measured twice a day at 9 and 18 o'clock and the temperature is measured on a self-registering thermometer. The measurements are performed with standard equipment except the wind velocity and the cloud cover which are estimated by the weather reporter, Mr Pétur Sumarlidason.

In the numerical analysis 4 values per day were used, at 3, 9, 15 and 21 o'clock. The values of the precipitation were obtained by interpolation and the values of wind velocity cloud cover and dew point at 3 o'clock were estimated from the observations and information from Mr Sumarlidason. In August 1966 a self-registering water-level recorder was put up at Jökulkrókur (Fig. 2). The observations of the water level were occasionally disturbed by pieces of ice from the glacier. The effect on the records was estimated and eliminated and as these disturbances did not last long, the errors incurred are not large.

In late summer 1968 it was noticed that the discharge partly began to run farther east as the glacier retreated and observations comparable with previous records could no longer be obtained.

We selected for analysis the periods when the water-level measurements continued relatively undisturbed. The following periods were included:

16 August–5 September 1966;
28 June–27 August 1967;
7 July–14 August 1968.

Figure 3 shows a sample of our data. A water-level recorder does not provide absolute values of the discharge, but no rating curve was available for the water-level recorder because of too few reliable measurements. In our analysis we used the following equation which is taken from Leopold and others (1964).

$$Q(t) = d(t)^{2.5} \qquad (1)$$

where $d(t)$ is the observed water level and $Q(t)$ is supposed to be proportional to the discharge.

Linear models and spectral analysis

A linear model of the relationship between the river $Q(t)$, and a meteorological variable, $X(t)$, can be written

$$Q(t) = \int_{-\infty}^{\infty} H(\tau)\, X(t-\tau)\, \mathrm{d}\tau + Z(t) \qquad (2)$$

or, if many meteorological variables are included,

$$Q(t) = \sum_i \int_{-\infty}^{\infty} H_i(\tau)\, X_i(t-\tau)\, \mathrm{d}\tau + Z(t). \qquad (3)$$

The effect of $X(t-\tau)$ on $Q(t)$ depends upon the time interval, τ, between them and is described by the weight function, $H(\tau)$, which is often called the impulse response.

It would obviously make good sense to use only past and present values of the meteorological series in explaining $Q(t)$. An apparently straightforward method of doing this would be to calculate the least squares estimates of the $H(k)$'s in

$$Q(j) = \sum_i \sum_{k=0}^{m} H_i(k)\, X_i(j-k).$$

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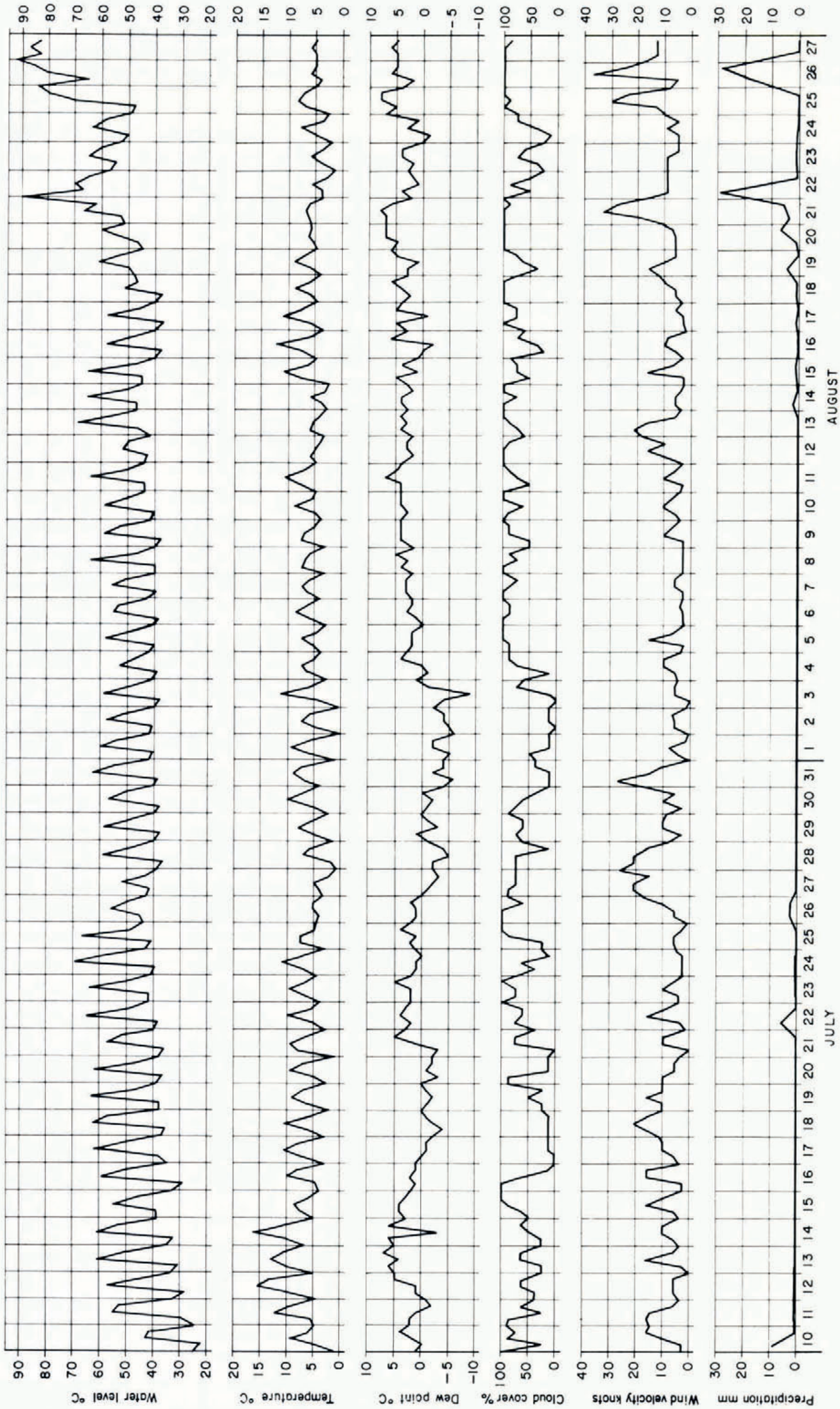

Fig. 3. A sample of the observations analysed in this study.

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Studies of run-off where special forms of $H_i(k)$ were estimated in this way were made by Lang (1968) and Mathews (1964). We have, largely for computational convenience, used an estimation procedure where both past and future values of $H(\tau)$ enter.

The main obstacle in the interpretation of estimates of the impulse response is that their statistical properties are awkward. The estimates at different values of k are heavily correlated and it is difficult to assess the accuracy of the estimates. Here the values of the estimates of $H(k)$ at $k < 0$ give some idea about the accuracy because we know that they should be zero. In view of the difficulties attached to the use of estimates of the impulse response we have based much of our investigation on spectral analysis. The computations involved in our estimates of $H(k)$ are mostly the same as in the spectral estimates.

Our data consist of time series. The main characteristic of spectral analysis is that the variation of the series is divided into frequency bands. This is familiar in the case of annual and diurnal variations, but here random variations are also transformed into the frequency domain and described as functions of frequencies or wavelengths. Individual series are investigated by means of the power spectrum, $S(u)$, where u denotes the frequency which will be given in cycles/day. The power spectrum shows how the variance is distributed among the frequency bands. The comparison of two series is also carried out separately for each frequency band by means of the cross-spectrum. An estimate of the phase difference, $\phi(u)$, is obtained and this can be converted into the time lag, $\Delta t = \phi(u)/u$, between the series at the corresponding wavelengths. Another function, called the "coherence", is estimated for each frequency band and shows the proportion of the variance of the series which can be attributed to a linear relationship between them. The coherence takes values between zero and one and is one in the case of a perfect linear relationship. It is analogous to a squared correlation coefficient.

The sample properties of estimates of power spectra, phases and coherences have been worked out for stationary Gaussian series. For non-coherent Gaussian series the median of our coherences is about 0.08 and 95% of the values would be $<$0.28 for the temperature, humidity and the river flow, but somewhat larger for the other series because of the interpolations. As however our series are neither stationary nor Gaussian we shall not quote any confidence levels for our results. Estimates of the phase from non-coherent series are uniformly distributed over the whole cycle. Each of our estimates is statistically almost independent of the others. A regular pattern in the phase estimates therefore indicates a correlation between the series.

When two series, X_i and X_j, are both coherent with a third series, X_k, the coherences of X_i and X_j may be large even when no independent relationship exists between them. The partial power spectra and cross-spectra of series X_i and X_j are based on the variations of these series after all variation which can be accounted for by a linear relationship with X_k have been eliminated. A more detailed discussion of the estimation procedures applied in this study is given by Gudmundsson (1970).

Results

Power spectra

Figure 4 shows the estimates of the power spectra of our series at frequencies between 0 and 2 cycles/day. All series were divided by their standard deviation before the spectral analysis was performed.

The values of the wind and the cloud cover at 03.00 h were usually obtained by interpolation between the values at 21.00 h and 09.00 h. This entails a reduction of the power of these series at the higher frequencies, but no bias is introduced in the estimates of phases and coherences. The estimates of the power spectrum of the precipitation are meaningless at frequencies over 1 cycle/day as only two observations were carried out per day.

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If each observation is independent of the others, the power spectrum is flat. Our power spectra are more than ten times larger in the lowest frequency band than in the higher frequencies. Normally this implies that large variations from the mean usually last considerably longer than the interval between the observations. The diurnal variation produces a large peak at 1 cycle/day in the power spectrum of the river and the temperature. Small peaks also appear at this frequency in the power spectra of wind and humidity, but there is no sign of a diurnal variation in the precipitation nor in the cloud cover. Apart from the diurnal variation about 60% of the variance of the river is confined to the lowest frequency band.

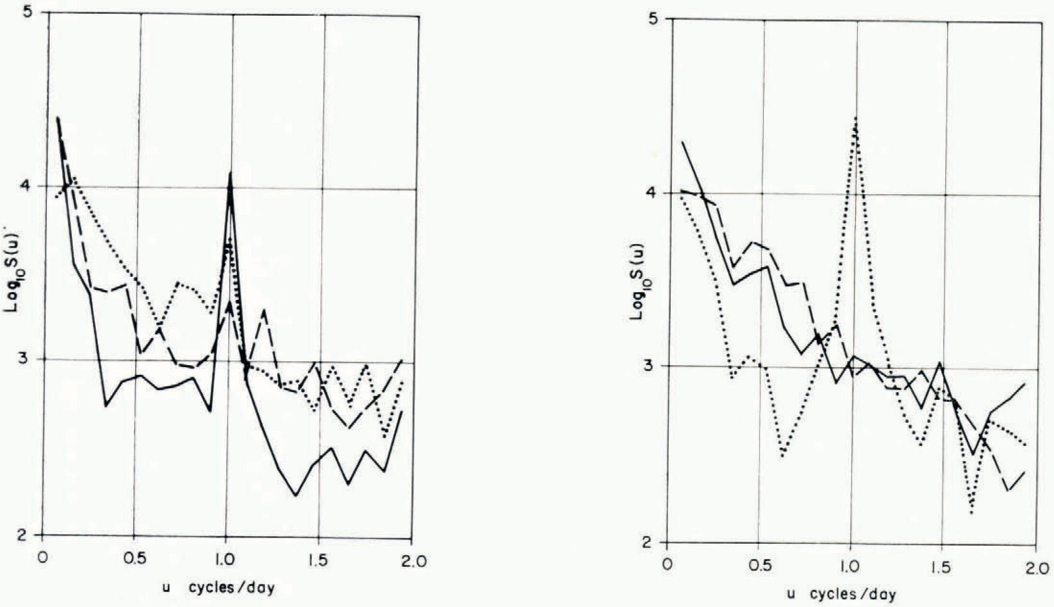

Fig. 4. Estimates of the power spectra of Tungnaá and the weather in Jökulheimar. In left-hand diagram, ... is wind, ———— is humidity, ———— is Tungnaá. In right-hand diagram .... is temperature, ———— is precipitation, ———— is cloudiness.

Phases and coherences

The phases and coherences of the river with the meteorological series are shown in Figures 5 and 6. In the lowest frequencies the largest coherences are with the precipitation. The phases correspond to time lags of 0.4–1.1 d.

There is a large coherence between the river and the temperature at 1 cycle/day. Two series with a strong periodic component at the same frequency will produce a large coherence regardless of whether there is any relationship between the series or not. In our case the diurnal variations of the river are in fact largely due to the diurnal variation of the temperature. The coherences in the neighbouring frequencies are also fairly large and the phases show a time lag of about 2 hours. At other frequencies our study shows little coherency between Tungnaá and the temperature.

In the frequencies below 1 cycle/day the coherences of Tungnaá and the wind are larger than the expected value from non-coherent series. The time lag is about 1 day in the lowest frequencies but drops to a couple of hours at 0.25 cycle/day and higher frequencies.

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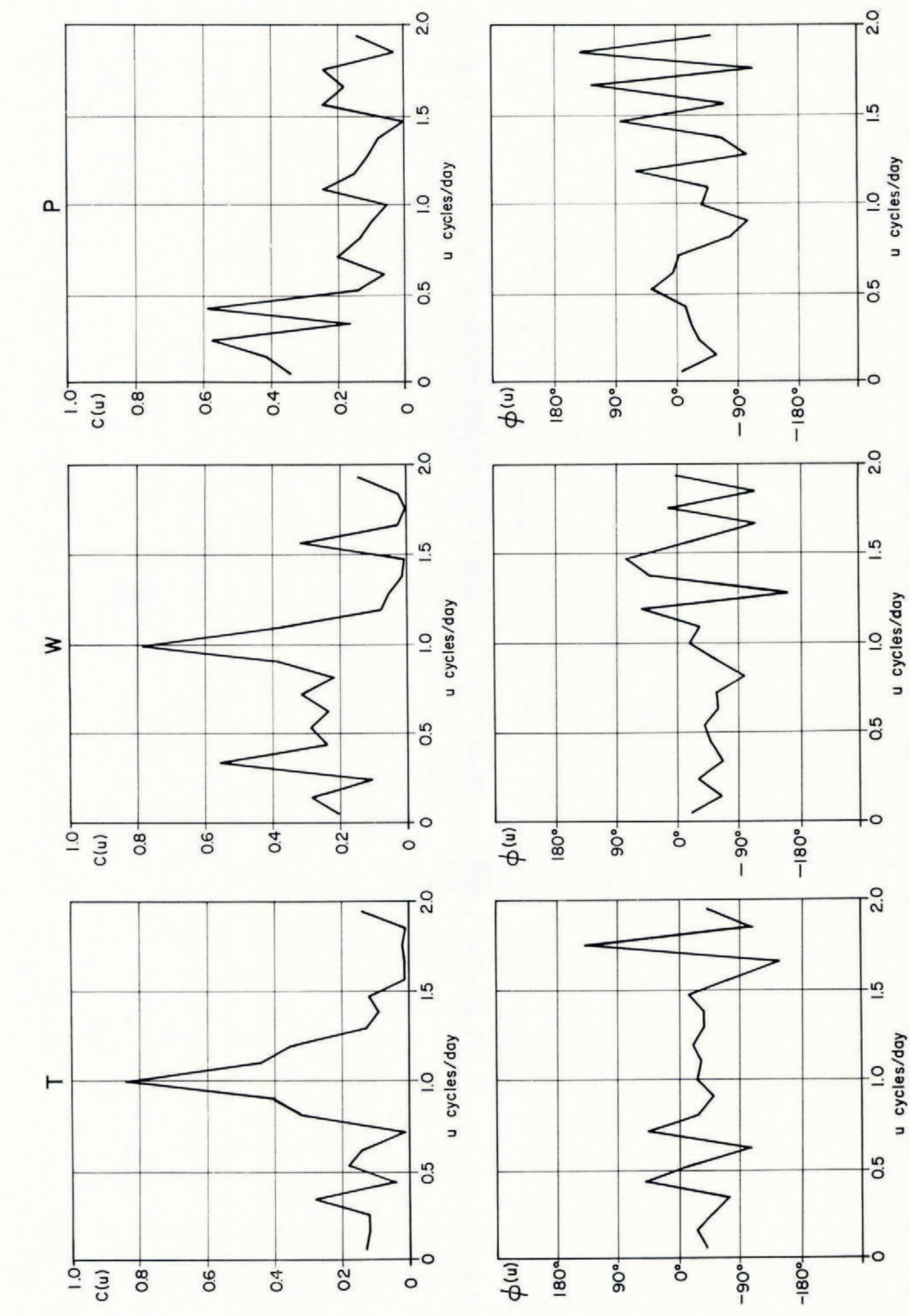

Fig. 5. Estimates of the phases and coherences of Tungnaá with temperature T, wind W and precipitation P.

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There is some coherence between the river and the humidity in the lowest frequencies, but otherwise the phases and coherences in Figure 6 hardly indicate any relationship between Tungnaá and the cloud cover or the humidity.

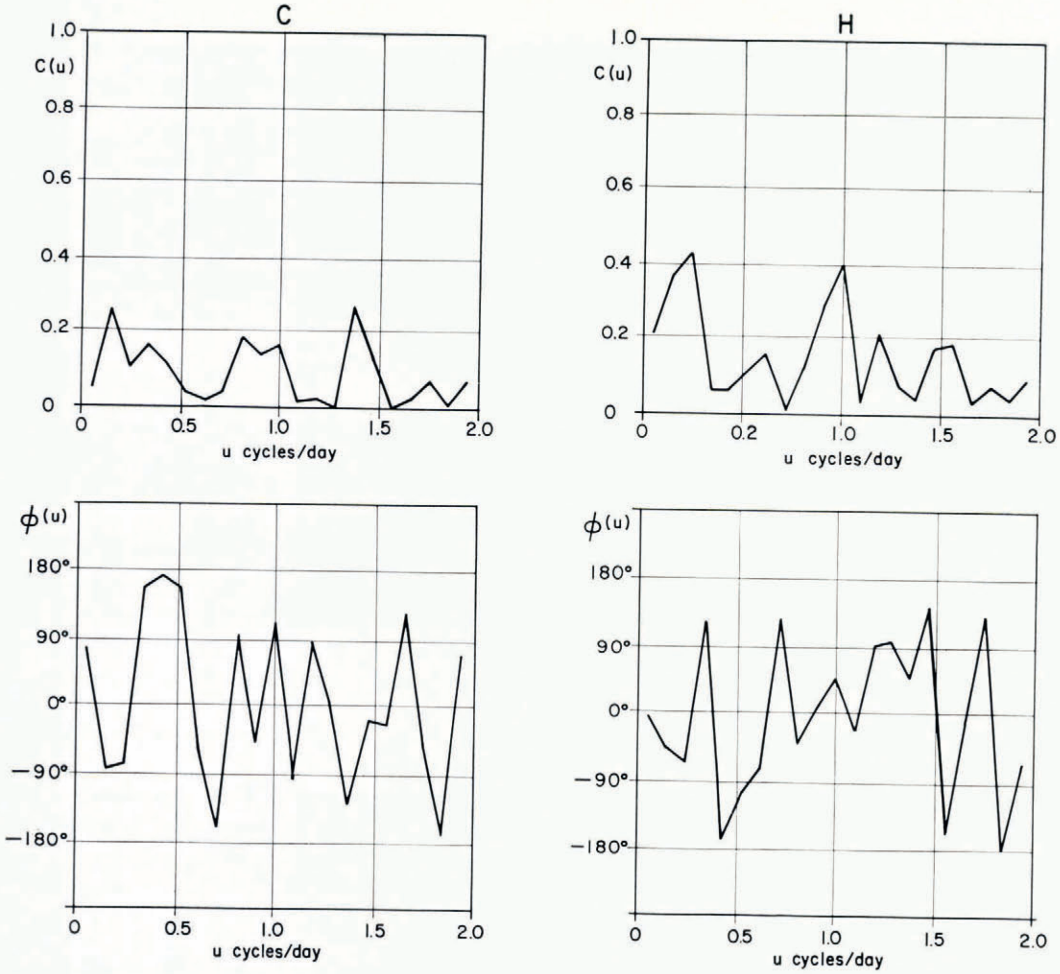

Fig. 6. Estimates of the phases and coherences of Tungnaá with cloud cover C and humidity H.

Figure 8 shows estimates of the phases and coherences of Þjórsá at Urriðafoss and the temperature and precipitation at Hæll in August (Gudmundsson, 1970). The power spectra are shown in Figure 7. The estimates were obtained from 12 years of daily observations and thus mainly cover a frequency range different from our estimates with Tungnaá. Apart from the delaying effect due to the distance between our point of observation and Urriðafoss the estimates are probably fairly representative for the relationship between Tungnaá and the meteorological components in low frequencies. At Urriðafoss only about 40% of the water of Þjórsá is melt water from glaciers. This implies reduced coherence with temperature compared with rivers consisting entirely of glacier melt water. In the case of Þjórsá at Urriðafoss and the weather at Hæll the river had larger coherences with the temperature than the precipitation in the frequency range between zero and 0.05 cycle/day.

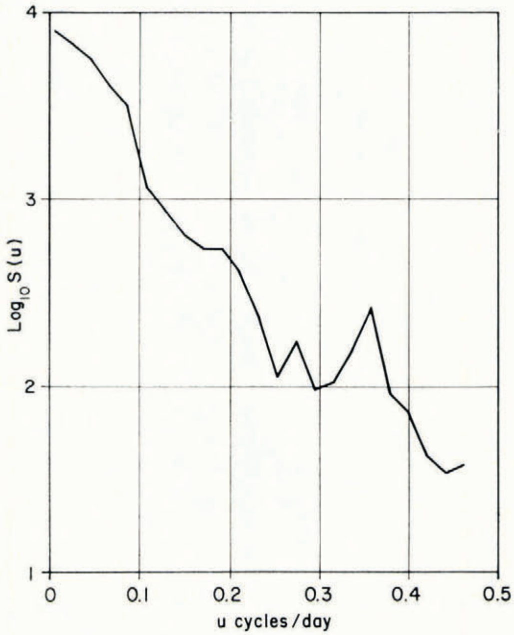

Fig. 7. Estimates of the power spectrum of Þjórsá at Urriðafoss.

Table I shows the phases and coherences of the meteorological series. The phases are rather irregular and most of the coherences are low. There appears to be a weak coherency between the temperature and the precipitation for the wavelengths between 1.5 and 3 d with the temperature leading by about 12 h. There is apparently no connection between the temperature and the wind. Cloud cover and temperature move in opposite directions, but the coherence is weak. There is some connection between temperature and humidity in the lowest frequencies and no significant phase lag. There is a positive correlation between the wind and the precipitation in the frequency range covered by our observations. The precipitation leads in the lowest frequencies, but the wind is ahead in wavelengths below 4 d. The precipitation has a positive connection with both the cloud cover and the humidity, but the coherences are rather low.

In these results there is hardly any detectable relationship between the wind and the other meteorological variables except for the precipitation. The largest coherences amongst the meteorological series are between the cloud cover and the humidity.

The coherences of the river with the meteorological series are often larger than the coherences between the various meteorological series. Elimination of variations that can be attributed to linear relationships with one meteorological series then results in improving the fit of the linear relationships with the other series without altering the phases. Elimination of the temperature produces larger coherences of the river with the wind, cloud cover and the precipitation in the lowest frequencies, but the relationship of the river with the humidity is apparently unaffected (Table II).

Elimination of the wind increases the coherences of the river with the temperature in the lowest frequencies. As a result of the coherence between the wind and the precipitation the coherences of the river with the precipitation are decreased by eliminating the wind.

Elimination of the cloud cover increases slightly the coherences of the river and the temperature in the lowest frequencies and has no clear effect on the relationship of the river

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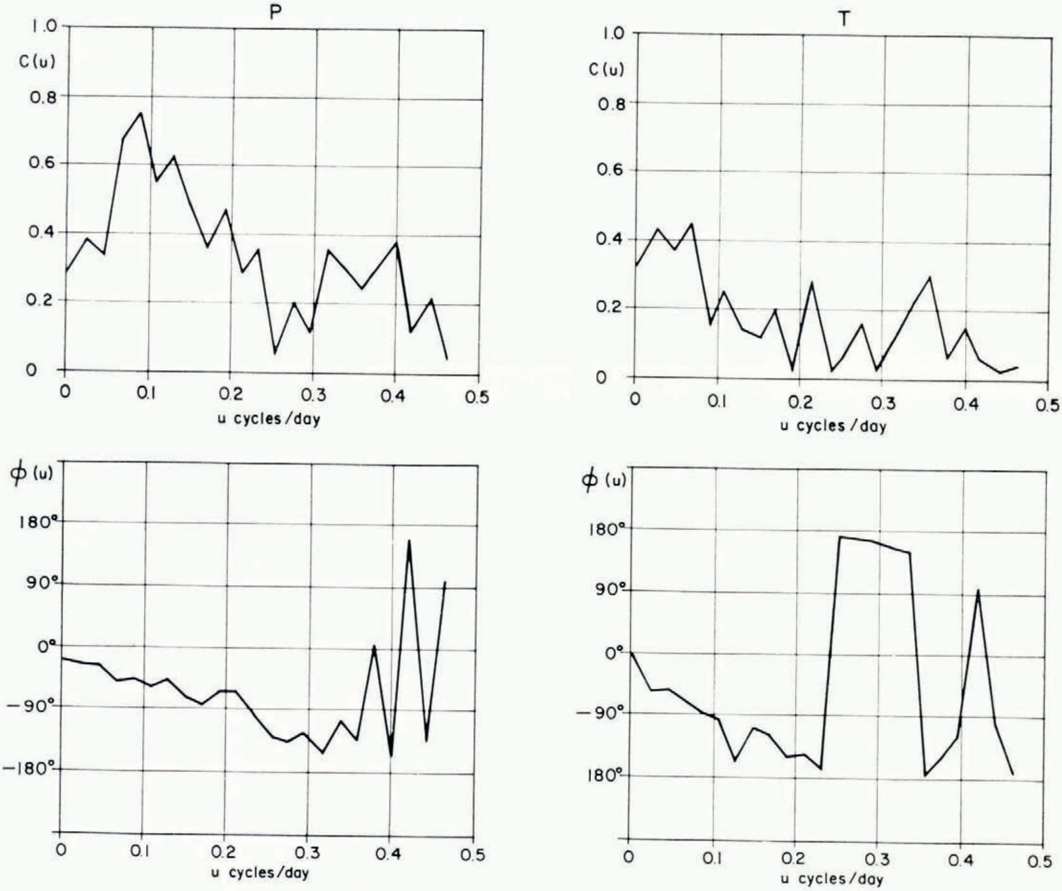

Fig. 8. Estimates of the phases and coherences in August of Þjórsá at Urriðafoss with the temperature T and precipitation P at Hæll.

with wind, humidity nor precipitation. After eliminating the humidity there is no sign of a significant relationship between the river and the temperature in the lowest frequencies. The relationship between the river and the wind is slightly improved and the coherences of the river and the precipitation are reduced.

When the precipitation is eliminated the coherences of the river and the temperature are increased, but no significant relationship remains between the river and the wind in the lowest frequencies. The coherences of the river and the humidity are reduced, but the phases and the coherences at 6.7 and 4.1 days still indicate some connection.

The impulse response

Figure 9 shows estimates of $H_i(k)$ from the model in Equation (3) with two series on the right-hand side. Here k represents an interval of 6 h. Only temperature, T, precipitation, P, and wind, W, were considered. The estimation was carried out for three versions of the equation, with two of the meteorological variables included each time.

When temperature and precipitation are used together as explanatory variables $H_i(k)$ is largest for $k = 0$ and $k = 1$ for both variables. The interval of temperature or precipitation

TABLE I. PHASES AND COHERENCES OF THE METEOROLOGICAL SERIES

		Temperature		*Wind*		*Cloud cover*		*Humidity*	
	λ	c	ϕ	c	ϕ	c	ϕ	c	ϕ
	18.3	0.00	−67						
	6.7	0.10	−127						
	4.1	0.20	109						
	3.0	0.17	−4						
	2.3	0.06	−175						
	1.9	0.07	26						
Wind	1.6	0.08	99						
	1.4	0.00	−2						
	1.2	0.02	139						
	1.1	0.09	−14						
	1.0	0.74	10						
	0.9	0.12	50						
	0.8	0.11	84						
	18.3	0.06	−157	0.10	−132				
	6.7	0.00	153	0.44	−23				
	4.1	0.16	−178	0.09	26				
	3.0	0.28	−167	0.21	−136				
	2.3	0.13	171	0.10	−141				
	1.9	0.06	−172	0.15	86				
Cloud cover	1.6	0.08	174	0.26	28				
	1.4	0.07	−54	0.06	−120				
	1.2	0.03	136	0.21	175				
	1.1	0.15	19	0.13	37				
	1.0	0.21	126	0.18	112				
	0.9	0.08	152	0.01	25				
	0.8	0.33	93	0.06	51				
	18.3	0.09	−20	0.05	−95	0.43	−3		
	6.7	0.35	27	0.10	29	0.32	21		
	4.1	0.22	−7	0.18	−98	0.07	−26		
	3.0	0.11	15	0.17	−177	0.34	−10		
	2.3	0.01	52	0.37	−129	0.33	26		
	1.9	0.04	−92	0.17	−12	0.19	−42		
Humidity	1.6	0.03	43	0.02	44	0.01	28		
	1.4	0.15	17	0.04	−92	0.19	63		
	1.2	0.14	29	0.09	59	0.02	−72		
	1.1	0.30	87	0.05	48	0.10	21		
	1.0	0.50	71	0.46	65	0.10	−72		
	0.9	0.03	114	0.25	58	0.07	11		
	0.8	0.29	123	0.16	38	0.29	41		
	18.3	0.01	−39	0.20	−6	0.08	−3	0.32	6
	6.7	0.00	−70	0.38	−19	0.48	1	0.18	−20
	4.1	0.02	155	0.22	7	0.13	3	0.17	27
	3.0	0.21	101	0.09	33	0.01	−44	0.01	155
	2.3	0.02	−22	0.16	54	0.01	−108	0.02	−52
	1.9	0.14	152	0.26	93	0.24	20	0.08	129
Precipitation	1.6	0.31	175	0.23	59	0.09	−3	0.07	168
	1.4	0.06	168	0.00	71	0.00	−149	0.05	139
	1.2	0.03	−133	0.27	2	0.28	147	0.03	−175
	1.1	0.10	−73	0.14	−57	0.01	121	0.00	108
	1.0	0.03	−65	0.11	−38	0.06	152	0.21	−96
	0.9	0.04	−10	0.31	−11	0.07	48	0.05	−26
	0.8	0.29	93	0.07	−37	0.30	5	0.22	−28

corresponding to $k = 0$ partly represents the weather after the observation of the river. The results therefore indicate that both $H_T(\tau)$ and $H_P(\tau)$ have their maxima in the interval $0 \leqslant \tau \leqslant 3$ h. The values of $H_p(k)$ for $k > 1$ are considerably larger than those of $H_T(k)$, indicating that on average rainwater reaches the river more slowly than melt water. We see no reason why the paths of rainwater and melt water from the same place on the glacier should differ. The difference between H_P and H_T at $k > 1$ therefore suggest that the spatial distribution of the rainfall is different from the melting. There are good reasons why neither

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TABLE II. PARTIAL PHASES AND COHERENCES OF TUNGNAÁ WITH METEOROLOGICAL VARIABLES

Temperature eliminated

	Temperature		*Wind*		*Cloud cover*		*Humidity*		*Precipitation*	
λ			c	φ	c	φ	c	φ	c	φ
18.3			0.24	−15	0.07	55	0.21	6	0.38	−3
6.7			0.38	−51	0.32	−84	0.45	−67	0.45	−59
4.1			0.19	−54	0.26	−69	0.36	−67	0.68	−36
3.0			0.49	−59	0.12	−158	0.03	166	0.16	−58
2.3			0.26	−43	0.10	160	0.07	−164	0.58	−16
1.9			0.29	−47	0.01	143	0.08	−105	0.24	18
1.6			0.24	−69	0.06	−81	0.12	−76	0.09	−46
1.4			0.32	−57	0.05	−154	0.01	150	0.24	−0
1.2			0.45	−95	0.17	93	0.10	−77	0.19	−66
1.1			0.36	−49	0.05	−77	0.20	−30	0.04	−81
1.0			0.24	−8	0.11	−153	0.05	152	0.19	5
0.9			0.50	−50	0.15	−72	0.09	−52	0.24	−49
0.8			0.02	32	0.09	−119	0.05	67	0.02	14

Wind eliminated

	Temperature		*Wind*		*Cloud cover*		*Humidity*		*Precipitation*	
λ	c	φ			c	φ	c	φ	c	φ
18.3	0.17	−43			0.17	59	0.33	5	0.24	2
6.7	0.23	−50			0.06	−83	0.31	−54	0.29	−40
4.1	0.20	−25			0.12	−97	0.50	−52	0.53	−36
3.0	0.16	−91			0.01	144	0.01	−75	0.08	−11
2.3	0.06	18			0.04	174	0.01	−40	0.53	−21
1.9	0.17	0			0.17	−165	0.14	−147	0.03	3
1.6	0.13	−88			0.04	178	0.17	−85	0.00	66
1.4	0.02	62			0.01	−105	0.03	80	0.26	−3
1.2	0.51	−31			0.09	115	0.06	−41	0.03	−52
1.1	0.36	−54			0.09	−82	0.28	12	0.02	−90
1.0	0.43	−31			0.09	−173	0.01	47	0.06	95
0.9	0.55	−15			0.03	−126	0.12	−119	0.05	−62
0.8	0.31	−18			0.01	75	0.15	91	0.13	96

Clouds eliminated

	Temperature		*Wind*		*Cloud cover*		*Humidity*		*Precipitation*	
λ	c	φ	c	φ			c	φ	c	φ
18.3	0.15	−30	0.31	−11			0.40	−26	0.38	−13
6.7	0.20	−26	0.10	−65			0.24	−34	0.32	−36
4.1	0.28	−57	0.13	−13			0.42	−59	0.59	−26
3.0	0.25	−93	0.49	−64			0.02	53	0.21	−24
2.3	0.03	88	0.19	−49			0.01	179	0.64	−17
1.9	0.15	−16	0.39	−44			0.21	−98	0.27	31
1.6	0.17	−104	0.26	−54			0.14	−76	0.06	15
1.4	0.02	59	0.30	−59			0.04	114	0.20	−4
1.2	0.31	−26	0.14	−112			0.14	−50	0.08	−46
1.1	0.34	−44	0.36	−46			0.25	16	0.14	−105
1.0	0.83	−30	0.77	−19			0.33	51	0.04	−11
0.9	0.52	−33	0.39	−29			0.03	−14	0.23	−48
0.8	0.40	−23	0.07	56			0.23	84	0.15	51

Humidity eliminated

	Temperature		*Wind*		*Cloud cover*		*Humidity*		*Precipitation*	
λ	c	φ	c	φ	c	φ			c	φ
18.3	0.12	−61	0.33	−29	0.28	135			0.19	−11
6.7	0.23	42	0.22	−55	0.10	−115			0.29	−56
4.1	0.01	−0	0.22	−79	0.09	−111			0.50	−31
3.0	0.25	−79	0.54	−65	0.12	169			0.16	−18
2.3	0.04	47	0.20	−54	0.05	176			0.69	−15
1.9	0.14	−18	0.31	−23	0.14	139			0.10	45
1.6	0.11	−107	0.26	−53	0.02	−46			0.09	−17
1.4	0.01	23	0.33	−54	0.07	−143			0.22	0
1.2	0.30	−17	0.17	−99	0.19	102			0.19	−75
1.1	0.32	−29	0.39	−57	0.10	−71			0.17	−101
1.0	0.75	−31	0.65	−15	0.07	109			0.01	113
0.9	0.48	−29	0.44	−23	0.01	−120			0.22	−48
0.8	0.22	−12	0.02	55	0.04	−165			0.05	55

Precipitation eliminated

	Temperature		*Wind*		*Cloud cover*		*Humidity*		*Precipitation*	
λ	c	φ	c	φ	c	φ	c	φ		
18.3	0.18	−50	0.10	−37	0.11	115	0.04	7		
6.7	0.17	−28	0.14	−103	0.14	−145	0.24	−54		
4.1	0.34	−32	0.01	70	0.13	−136	0.33	−68		
3.0	0.28	−55	0.52	−66	0.21	159	0.06	113		
2.3	0.05	79	0.16	−23	0.25	−173	0.31	−160		
1.9	0.28	0	0.20	−28	0.19	177	0.07	−114		
1.6	0.17	−87	0.20	−60	0.02	−89	0.17	−65		
1.4	0.06	27	0.37	−56	0.04	−142	0.03	86		
1.2	0.37	−34	0.14	−109	0.13	74	0.18	−47		
1.1	0.36	−53	0.33	−55	0.17	−48	0.34	6		
1.0	0.87	−28	0.79	−16	0.15	108	0.38	48		
0.9	0.44	−31	0.23	−25	0.00	80	0.01	−22		
0.8	0.26	−13	0.06	35	0.02	−170	0.12	94		

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should be uniformly distributed on the drainage area. The temperature decreases with height so that most of the melting takes place relatively close to the river outlet while the amount of precipitation increases with height. Variations in the temperature below the freezing point are relatively unimportant for the run-off. These factors all speed up the impulse response to

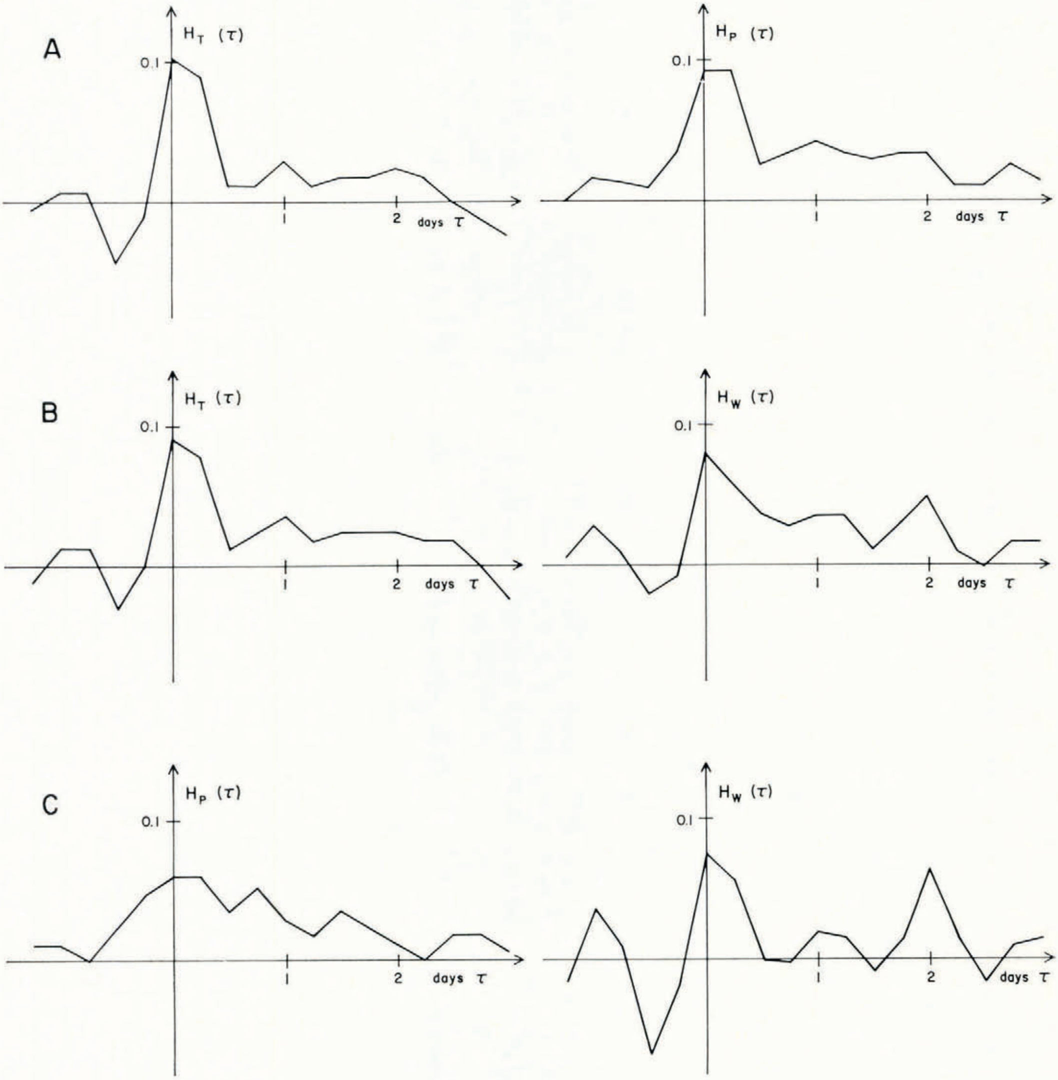

Fig. 9. Estimates of the impulse response of Tungnaá to meteorological variables. In A, the independent variables are temperature and precipitation, in B temperature and wind, and in C precipitation and wind.

the temperature compared with that of the precipitation. The temperature of the rain does not much affect its passage through the glacier as it is temperate (Rist, 1961). Precipitation which falls as snow is practically incoherent with run-off in our frequencies. Rain constitutes a smaller proportion of the precipitation at the higher altitudes. Therefore, $H_p(\tau)$ is more concentrated at low values of τ than would be the case if all precipitation fell as rain.

In the analysis of the river with the wind and the temperature H_W closely resembles H_P in the analysis with the temperature and the precipitation. The estimates of H_T in both cases are rather similar, but here the values at $k > 1$ are somewhat larger. It is however doubtful whether any conclusions should be drawn from this. If we return to the frequency domain we see that the increase is caused by the result from the estimation of the cross-spectrum of T and W at 6.7 and 4.1 d (Table I). Neither of the coherences is large enough to be regarded as significant on its own and the phases differ by more than $\pi/2$ so that the estimates lend little support to each other. However, the phases are both closer to π than to zero which implies a negative correlation between the observations of T and W and this enhances the values of $H_T(k)$.

The coherence between wind and precipitation renders joint estimates of H_P and H_W inaccurate and comparison of the values of $H_W(k)$ at positive and negative k indicates that this function contains little reliable information about the effect of the wind on the glacier.

The positive values of $H_P(-1)$ are probably the result of the fact that k corresponds to intervals of 6 h but precipitation was only observed twice each day.

Discussion

Anyone familiar with Icelandic rivers is aware that temperature is the most important factor in determining the run-off from the glaciers. Nevertheless precipitation explains a larger proportion of the random variations analysed here than the temperature. The reason for this is that most of the variation of both the river and the temperature is confined to diurnal and seasonal frequencies. The power spectra of the random variations of river and temperature decrease rapidly with increasing frequencies. On the other hand the variance of the precipitation is fairly evenly spread on the frequencies so that although its effect on the total variations of the river is small compared with the temperature, it explains more of the variations in certain frequency intervals. In the study of the variations of daily averages of Þjórsá at Urriðafoss and the temperature and precipitation at Hæll (Fig. 1) the variations could be divided into narrower frequency bands than was possible here. The coherences of the river and the temperature were larger than the coherences of the river with the precipitation in wavelengths larger than 20 d, but the precipitation dominates in the higher frequencies. This change takes place in the middle of the lowest frequency band in the present study. Variations in the relationship between two processes lead to underestimates of the coherences unless the variation is small within the bandwidth of the frequency band used in the estimation. The comparison with the analysis of Þjórsá therefore indicates that a linear model of Q, T and P could explain more of the variations of Tungnaá than the coherences suggest. The coherence of Tungnaá and the temperature is of course very large (0.84) in the frequency band containing the diurnal frequency. But the coherences in the neighbouring frequency bands are also clearly significant. This is probably because the diurnal peak in the power spectrum of the temperature extends into these frequencies and its effect on the river is therefore discernable although the coherence is hardly strong enough to produce a similar broadening of the peak for the river. The reason for this broadening of the peak in the temperature is probably that the shape of the diurnal variation varies according to the weather and the season. The investigation of Þjórsá indicated that the observations at Hæll were a fairly satisfactory indicator of variations of temperature and precipitation on the drainage area in wavelengths over a couple of days (Gudmundsson, 1970). Local variations in the weather are probably more pronounced in the shorter wavelengths, but the station is close to the drainage area.

The actual relationships between glacier run-off and meteorological variables are by no means linear. Non-linear effects are mainly due to the singularity in the behaviour of water at the freezing point. The effect of changes in all the meteorological variables included here are different at $+5^\circ$ C from what they are at -5° C.

The coherences of Tungnaá with the temperature and precipitation at Jökulheimar are lower than those of Þjórsá with the weather at Hæll. We have no reason to suspect that our observations contain a less accurate information about the river and the weather on its drainage area than the observations at Hæll and Urriðafoss, rather the contrary. We have already mentioned that in the lowest frequencies the larger bandwidths used here reduce the coherence. But the main reason for the low coherences in the higher frequencies is that our method of analysis, based on the assumption of linearity, is inadequate.

Another false assumption is that the relationships are independent of time. Both the glacier and the river channel change from year to year. Seasonal changes in the physical condition on the glacier surface during the melting season affect its response to changes in the weather. The snow first melts at the front and the snow-free area gradually reaches higher altitudes (Meier and Tangborn, 1961). The snow holds a certain amount of water and probably increases the time lag between melting or precipitation and the river. The relationships between melting and the meteorological components is different for snow than for glacier ice.

Our present analysis has not thrown up any very unexpected results, but it provides a convenient way of making some of our present knowledge more quantitative. More general methods of analysis exist where the assumption of linearity is abandoned, but the computations are cumbersome and the interpretation of the results difficult. A more promising approach is to take explicitly into account existing knowledge about melting and the distribution of precipitation between rain and snowfall. Thus instead of using the meteorological observations as independent variables in Equations (2) and (3) we could try series derived from the meteorological series by use of the physical laws of melting and the variation of meteorological variables with altitude.

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REFERENCES

Ahlmann, H. W., *and* Thorarinsson, S. 1938. Vatnajökull. Scientific results of the Swedish–Icelandic expeditions 1936–37–38. *Geografiska Annaler*, Årg. 20, Ht. 3–4, p. 171–233.

Björnsson, H. In press. Energy budget study of Bægisárjökull, north Iceland. *Jökull*, Ár 21, 1971.

Freysteinsson, S. 1969. Tungnárjökull. *Jökull*, Ár 18, 1968, p. 371–88.

Gudmundsson, G. 1970. Short-term variations of a glacier-fed river. *Tellus*, Vol. 22, No. 3, p. 341–53.

Lang, H. 1968. Relations between glacier runoff and meteorological factors observed on and outside the glacier. *Union de Géodésie et Géophysique Internationale. Association Internationale d'Hydrologie Scientifique. Assemblée générale de Berne, 25 sept.–7 oct. 1967.* [*Commission de Neiges et Glaces.*] *Rapports et discussions*, p. 429–39.

Leopold, L. B., *and others.* 1964. *Fluvial processes in geomorphology, by L. B. Leopold, M. G. Wolman and J. P. Miller.* San Francisco and London, W. H. Freeman and Co.

Mathews, W. H. 1964. Discharge of a glacial stream. *Organisation Météorologique Mondiale et Association Internationale d'Hydrologie Scientifique. Symposium. Eaux de surface, tenu à l'occasion de l'assemblée générale de Berkeley de l'Union Géodésique et Géophysique Internationale, 19–8—31–8 1963*, p. 290–300.

Meier, M. F., *and* Tangborn, W. V. 1961. Distinctive characteristics of glacier runoff. *U.S. Geological Survey. Professional Paper* 424-B, p. B-14–B-16.

Rist, S. 1961. Rannsóknir á Vatnajökli 1960 [Investigations on Vatnajökull in 1960]. *Jökull*, Ár 11, p. 1–11.

Sigbjarnarson, G. 1971. On the recession of Vatnajökull. *Jökull*, Ár 20, 1970, p. 50–61.

Sumarlidason, P. 1965. Vatnshæðarmælingar í Tungnaá við Félaga sumarið 1965 [Oscillations of the river Tungnaá at Jökulheimar, summer 1965]. *Jökull*, Ár 15, p. 144–47.

Thorarinsson, S. 1964. Sudden advance of the Vatnajökull outlet glaciers 1930–1964. *Jökull*, Ár 14, p. 76–89.

Thorarinsson, S. 1969. Glacier surges in Iceland, with special reference to the surges of Brúarjökull. *Canadian Journal of Earth Sciences*, Vol. 6, No. 4, Pt. 2, p. 875–82.