(3) The triangle C′B′C″.

Now, since the planimeter in its circuit returns to its original position, the areas (1) have a sum zero; so have the areas (3); and the area of the closed figure is, therefore, equal to the sum of all such parallelograms as BCC″B′; and this sum equals their common side BC, multiplied by their total heights.

Next, let P be any fixed point on BC, then its (elementary) motion at right angles to BC consists of the line P′M + MN, which is perpendicular to BC and B′C′; the sum of these elements is the registration by a wheel at P, free to revolve on BC as an axis; but the sum of the elements P′M vanishes (as is the case with the areas (1) and (3)) from its twofold description, and, therefore, the registration of the wheel is equal to the sum of all the heights of the parallelograms BCC′B″. Hence the area of the closed curve is equal to BC × registration of wheel, where "registration of wheel" means number of revolutions × circumference.

If the curve enclose the point A, as the instrument is not constructed to allow BC to cross over AB, we must note that our curve involves, besides the parallelograms, the areas of the circles whose radii are AB, BC; and the registration only gives us the heights of the parallelograms, together with the circumference of the circle of radius BP. Thus we must add to our result obtained by the usual reading, the expression $\pi AB^2 + \pi BC^2 - 2\pi BP.BC$. If a circle of known area is described about A, this quantity can be at once found for any particular instrument.

Fifth Meeting, March 8th, 1889.

George A. Gibson, Esq., M.A., President, in the Chair.

An Elementary Discussion of the Closeness of the Approximation in Stirling's Theorem.

By Prof. Chrystal.

[The substance of this paper will appear in the second volume of Professor Chrystal's Algebra.]

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