FELIX ADALBERT BEHREND

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Felix Behrend was born at Berlin-Charlottenburg, Germany, on 23 April, 1911, the eldest of four children of Dr. Felix W. Behrend and his wife Maria, nee Zöllner. Felix Behrend senior was a mathematics and physics master at the Herderschule, a noted "Reform-Realgymnasium" in one of the western suburbs of Berlin; he was a widely known educationalist, and later headmaster of an important school elsewhere in Berlin, until demoted and finally dismissed by the Nazis, partly because of some Jewish ancestry, partly because of his liberal political views.

Felix Behrend junior also went to the Herderschule, and passed out of it in 1929, with high distinction, to study mathematics and theoretical physics at the Universities of Berlin and Hamburg. His chief teachers were I. Schur, E. Schmidt, E. Artin, H. Hopf and (for physics) E. Schrödinger. He also attended lectures in philosophy by W. Köhler and A. Görland. He soon showed himself a pure mathematician of originality and imagination; his first three papers in the theory of numbers were published in quick succession before he was 23 years old. After taking, in 1933, his Dr. phil. at the University of Berlin, he emigrated from Nazi Germany in 1934, first to Cambridge, then to Zürich and Prague where he worked as an actuary at the Merkur life insurance company, at the same time continuing his research in pure mathematics. He took the degree of Doctor of Science at the Charles University of Prague in 1938 in the hope of becoming, eventually, a Privat-Dozent of that University; but Czechoslovakia became unsafe in 1939, and he returned to Zürich and then to England just before the outbreak of war.

During the brief scare in the summer of 1940 when Holland was overrun by the German armies, Felix Behrend, like most adult male "enemy aliens" in Britain, whether friends of foes of Hitler, was interned, and not long afterwards he was transported to Australia. At the instance of G. H. Hardy, J. H. C. Whitehead, and other prominent British mathematicians, his release from internment was authorised before the end of 1940, but he chose not to return to Britain — the journey to Australia on the *Dunera* had been a harrowing experience. But this remaining time in the internment camps at Hay and Tatura was not lost; he gave courses of lectures in the "camp university", and awakened an abiding enthusiasm for mathematics in several younger fellow internees, among them Walter F. Freiberger, F. I. Mautner, and J. R. M. Radok. The students were prepared for examinations of the University of Melbourne, without textbooks because none were available: in spite of this, or because of this, the teaching was highly successful.

He was released from internment in 1942, and was appointed Tutor in Mathematics at the University of Melbourne. There he remained, being successively promoted Lecturer (1943), Senior Lecturer (1948), and Associate Professor (1954). In 1960 the University instituted a category of Personal Professorships, and he would have been a strong candidate for one of these; but the illness intervened that led to his premature death on 27 May 1962, at the age of 51 years. He is survived by his widow Daisy, née Pirnitzer, whom he had married in 1945, and their two daughters.

Felix Behrend was, like his father, an outstanding teacher. He knew, and appreciated, both the British and Continental traditions, and was able to combine the best features of both. From time to time he carried responsibility for classes ranging from the elementary level to the most advanced; he discharged this to the full, with a sure feeling always for the level of his audience. For a number of years, also, he acted as Chairman of Examiners at the matriculation level; he knew about school work and took a constructive interest in it, and as an examiner he had a flair for asking the sort of question that was penetrating but remained of the fair standard.

Among his recreations, classical music ranked high (he sang bass with the Philharmonic choir during his early years in Melbourne), but much the most important was creative writing. He had as a young man been deeply impressed by the novels of Thomas Mann, and he continued to admire him and to acknowledge him as his master; but in his later writings the influence of Thomas Mann becomes less noticeable, and his style more individual. Most of his essays and short stories were written in German and circulated privately among his friends. But he also wrote in English, and his last work, completed only shortly before his illness took its final turn, is a book, "Ulysses' Father", published shortly after his death: it is a children's book of very great charm, which should surely, like "Alice in Wonderland", captivate grown-ups, too.

Felix Behrend was a man of integrity and modesty, enlivened at times by dry humour and suffused by a warm personality. He put down permanent roots in Australia, and enriched her by becoming her citizen. He will be remembered with respect, gratitude and affection.

Behrend was an unusually versatile mathematician, full of ideas, which were always expounded with an elegant and well judged economy of style. In his earlier years he published chiefly on Theory of Numbers, and in his later years on Foundations and Topological Spaces (axiomatics), but there is throughout a sprinkling of papers on Analysis and Geometry, and there is one paper, of great interest, on Algebraic Equations. One is tempted to speculate on what unity of motive may have underlain this diversity; there is a genuine breadth of interest and, it seems, a passion for problem-solving; but it is possible that the deep disturbances of his life during the pre-war and war years — he was six times uprooted — may have prevented the concentration of his energies into one deep channel. His work was, in the main, in territory already explored, where he worked usually "from first principles" by the injection of some new idea or device; a knowledgeable critic has remarked how often Behrend was able to find a new approach to questions already deeply studied — an approach which, when one sees it, is recognizable as relevant and fruitful.

Theory of Numbers [2, 3, 5, 8, 10, 11, 13, 18]. Behrend wrote his doctoral thesis on abundant numbers, i.e. those positive integers n for which the sum of the divisors (including 1 and n) is not less than $2n$. He first proved that the number $A(n)$ of abundant numbers less than n satisfies $A(n) < 0.47n$; the factor 0.47 was established by an asymptotic estimate valid for $n > 6230$, supplemented by a direct count for $n \leqq 6230$. He then showed that, for all sufficiently large n,

$$0.241n < A(n) < 0.314n,$$

and he had a proof (unpublished) that $A(n)/n$ tends to a limit as $n \to \infty$. His results led to much further work by Davenport, Chowla and Erdös on this topic.

The remaining papers in this series are on the density (relative to the sequence of all positive integers) of the sequence of primes and of sequences that are to some extent analogous thereto, such as those which contain no set of terms in arithmetic progression. For a sequence of numbers not exceeding x, let $r_k(x)$ be the largest number of terms such that no k of them are consecutive members of an arithmetic progression. In [8] Behrend proved that $r_k(x)/x$ has a limit ρ_k as $x \to \infty$, and that for all x, $r_k(x) > \rho_k x$; and further, that as $k \to \infty$, ρ_k has a limit, which limit must be either 0 or 1. In [13], by improving on a method used by Salem and Spencer, he proved that for every positive ε,

$$\frac{\log r_3(n)}{\log n} > 1 - \frac{\sqrt{8 \log 2} + \varepsilon}{\sqrt{\log n}},$$

provided $n \geqq n_0(\varepsilon)$. This problem has attracted the attention of several powerful mathematicians; in 1960 Davenport told us that, as far as he knew, Behrend's results still then "held the record".

On the frequency of the primes, Behrend in [10, 11] added improve-

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ments to ideas and methods of Broderick and Landau. By strictly "elementary" means he proved that, for $n \geqq 2$,

$$\tfrac{1}{2}n < \pi(n)\lambda(n) < 3n,$$

where $\pi(n)$ is the number of primes not exceeding n and $\lambda(n) = \sum_1^n 1/\nu$; also that, for any positive ε,

$$(\log 2-\varepsilon)n < \pi(n)\lambda(n) < (2 \log 2+\varepsilon)n$$

provided $n > n_0(\varepsilon)$.

In [18] he proved that

$$T(a_1, \cdots, a_n, b_1, \cdots, b_m) \geqq T(a_1, \cdots, a_n)T(b_1, \cdots, b_m),$$

where $T(p, q, \cdots, r)$ is the density of the set of all integers not divisible by any of $p, q, \cdots, r$. Heilbronn and Rohrbach had previously proved what is essentially the case $m = 1$ of this.

Algebra. Let $f_\nu(x_1, \cdots, x_r; y_1, \cdots, y_s)$, $\nu = 1, \cdots, n$ be n polynomials which are homogeneous in both the x and the y, the degrees in each set being odd. This system of polynomials is called "definite" if the equations $f_\nu = 0$ have no real solutions other than the trivial ones $x_i = 0$ and $y_i = 0$; for example the system $f_1 = x_1y_1-x_2y_2$, $f_2 = x_1y_2+x_2y_1$ is definite. For given values of r, s there is a smallest value of n for which a definite system of such polynomials exists, and the question is to determine this value $N(r, s)$, or to find criteria for it. There are crude estimates

$$\max(r, s) \leqq N(r, s) \leqq r+s-1$$

and by an essentially topological argument, E. Stiefel and H. Hopf established the deeper result, that for the system of n polynomials to be definite it is necessary that all the binomial coefficients $\binom{n}{k}$ for which $n-r < k < s$ be even. It is stated by Hopf (*Math. Rev.* 1, 36 (1940)) that he, and others, had tried without success to find a purely algebraic proof of this result. In [9] Behrend showed how this could be done, by using the ideas in algebraic geometry of van der Waerden which were at this time new. For the bi-linear case Behrend gave also another proof, resting on the apolarity theory of Reye; and he found the exact value of $N(r, r)$ for $r \leqq 8$, and improved inequalities for $9 \leqq r \leqq 17$.

Analysis. [1, 4, 15, 16, 17, 26]. These are short papers on isolated topics. [4] gives an elegant proof that any bounded set of points in R_m which is the union of a set of convex sets K has a determinate Jordan content, provided that for some fixed positive ε each K contains an m-sphere of radius ε. [17] and [26] generalize Weierstrass's construction of continuous non-differentiable functions.

Geometry [6, 7, 12, 20, 28]. From the metric invariants, such as the diameter, circumference, area, of a plane convex region, Behrend simply defines affine invariants by choosing a suitable function of two of the quantities, and considering its least upper bound as the region is subjected to affine transformations. In [6] he investigates such invariants on the supposition that the convex region has a centre of symmetry, in [7] he extends his results considerably by dropping this restriction. Roughly speaking it turns out that all plane convex regions with centre fall, in regard to the affine invariants Behrend considers, between the ellipses and the parallelograms as extremes; and those without assumption of a centre between the ellipses and the triangles. The numerous results are almost without precedent, and barely any have been improved or generalized since: Only Fritz John, with whom Behrend had discussed the problems when they met at Cambridge, had some partially sharper results and some partial extensions to more than 2 dimensions, arrived at by quite different methods [Duke Math. J. 2, 447—452 (1936); University of Kentucky Research Club Bulletin 6, 26 (1940); ibid. 8, 8—11 (1942)]. The stimulus of Behrend's work can also be found in B. H. Neumann's papers in J. London Math. Soc. 14, 262—272 (1939), and ibid. 20, 226—237 (1946). However, a full generalization of Behrend's results in [7] to more than 2 dimensions has not, as far as we know, been carried out yet.

In [20], Behrend returns to a problem that had exercized him already as a student, namely the best upper bound c_n for the radius of an n-dimensional sphere into which a closed polygon can be squeezed if it starts and ends in the centre, and the sides (taken as given vectors all of at most unit length) may be permuted. It had been proved by Ernst Steinitz [J. reine angew. Math. 143, 128—175 (1913); ibid. 144, 1—40 (1914)] that such a bound c_n — depending only on the number of dimensions but not on the number of sides of the polygon — exists; and he had shown the inequality

$$c_n \leqq 2(n+1).$$

Other authors had obtained slightly better estimates for c_n when $n \leqq 4$. Behrend uses a refinement of Steinitz' method to prove a very much better estimate, namely

$$c_n < n$$

for $n \geqq 3$, and $c_3 \leqq (5+2\sqrt{3})^{\frac{1}{2}}$.

In [12] and [28], Behrend returned to a problem that had caught his fancy early in his student days, namely the construction of finite models in euclidean 3-space of the real projective plane; [28], written during his final illness and published posthumously, gives a simple parametric representation of the Klein bottle.

Topology. [23] gives a simple method of obtaining for any uniform space S a uniform structure which is totally bounded and compatible with the topology of S. This result is useful because then, if the space is separated, its completion is compact. In [24] a related question is subjected to deeper study; four different but equivalent criteria are given for the uniformizability of a topological space S. They are intrinsic in that they use no mappings of S into comparison spaces, such as the real line, and in this respect they improve upon a criterion of Bourbaki. If such a space S is a T_0-space, it is compactifiable, whence Behrend's criteria at once give corresponding criteria for compactifiability.

Foundations. [19, 21, 22, 27]. During the last ten years Behrend became increasingly interested in questions of axiomatics, especially in axiom systems for "natural" mathematical disciplines, like real analysis [22], real vector spaces [27], magnitudes [22], [21]. Although much work had already been done in this field, Behrend brought new ideas to it. The fundamental notions he uses are familiar: binary or ternary relations (order or betweenness) and binary operations (addition or multiplication); but the axioms are new, and their independence is studied as well as their sufficiency for the purpose in hand; and one aim is always to match the naturalness of the discipline with the naturalness of the axioms.

Behrend's ideas also proved fruitful for others: out of a question posed in [21] there arose the very delicate investigation of sesquilateral division semigroups by P. M. Cohn [J. London Math. Soc. 31, 181—191 (1956); Proc. London Math. Soc. (3) 8, 466—480 (1958)]; and paper [27] is immediately followed by a paper in which W. Greve, at Behrend's suggestion and in close connection with Behrend's work, investigates partial betweenness groups [Math. Zeitschr. 78, 305—318 (1962)].

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