SECTION V

DYNAMICS OF COMETS

STATISTICAL AND NUMERICAL STUDIES OF THE ORBITAL EVOLUTION OF SHORT-PERIOD COMETS

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ABSTRACT. The evolutionary paths connecting cometary reservoirs in the outer solar system and the population of periodic comets involve stellar and planetary perturbations. Close planetary encounters play a special role, making the dynamical evolution slow or fast depending on the orbital elements of the comet at various stages of the process. In this paper numerical work done in the last decades on this subject, using both real and fictitious objects, is reviewed.

1. INTRODUCTION

It is well known that the lifetime of comets in the inner solar system is limited to very much less than the age of the system itself both physically, because of progressive gas and dust loss from the nucleus, and dynamically, due to the instability of their motion against ejection on hyperbolic orbits; in fact, these arguments hold for all comets, no matter if of long or short period, and the conventional explanation for the mere fact that we do observe comets is that reservoirs sufficient for the replenishment of both cometary populations exist in the outer solar system (Oort cloud, Uranus-Neptune planetesimals).

The formation of the Oort cloud is reviewed in this book by Fernandez, and its dynamical evolution is discussed by Weissman. Doubts have been recently cast on its survival over the age of the solar system (Napier and Staniucha, 1982) because encounters with giant molecular clouds would strip comets from the cloud repeatedly and with great efficiency, but Bailey (1983a) has shown that an appropriate revision of the extension and especially the structure of the cloud could ensure the necessary stability (more on this can be found, in this book, in the above mentioned papers by Fernandez and Weissman, and also in those by Clube and Napier).

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Planetesimals scattered to orbits half-way between the planetary region and the Oort cloud by Uranus and Neptune during their growth have been proposed by Fernandez and Ip (1983) as a likely source of short-period comets; this suggestion seems to support the above mentioned results by Bailey (1983a) on the structure of the cometary cloud necessary to ensure its survival over the age of the solar system (Bailey, 1983b).

In this review we will first outline the dynamical channels connecting the proposed reservoirs to periodic comets; there have been many studies on various aspects of this problem, using different techniques, and a general framework has emerged, although still needing improvement. We will then examine the numerical work on strong perturbations at close encounters with the giant planets and their consequences on the orbits of short-period comets.

2. FROM THE OORT CLOUD TO PERIODIC COMETS

To fix ideas about the evolutionary channels between Oort cloud and periodic comets, we may refer to the schematic representation of the solar system given in Figure 1. In its left half a plot of the quantity $-1/a$, proportional to the orbital energy per unit mass, versus e, the orbital eccentricity, is given; it covers almost the whole solar system, from the asteroid belt out to hyperbolic orbits, but obviously lacks the information relative to the angular elements. Note that the regions ($0 < e < 1$; $0 < -1/a$) and ($1 < e$; $-1/a < 0$) are forbidden by definition. The plot on the right has the same ordinate, whereas the abscissa is the Tisserand invariant J, given by

$$J = A/a + 2(a/A(1-e^2))^{1/2}\cos(i) \qquad (1)$$

(where A is the semimajor axis of the planet perturbing the comet, Jupiter in the case of Figure 1b, and the inclination is with respect to the orbital plane of the planet).

The value of J is related to the unperturbed planetocentric velocity at encounter of the comet U (in units of the planet's orbital velocity) by

$$U = (3-J)^{1/2} \qquad (2)$$

The above formulae refer to a circular planetary orbit; in that case J, and hence U, is conserved if the comet is perturbed, even strongly, by the planet under consideration.

Opik (1972) discusses, semi-analytically, the effects of elliptic planetary orbits, which introduce a systematic decrease of J as a conse-

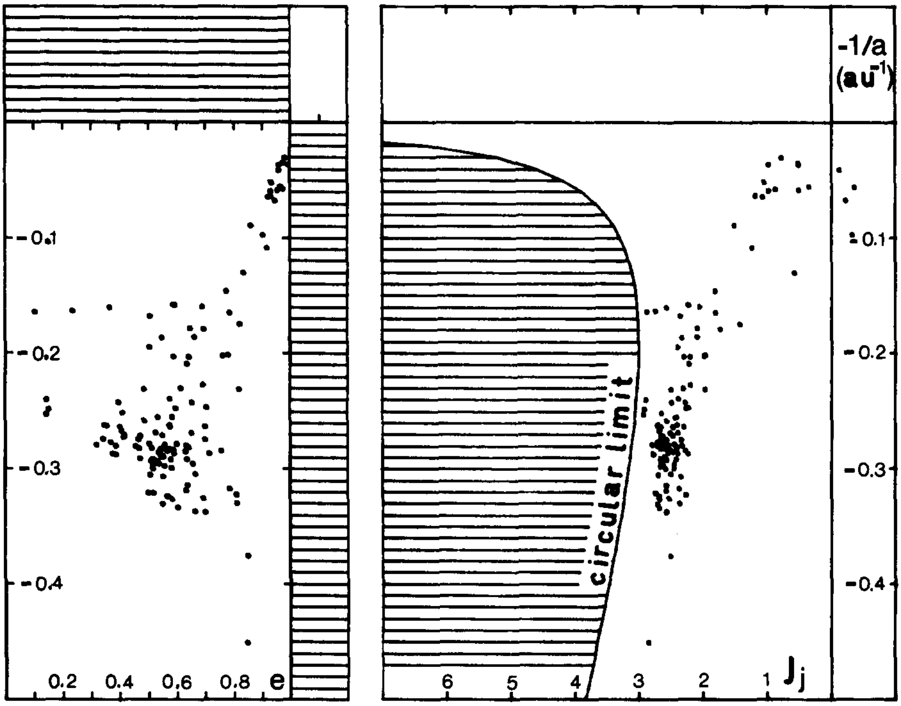

Figure 1. Left: The orbital energy (-1/a) of all periodic comets discovered until the end of 1983 is plotted versus their orbital eccentricity e. Right: The same quantity is plotted versus the Tisserand invariant with respect to Jupiter J_j. Forbidden regions are shaded in both plots.

quence of close encounters; he tabulates the number of revolutions of the small body after which a given decrease of the Tisserand invariant is to be expected. Experimental work, based on long integrations in the restricted elliptical three-body problem, would be very useful both to confirm Opik's results and to extend them to cases in which the orbit of the small body does not cross that of the planet, but still allows close encounters.

From Opik's results one can see that, in the time scales typical for the orbital evolution of periodic comets, the changes in J due to the eccentricity of the controlling planet are rather small; Kresák (1972) has shown that they are anyway much smaller (20 times less, on the average) than those in orbital energy undergone in the same time span by observed short-period comets. Also the error made by using the inclination with respect to the ecliptic is normally negligible; the cause of the greatest changes in J relative to a planet are close encounters with

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another one, when they are possible. This prevents the use of J computed with respect to a specific giant planet, even Jupiter, as a means to classify orbits permanently (Everhart, 1979), but one can still recognize, on a plot like that of Figure 1b, the possible dynamical evolutionary tracks of comets (see later).

In Figure 1 are also plotted all the periodic comets listed in Marsden (1982), plus those discovered up to the end of 1983. We will make use of various versions of this basic plot in the following discussions.

The region in the phase space of orbital elements filled by the Oort cloud comets can be roughly identified considering that the outer radius of the cloud (that is, the typical aphelion distances of new comets) should be about 50,000 AU, while the perihelion distances should be outside the planetary region; inclinations and orientations of the orbital planes, as well as the directions of the aphelia, are thought to be randomly distributed over all possible values.

In Figure 2 the outer part of the solar system is represented, in the same fashion of Figure 1. The diagonal lines in Figure 2a represent orbits with perihelion distance of 30 AU (Neptune's orbital radius, leftmost line), 19 AU (Uranus), 9.5 AU (Saturn) and 5.2 AU (Jupiter, rightmost line). New comets are situated very close to $-1/a = 0$ and $e = 1$, while unobserved Oort cloud comets, still being very close to the line $-1/a = 0$, are displaced more to the left, not penetrating into the planetary region.

Stars passing through the cloud can impart to the comets small displacements in all directions (but not into the forbidden regions!). Therefore, if pushed towards the upper right of Figure 2a, through $e = 1$ and $-1/a = 0$, a comet will be lost to interstellar space; if the displacement is horizontal and to the left, it will remain in the cloud, with a larger perihelion distance; finally, if moved downwards, it may enter the region below one or more diagonal lines of Figure 2a.

At this stage the further evolution depends essentially on the orbital elements of the comet. In fact the orbit now crosses those of one or more planets and, for suitable values of the angular elements, planetary encounters become possible. These, unlike the stellar perturbations mentioned before, can move comets in the phase space only along peculiar tracks, which depend on the planet involved, and are constrained by the approximate conservation of the Tisserand invariant relative to the planet with which the comet interacts (Kresak, 1982). These tracks are represented by vertical straight lines in Figure 2b, in the case of Jupiter; interactions with other planets give other straight lines, some of which are also shown in Figure 2b. They refer to $J = 2$ and $J = 2.83$ for all four outer planets.

Using (2) we see that $J = 2$ means that at the encounter the comet's velocity relative to the planet is as large as the heliocentric velocity

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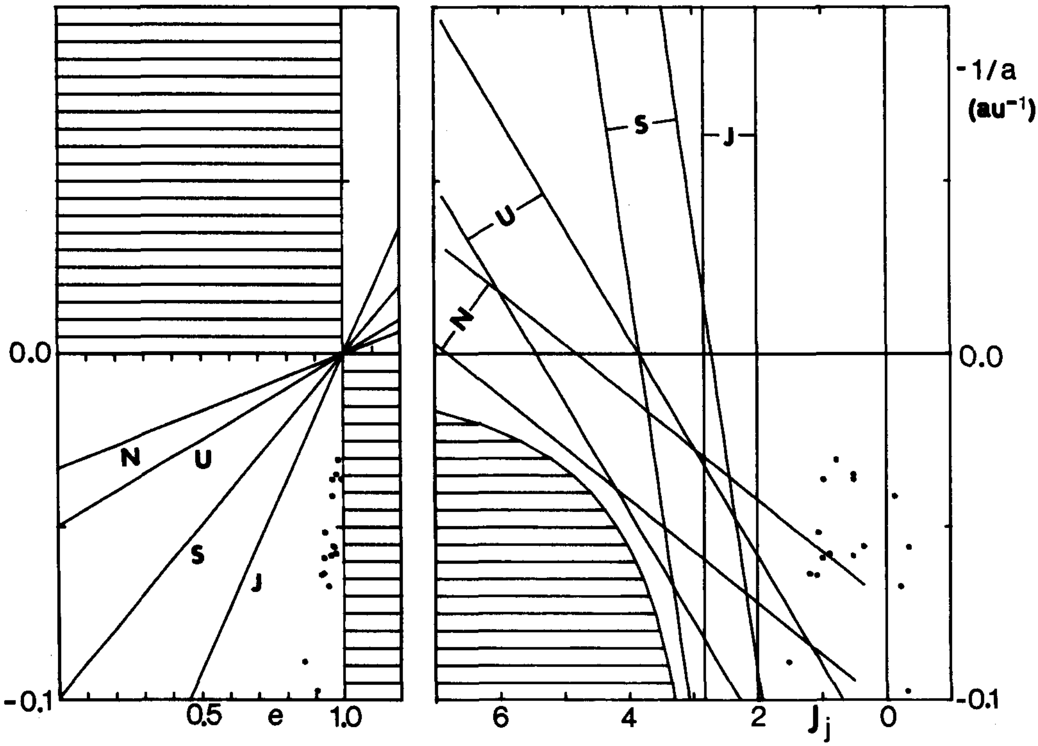

Figure 2. The same as Figure 1, the ordinate being restricted to the outer part of the solar system. The slant lines in the left plot represent orbits tangent in perihelion to those of the planet whose initial labels the line. The pairs of straight lines in the right plot represent orbits whose Tisserand invariant is equal to 2 (line on the right of each pair) and 2.83 (line on the left) with respect to the four outer planets (labels as before).

of the planet itself; it is easily understandable that in this case planetary encounters are not very effective unless they are particularly close. J = 2.83 implies that the encounter velocity of the comet is sufficiently high to allow escape from the solar system if the planetary encounter causes a particular reorientation of the velocity vector. On the other hand, if J is greater than 2.83, the comet cannot be ejected on (or captured from) a parabolic or hyperbolic orbit by encounters with the planet under consideration (Kresák, 1982).

Coming back to the fate of comets penetrating into the planetary region, the orbital elements determine what is (or what are) the planet that can encounter it, if any, and what is the value of J relative to that planet. If close encounters are not possible, and the perihelion distance is great, the comet may remain for a long time stored in its

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"parking orbit"; if encounters with a planet are possible, and J relative to it is greater than 2, the comet can end up in a short-period orbit (we will return to this case in more detail later); the last case, i.e. the possibility of encounters with J < 2, can lead, after many revolutions, to Halley type comets. These are comets of any inclination, period between 20 and 200 years, and J_j smaller than 2 (note that there can be comets with P < 20 years and $J_j < 2$, as is the case of P/Tuttle).

In Figure 3 all observed Halley type comets are shown. 17 of them have been taken from Marsden (1982), and P/Hartley-IRAS, discovered in 1983, has been added. As it can be seen, many of these comets have also the Tisserand invariants relative to Saturn, Uranus and Neptune smaller than 2. It appears that they have had their periods shortened directly by Jupiter, probably as a consequence of a very deep encounter; after that, the orbit should have evolved only rather little. Had they been captured by other outer planets, to reach their present locations in Figure 3 they should have had to pass through the region $J_j > 2$; in that region the effect of Jupiter would have surpassed those of the other planets, and the comet would have not been allowed to reach smaller J_j values.

The dynamical evolution of the other short-period comets can be more complex, since the reduction of their revolution periods can be due to the action of more than one giant planet, in succession; if, at the first entrance in the planetary region, the perihelion distance allows only encounters with Neptune, it is this planet that controls the evolution until the comet reaches a perihelion distance small enough to allow encounters with Uranus, which then takes the control; in the meantime collisions with Neptune, as well as ejections by this planet to interstellar space will always be a possibility.

Everhart (1977) modelled numerically the process of multistage capture just described. He found that a minority of comets reaches short-period orbits without being ejected (he did not consider collisions with planets as possible end states), the fraction being about one in several thousand when the capture process starts with Neptune, something more in the case of Uranus, and in the case of Saturn and Jupiter one order of magnitude more. To quote precise figures is not very wise, since the results of the numerical experiments are different for different initial conditions; the time scales found by Everhart for the process are of the order of hundreds of million years starting at Neptune and Uranus, millions of years starting at Saturn, and hundreds of thousand years if comets are captured directly by Jupiter.

Examples of possible evolutionary tracks leading to captures, either multistage or single-stage, are sketched in Figure 4. There it is possible to see that these tracks converge before comets reach perielion distances small enough to allow their discovery (Kresák, 1982). There-

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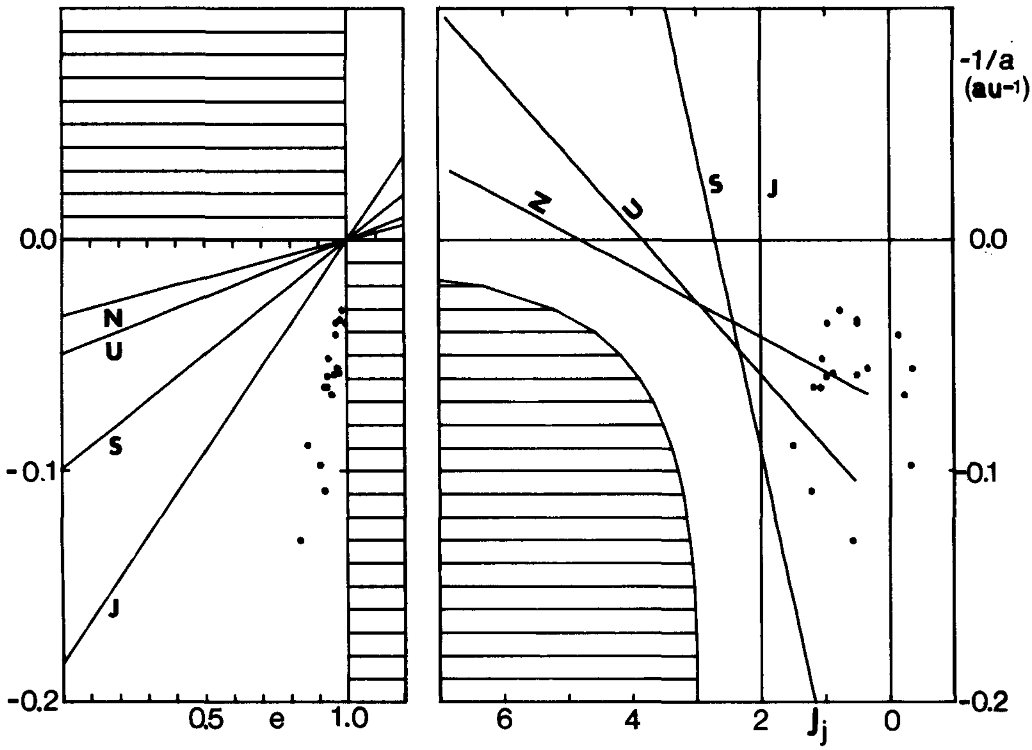

Figure 3. In the same style of Figures 1 and 2, all Halley type comets discovered up to the end of 1983 are indicated. In the right plot only the lines for Tisserand invariant equal to 2 are reported.

fore one should resort to numerical integration of the past motion of comets to establish which of the possible tracks they have actually followed; unfortunately, the motion of a comet cannot in general be reliably computed beyond one or two close planetary encounters (Carusi, Perozzi, Pittich and Valsecchi, this volume), thus rendering this reconstruction practically impossible.

3. FROM URANUS-NEPTUNE PLANETESIMALS TO SHORT-PERIOD COMETS

An alternate source for the short-period comets has been proposed by Fernández and Ip (1983); according to them, the formation of Uranus and Neptune by accretion of planetesimals, in the framework of the accumulation theory of Safronov (1969), would have had as a natural by-product the scattering from the Uranus-Neptune region of an amount of mass larger than the present masses of the two planets, in the form of small bodies mainly composed of volatiles, in other words of possible future

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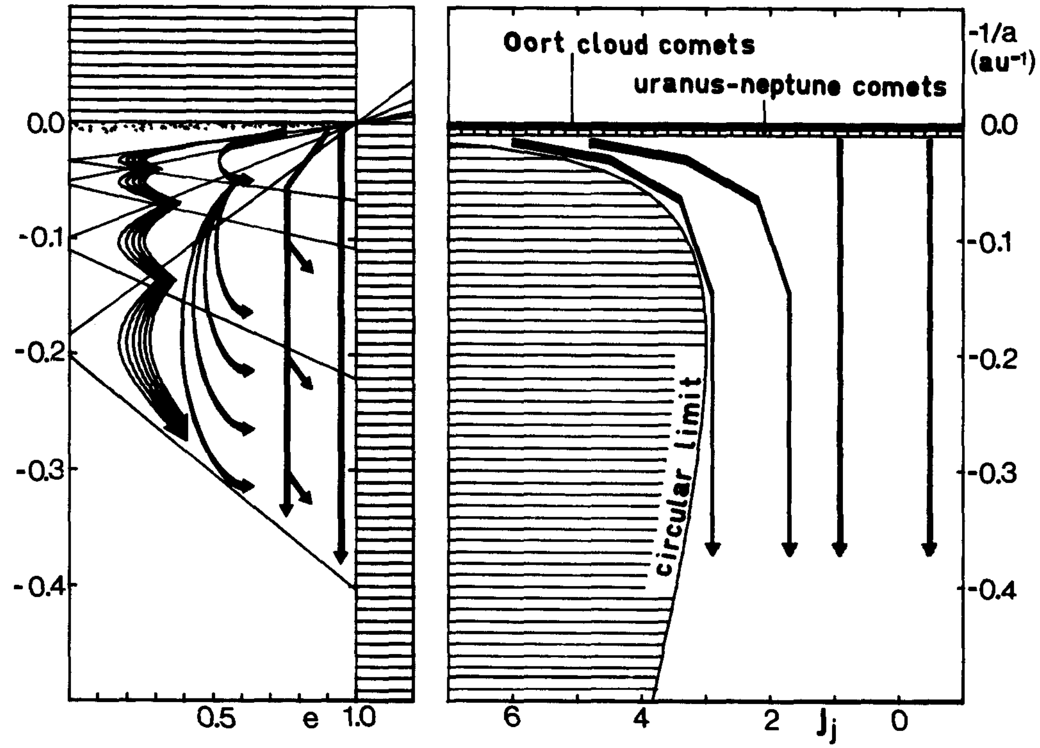

Figure 4. Possible evolutionary paths in the energy-eccentricity and in the energy-Tisserand invariant planes. The leftmost ones in both plots can imply low velocity encounters with all the outer planets in succession; the rightmost ones, leading to Halley type comets, imply mainly encounters with Jupiter; between these two extrema, single-stage captures are possible. The paths are only indicative, and real objects would not follow them strictly. The cometary reservoirs of the outer solar system are indicated by dots in the left plot.

comets.

The majority of the scattered planetesimals would be ejected from the solar system by the giant planets, especially Jupiter and Saturn, (Fernández, this book); another fraction would have their aphelia diffused out to distances allowing passing stars to decouple their perihelia and orbital planes from the planetary region, thus forming the Oort cloud; finally, those bodies not undergoing either of the described fates would remain stored for very long times in orbits with perihelia close to, or slightly outside, Neptune's orbit, and aphelia out to several thousand AU. These comets would be more tightly bound to the solar system than those in the Oort cloud, and their orbital planes would not be completely randomized. As Fernández and Ip point out, quantitative statements are difficult to make because of the uncertainties involved in the model; however, the evolutionary path connecting these comets to the short-period ones is faster than that from the Oort cloud described before.

In terms of the description exemplified in Figure 4, the dynamical evolution in this case is essentially the same, the main difference being that now comets start, as far as the orbital energy is concerned, slightly closer to the arrival than those coming from the Oort cloud. Note also that the orbits in which the Uranus-Neptune planetesimals are stored for very long time resemble those intermediate orbits in which comets from the Oort cloud can be put by stellar encounters, the difference being that the former are more closely coupled to the planetary system and decoupled from the stellar environment.

4. ORBITAL EVOLUTION IN THE PLANETARY REGION

Once comets have come back to orbits in the planetary region, the time scale of their evolution is greatly accelerated. The dynamics can be controlled by one or more planets, depending not only on the perihelion and aphelion distances, but also on the geometry of the orbit. When interactions with, for instance, two giant plenets are possible, in a majority of case it will be the inner one to take sooner or later the control, mainly because the synodic period of the comet with respect to it will in general be shorter than that with respect to the outer one; when encounters with only one planet are possible, this can happen either because the comet's orbit is rather inclined and the nodes are far from from any other planet, or because the perihelion and aphelion distances do not allow crossing of other planetary orbits. In the first case, one has to wait for a sufficient rotation of the argument of perihelion, if this is not prevented by libration mechanisms; in the second, the perturbations of the planet controlling the dynamics of the comet result in a random walk of the perihelion distance that can, after enough time, allow near tangency of the comet orbit to that of the planet immediately inside. In this case, the shorter synodic period already mentioned, together with a great efficiency of the slow encounters on nearly tangent orbits (Carusi and Valsecchi, 1982a) will probably decouple the comet from the outer planet, leaving again the control to the inner one.

It may also happen that a comet orbit crosses (or is tangent to) two planetary orbits, and encounters with one planet are prevented by libration involving the planetary and cometary longitudes and mean motions. This situation cannot last too long, however, since the revolution periods of pairs of neighbouring planets (excluding Neptune-Pluto) are not in exact ratios of small integers, so that a comet cannot be in libration with both the planets whose orbits it crosses.

The multistage capture process ends when the control is passed to Jupiter: at this point, the planets within Jupiter's orbit are by far too little massive to cause significant orbital evolution in the short time span covered by the life of the comet as an active object.

It must be noted that, although the prevailing direction of the dynamical evolution just discussed is inwards, also the evolution outwards, although less frequent, is possible.

For quite a long time in the past people like Laplace, LeVerrier, Tisserand and Callandreau thought that short-period comets come from long-period ones which in a single encounter with Jupiter lose enough orbital energy. Newton (1893) showed that the probability of this process is too low, and Everhart (1969) found that this mechanism would have implied that one quarter of the short-period comets should be in retrograde orbits, a fact in contrast with observations. Everhart (1972), moreover, studied in great detail the evolutionary path starting from parabolic comets; he identified a "capture zone" delimited by low orbital inclination ($i < 9°$) and perihelia close to the orbit of Jupiter ($4 < q < 6$ AU). Decelerating encounters with this planet were shown to be able to produce qualitatively the short-period comets that we observe.

Much of the numerical work done in the last 20 years has been devoted to the last part of the orbital history of short-period comets. Massive computations are in fact necessary to achieve a global picture of the dynamics of observable periodic comets, and the task of doing them has been alleviated by the increase of the performance of electronic computers.

Everhart (1973) has studied the orbital evolution of fictitious comets starting from randomly chosen circular, inclined orbits in the region of Jupiter and Saturn, and following these objects for thousands of revolutions, in a simplified solar system composed only of the Sun and the two planets. He found that many of these objects could enter different regimes of motion, including temporary trojans and horseshoes, generalized trojans and horseshoes, temporary satellite captures (see later), orbits of moderate inclination changing irregularly because of planetary close encounters, nearly circular orbits of longer persistence etc.; he named these orbits "chaotic": an object in a chaotic orbit can pass from one orbital pattern to another, its long-term evolution being generally unpredictable and, if it does not collide with a planet or disintegrate, it will in the end be put by a planet on a hyperbolic orbit.

Pioneering work regarding the dynamics of observed short-period comets was done in the sixties in USSR by Kazimirchak-Polonskaya (1967), Belyaev (1967) and coworkers, and in USA by Marsden (1967). They integrated the orbits of many short-period comets backwards in time for a few centuries and, in the case of Kazimirchak-Polonskaya and Belyaev, also forwards for one century. Over such time spans they were able to identify many close encounters with Jupiter and a few with Saturn, that changed in some cases even dramatically the orbits of comets.

Other researchers investigated the problem either using Monte Carlo techniques or concentrating on the dynamics of close planetary encounters.

The first approach was pursued by Rickman and Vaghi (1976) and by Froeschlé and Rickman (1980), the latter paper being mainly an improvement of the former. They divided the q-Q plane into a number of cells, and computed a set of jovian perturbations acting on fictitious comets in each cell. Then they started populating some cells, which therefore behaved as source regions, and followed the overall evolution, taking also into account the limited lifetime of the bodies in the inner part of the solar system. Rickman and Vaghi (1976) found a too low efficiency of captures by Jupiter, but it turned out that their numerical procedure underestimated the largest perturbations, which in this case are the most important. Froeschlé and Rickman (1980) corrected this shortcoming, and investigated also the effects of different lifetimes of comets.

The dynamics of objects at close encounters with giant planets has been extensively studied in Italy (Carusi and Pozzi, 1978; Carusi et al., 1979; Carusi and Valsecchi, 1979, 1982a,b; Carusi et al., 1983). In these works various populations of fictitious objects were investigated, having different distributions of starting orbits; each object was followed through only one encounter with one of the outer planets, disregarding completely the previous and subsequent histories of the motion; the phase space of the orbital elements was filled as uniformly as possible, so that differences in the outcomes of the encounters could be related to specific regions of that space. The only elements not taken at random were the true anomalies of the planet and of the fictitious comet: they were chosen so as to allow the closest possible approach.

The main finding of these studies is that, in order to be most efficient in transforming the minor bodies' orbits, the encounters must be either very deep or very slow. The last case implies a high value (close to 3, or even larger) of the Tisserand invariant; as a result, temporary satellite captures and transformations of the orbits without crossing of the initial and the final ones are possible (Carusi et al., 1983).

Temporary satellite captures (TSC: i.e. occurrence of elliptical planetocentric osculating elements of the comet during the encounter) have been already observed both among real objects (Chebotarev, 1967; Rickman, 1979) and fictitious ones (Kazimirchak-Polonskaya, 1972; Everhart, 1973), but the initial conditions leading to them were not clearly recognized.

Carusi and Valsecchi (1981, 1982c) then integrated backwards in time the orbits of all short-period comets with $J_j > 2.9$ in order to find other events of this type. In a sample of 22 comets, 7 turned out to have been temporarily captured as satellites by Jupiter, for time spans between several months and several years, during the last 120 years. It must be added that, although a clearcut distinction cannot be made, TSCs seem to be of two main types (Carusi et al., 1983): one in which the planetocentric energy has negligible variations compared to those of the heliocentric energy, and another in which the two energy variations have similar amplitudes (see Figure 5).

TSCs are possible with all the giant planets; however, the perihelion distances of comets that can be captured as satellites by Saturn, Uranus and Neptune are far too large to make them observable (Carusi et al., 1983).

A process often accompanying TSCs is the transition from completely outside to completely inside the orbit of a planet, or vice-versa, with the initial and final orbits of the comet far from crossing each other. The two events are not necessarily related, in the sense that not always they occur during the same encounter; however, the initial conditions leading to both of them are essentially the same (Carusi et al., 1983), characterized also in this case by high values of the Tisserand invariant. This point needs to be stressed, since it can be very important for the orbital evolution: in fact, the perihelion distances of comets on these orbits can be substantially reduced even if the pre-encounter minimum distances between the cometary and planetary orbits are large (up to 0.6 AU in the case of the 1937 encounter with Jupiter of P/Oterma). This greatly increases the mobility across the planetary system of comets in these high Tisserand invariant orbits.

5. POSSIBLE EVOLUTIONS FROM PECULIAR ORBITS

A different approach to the study of cometary dynamics involves the integration of the motion of many varied versions of the same orbit, in order to evaluate the probabilities pertaining to different evolutionary tracks. Also in this case the progress in the performance of electronic computers has been crucial in making possible the computations and the processing of the outputs.

Following the discovery of 2060 Chiron, the first object found between the orbits of Saturn and Uranus, Kowal et al. (1979) integrated its motion backwards and forwards over nine millennia, showing the possibility of encounters with Saturn and Uranus, and revealing the chaotic nature of its orbit. This led Oikawa and Everhart (1979) and Scholl (1979) to investigate the long-term motion of Chiron in more detail.

Oikawa and Everhart made an accurate numerical integration of the motion of Chiron in a solar system composed by the Sun and the five outer

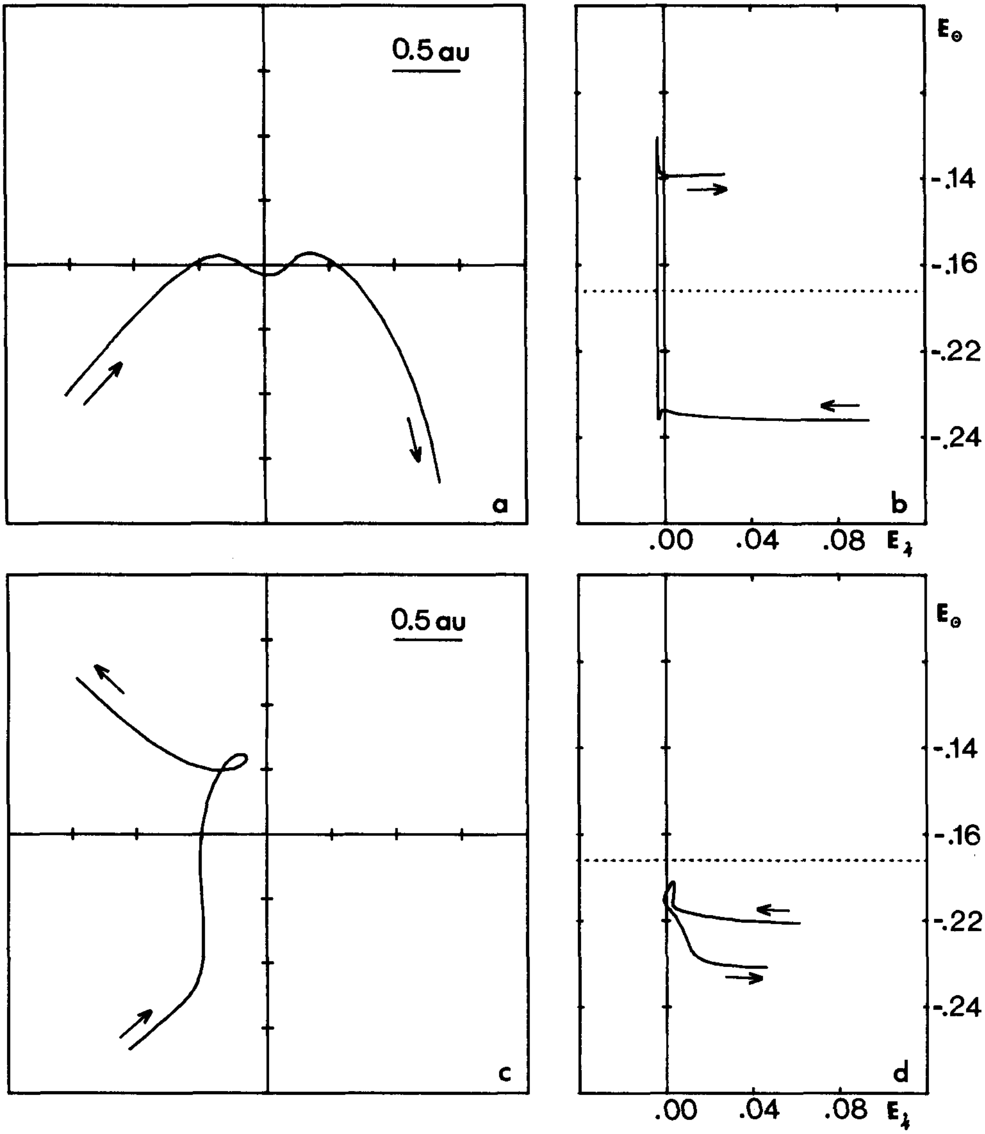

Figure 5. Left: Jovicentric paths in a rotating frame (Sun on the negative x-axis) of P/Oterma (a) during the 1960-1967 encounter and of P/Gunn (c) during the 1870-1884 encounter; tic marks: 0.5 AU. Right: For the same encounters (b: P/Oterma; d: P/Gunn) the heliocentric orbital energy $E_{\odot} = -1/a$ versus the jovicentric one $E_{♃} = -m/(Ma')$ are given, where m is the mass of Jupiter, M that of the Sun, and a' is the jovicentric semi-major axis.

planets, covering nearly 14 millennia roughly centred on the present epoch. Encounters with Saturn and Uranus were confirmed, showing that it is the former planet to control mainly the evolution (the current values of the Tisserand invariants relative to the two planets are $J_S = 2.9$ and $J_U = 2.95$); successive large perturbations were often found to be correlated in size and sign. Since the accuracy of integration is lost after the first very close planetary encounter, to assess the probability of inward or outward evolution of Chiron these authors integrated 60 fictitious "Chirons", obtained by changing at random the 5th significant digit of the position-velocity vector of the real body, in a simplified 3-body model in which only the Sun and Saturn were retained. In 7 out of 8 cases the evolution was found to be inwards, towards Jupiter's control, showing also the interesting result that, in the region of the phase space of orbital elements in which Chiron is, the largest perturbations of orbital energy, which in the end determine the evolution, are negative; the time scale for passing to Jupiter's control was found to be of the order of a hundred thousand years.

Similar results were obtained by Scholl (1979), who integrated 10 "Chirons" for about 24,000 years, two thirds of the time span being in the future, keeping the four giant planets as perturbing bodies. He found 5 future evolutions inward, 3 outward, and in the remaining 2 cases the objects did not leave the region in which Chiron is; going backwards, the same figures are 0, 5 and 5, confirming that probably Chiron will reach Jupiter's control in tens or hundreds of thousand years.

Other researchers concentrated on some peculiar close encounters of short-period comets, again applying the method of slightly varying the initial conditions, in order to determine the probabilities of specific outcomes.

Carusi et al. (1981) modelled the strong perturbations exerted by Jupiter on a chain of fictitious comets in the orbit that P/Oterma had just before its 1937 encounter with the planet. The 80 objects were equally spaced in mean anomaly, and their motion was followed, in some cases, for 250 years. The main aim was to find if an evolutionary channel exists connecting the orbit of P/Oterma between 1939 and 1961 and one of the type of that of P/Encke, both characterized by very high Tisserand invariant and low aphelion distance, perhaps with the help of nongravitational forces. It turned out that only three objects were placed by Jupiter on rather long lasting orbits of low aphelion distance (4.5 AU) and moderate perihelion distance (2.5 AU); whether or not these values could be further reduced by nongravitational forces, to end in Encke type orbits, is still an open question. This work, however, showed other important features typical of this kind of orbits: first, all the 80 "Otermas" were temporarily captured by Jupiter as satellites - indicating a very high probability per revolution of this event (0.026) - and in one case the duration of the binding was of the order of 100 years; second, in spite of the large unperturbed minimum distances between the pre-encoun-

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ter orbits of the "Otermas" and that of Jupiter (> 0.6 AU), the actual minima of the encounter distances were much smaller, even by one or two orders of magnitude, thus rendering problematic the definition of a "sphere of action " of the planet independent of the parameters of the encountering body; third, the high probability for orbits tangent to that of Jupiter to change the type of tangency (perihelion to aphelion tangency, or vice-versa) without even approximate crossing of the initial and final orbits, was further assessed.

In a subsequent work, Carusi, Kresák and Valsecchi (1982) examined the patterns followed during the encounter with Jupiter by P/Oterma and its 79 variations. They found that these patterns are arranged in a sequence (see Figure 1 in that paper) in which the first and the last, the second and the last-but one, etc., are related by symmetries; it appeared that different patterns were separated, at least in some cases, by values of the difference in mean anomaly leading to long-lasting satellite captures (up to 100 years). Moreover, the span in M covered by individual patterns varied widely, and one of them was identical to that followed by P/Gehrels 3 during its 1963-1976 encounter with Jupiter. The authors also compared these patterns with those followed by other 100 fictitious bodies on orbits of different semiaxis and eccentricity but still tangent either in their perihelion or aphelion to that of Jupiter, and thus having very high values of the Tisserand invariant. Similar patterns were again found, together with new ones, mainly exhibited by objects with values of J_j somewhat smaller than that of P/Oterma. They concluded that the basic types of paths followed by objects having low velocity encounters with Jupiter were included among these 180 cases.

Rickman and Malmort (1981) studied the effects of varying the starting orbital elements of P/Gehrels 3 to strengthen the evidence of its 1967-1974 TSC by Jupiter. For orbits varied up to several degrees in angular elements, up to 0.05 in eccentricity and up to 0.2 AU in semiaxis, they found an extended interval of gravitational binding to the planet; moreover, a smaller domain (about 1° in angular elements, 0.01 in eccentricity, and 0.05 AU in semiaxis) in which at least one revolution around Jupiter was performed, and some cases of long-lasting TSCs (up to 60 years), were found.

Carusi, Kresákova and Valsecchi (1982) followed the motion of particles ejected from the nucleus of P/Lexell at its 1770 perihelion passage through the very close approach to Jupiter in 1779, to examine the dynamics of the peculiar meteor stream possibly associated with the comet. In some sense this work is analogous to the one on P/Oterma; this time the modulus of the velocity vector at the perihelion passage of 1770 was slightly varied, and again a sequence of orbital patterns at the close encounter was recognized. In this case, however, the trajectories were much simpler than in that of P/Oterma, due to the much higher velocity of the bodies relative to Jupiter, a velocity sufficiently high (J_j < 2.83) to allow ejection from the solar system on a hyperbolic orbit.

6. CONCLUSIONS

Like other problems regarding comets, e.g. those of their origin, composition, relation to the outer planets, lifetime, etc., also that of their evolution into short-period orbits still demands much work. The last 20 years have seen a great increase of our knowlodge, especially for what concerns the main evolutionary channels and the dynamics of close planetary encounters, a subject that has had to wait the availability of fast computing tools before being tackled. Although the principal evolutionary paths are probably all identified, the efficiencies are not known for all of them, and new numerical experiments, modelling for instance the transfer from low eccentricity orbits just outside Neptune to ones like those of Chiron and P/Schwassmann-Wachmann 1, would be useful.

However, a quantitative assessment of the efficiencies of the various evolutionary channels, as well as a better understanding of the dynamics of close encounters, are still insufficient for a global picture: we do not know how densely the various regions of the phase space of orbital elements are populated, and how the present situation differs from the past ones.

As we have seen, the orbits of periodic comets are chaotic, that is they can pass through different regimes of motion in an unpredictable way. This obviously complicates the numerical work on the reconstruction of past orbital evolutions; moreover, apart from the orbits of Halley type comets, and of those of the so-called Jupiter family, all other possible regimes of motion for chaotic orbits imply large perihelion distances, rendering bodies in these phases of evolution unobservable.

Another factor that needs to be taken into account is the physical nature of the objects; this can have several consequences, since the finite lifetime in the inner regions of the solar system can render practically impossible those transitions between types of orbits whose low probability implies long time scales. Moreover, the number of candidates for potential evolution into short-period orbits can vary also in the other direction, i.e. increase because of splitting events that leave long lasting fragments, like in the case of the recently suggested common origin of P/Van Biesbroeck and P/Neujmin 3 (Carusi et al., 1984).

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DISCUSSION

Milani: The variation of orbital behaviour you find in TSCs by varying one angular variable reminds me of the theory of "Cantori", that is invariant subsets in hamiltonian systems whose projection on one axis is a Cantor set. Do you find anything suggesting this kind of behaviour?
Valsecchi: The spacing of our objects was possibly large compared to the scale of the phenomena you mention. However, we found the longest TSCs, and the most complex orbital patterns, between neighbouring "families" of similar, simple patterns, and this may well be due to what you suggest.

Mignard: Among the TSCs by Jupiter, are the comet orbits comparable to those of Jupiter outer satellites?
Valsecchi: They are less tightly bound, and their planetocentric elements vary considerably even during a single revolution. Even when you have a TSC of long duration (10-100 years), the eccentricity is quite large, larger than those of the irregular jovian satellites.

Fernández: Are there possible evolutionary paths that bring retrograde-orbit comets captured by the outer jovian planets to small perihelion distances under the dynamical control of Jupiter?
Valsecchi: I cannot support my opinion quantitatively, since numerical experiments on the process you mention have not been carried out, but I think that this channel should be inefficient.