

# RISK-BEARING AND CONSUMPTION THEORY

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## ABSTRACT

A simple risky situation is studied in the framework of consumption theory. Saving is shown to be a substitute to insurance. Two new concepts, risk-bearing budget and effective risk coverage, are introduced in order to give a more accurate insight into the optimal risk-bearing decision. The effect of a variation in current consumption and in wealth upon the optimal insurance coverage is analysed.

## I. INTRODUCTION

The problem of determining the optimal insurance coverage, when facing a risky situation, has raised considerable interest following the work of Arrow [1] on the economics of medical care in 1963. However, it should be noted that few years earlier, Borch [3] had studied extensively the problem of optimal risk retention in a reinsurance context.

In 1968, Mossin [5] and Smith [10] came independently to rather surprising conclusions about the optimal decisions of a rational insurance buyer (a rational individual being understood as a so-called expected-utility-maximiser). Their most striking finding was perhaps that it suffices that the premium is actuarially unfair to the buyer to make a full coverage non optimal. Mossin was particularly puzzled by the real-life fact that "some of his best friends do take full coverage", his own comments about this observed behaviour are certainly worth reading (see [5] p. 558).

While the authors mentioned before concentrated mainly on the theoretical aspects of the problem, there has been over the same period, a few attempts to test the expected-utility hypothesis against objective data. As far as this author knows, the results have

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been rather disappointing despite all the seriousness of the effort made. The interested reader is particularly referred to Murray [6] and to Pashigian, Sckade and Menefee [7].

The approach traditionally adopted in attempting to build a decision model to determine the optimal insurance coverage, is based on the maximisation of the expected utility of terminal wealth. This approach leaves aside completely the saving element. It is assumed tacitly that the optimal strategy in dealing with a risky situation involves only a decision about insurance buying. By studying a very simple risky situation, it will be shown that saving can, in fact, be a substitute to insurance.

The introduction of the saving element into the model, will lead us to a formulation of the problem in terms of consumption theory; see for example, Sandmo [9]. This new formulation will illustrate that both current consumption and terminal wealth are determining factors of an optimal risk-bearing decision.

## 2. PROBLEM STATEMENT

Consider an individual who owns a certain asset worth  $N$  that can either be completely lost during the forthcoming period with probability  $q$  or remain unaltered with probability  $p = 1 - q$ . This risky situation can be dealt with by buying insurance, by saving, or by merely accepting the eventuality of a decrease in the terminal wealth.

It is assumed that insurance is available at a premium rate of  $\lambda$  per dollar of coverage, and that the rate of return on riskless investments is  $i$  (which is also the rate of interest at which money can be borrowed). For the sake of simplicity, it will also be assumed that any claim settlement occurs at the end of the period.

The decision-maker faces a twofold problem. He must decide by how much he is willing to reduce his current consumption specifically for the purpose of risk-bearing and at the same time, he must find an optimal allocation of his risk-bearing budget between insurance and saving.

## 3. THE MODEL

Before we formulate the problem formally, let us summarise the notation that will be used throughout our analysis:

$Q$	risk-bearing budget taken out of current consumption
$C$	current consumption for the period
$Y$	terminal wealth or value of all marketable assets at the end of the period
$N$	value of a particular asset at risk during the period
$X$	amount of insurance coverage
$q$	probability that $N$ will be lost during the period
$p = 1 - q$	probability that $N$ will remain unaltered during the period
$\lambda$	cost of insurance per dollar of coverage
$i$	rate of interest on riskless investments
$S$	accumulated value of the saving at the end of the period, $S \equiv (Q - \lambda X)(1 + i)$ .

Let  $A$  and  $W$  be the current consumption level and the value of the terminal wealth respectively, if there was no risk of losing  $N$ . In such a case,  $Q$  and  $X$  would be null. We now have the following basic relationships:

$$\begin{aligned} C &= A - Q \\ Y &= W + S && \text{if there is no loss} \\ Y &= W - N + X + S && \text{if there is a loss} \end{aligned}$$

It is realistic to assume that the insurance company includes into the calculation of the premium rate a loading to allow for a profit and to cover administrative expenses; furthermore the premium should be smaller than the discounted value of the coverage. Consequently we shall work with

$$\boxed{\frac{q}{(1+i)} < \lambda < \frac{1}{(1+i)}}$$

We shall assume that the individual's preferences are representable by a utility function  $V(Q)$  having the following form:

$$V(Q) = g(C) + E[u(Y)],$$

where  $g(C)$  and  $u(Y)$  are utility functions, in the von Neumann-Morgenstern sense, associated with current consumption and termi-

nal wealth respectively. It will be assumed also that the individual is a risk averter in both  $C$  and  $Y$ , that is:

$$\begin{matrix} g'(C) > 0 & g''(C) < 0 \\ u'(Y) > 0 & u''(Y) < 0 \end{matrix}$$

Let us now introduce the Arrow-Pratt (see [2] and [8]) absolute risk aversion measure defined as follows:

$$R_A(Y) = - \frac{u''(Y)}{u'(Y)}$$

We shall assume finally a decreasing absolute risk aversion, that is  $R'_A(Y) < 0$ . Which according to Arrow ([2] p. 35) "amounts to saying that the willingness to engage in small bets of fixed size increases with wealth". It is easy to verify that  $R'_A(Y) < 0$  implies  $u'''(Y) \cdot u'(Y) > [u''(Y)]^2$ , thus the existence of  $u'''(Y)$  is required.

The reader should be warned that convenient utility functions for which  $R_A(Y)$  is decreasing might be difficult to obtain. As a matter of fact, a quadratic utility function despite its operational attractiveness is not acceptable. Very fortunately, Pratt ([8] p. 133) gives us all the strictly decreasing risk-averse utility functions, just in case one is needed.

The problem can now be formulated as follows:

$$\text{Max}_{Q, X} V(Q) = g(A - Q) + pu[W + S] + q[W - N + X + S]$$

subject to

$$0 \leq X \leq N$$

#### 4. ANALYSIS OF THE OPTIMAL SOLUTION

Let  $Z(Q)$  be the maximum value of  $E[u(Y)]$  for a given value of  $Q$ . We can write

$$Z(Q) = \text{Max}_{X|Q} \{p u[W + S] + q u[W - N + X + S]\}$$

The optimum values of  $Q$  and  $X$  must satisfy the following first-order conditions:

$$g'(A - Q) = Z'(Q) \tag{1}$$

and

$$\frac{[1 - \lambda(1 + i)] q}{\lambda(1 + i) p} = \frac{u'[W + S]}{u'[W - N + X + S]} \tag{2}$$

*Theorem 1*

If  $q < \lambda(1 + i) < 1$  then  $X < N$

*Proof*<sup>1)</sup>

A necessary condition for  $X = N$  is

$$\left. \frac{dZ}{dX} \right|_{X=N} \geq 0$$

which is equivalent to requiring

$$\frac{[1 - \lambda(1 + i)]q}{\lambda(1 + i)p} \geq \frac{u'[W + (Q - N)(1 + i)]}{u'[W + (Q - N)(1 + i)]}$$

Since  $q < \lambda(1 + i) < 1$ , it is easy to see that the L.H.S. is strictly smaller than one. However, the R.H.S. is obviously equal to one. Thus, the necessary condition for  $X = N$  can not hold no matter the value of  $Q$  and  $N$ .

*Comment*

This is one of the main results of the traditional approach, its robustness is therefore confirmed. However, it should be noted that this result is based upon the existence of  $u'(Y)$  over the whole domain of  $Y$ . It is certainly possible to find some people who use a decision rule that cannot be represented by a utility function of the type we have described. Suppose for example, that an individual states the problem as follows:

$$\text{Max } V(Q) = g(A - Q) + pu[W + S] + q[W - N + X + S]$$

subject to

$$0 \leq X \leq N$$

and

$$0 \leq X + S - N$$

The last constraint just says that the individual will not accept any decrease of his terminal wealth. It is easy to verify that there are two possible cases for an optimal solution to the modified problem:

1.  $Q = \lambda N, \quad X = N, \quad S = 0$
2.  $Q > \lambda N, \quad 0 \leq X < N, \quad S > 0$

<sup>1)</sup> This proof employs the method of Mossin [3].

The first of these cases corresponds to buying a full coverage. An interesting discussion of a similar decision rule is presented in Borch [4] pp. 41-42.

*Theorem 2*

If  $u'(Y) > 0$ ,  $u''(Y) < 0$  and  $R'_A(Y) < 0$ , then

$$\frac{(1+i)}{[\lambda(1+i) - 1]} < \frac{dX}{dQ} < 0$$

That is, the optimal insurance coverage is lesser, the greater the optimal risk-bearing budget is.

*Proof*

By implicit differentiation of (2) we obtain

$$\frac{dX}{dQ} = - \frac{\{-\lambda(1+i)^2 pu''[W+S] + (1+i)[1-\lambda(1+i)] qu''[W-N+X+S]\}}{\{\lambda^2(1+i)^2 pu''[W+S] + [1-\lambda(1+i)]^2 qu''[W-N+X+S]\}} \quad (3)$$

Let

$$G = -\lambda(1+i)^2 pu''[W+S] + (1+i)[1-\lambda(1+i)] qu''[W-N+X+S] \quad (4)$$

Since  $u''(Y) < 0$ , it follows that the denominator of (3) is strictly negative, which implies that  $(dX/dQ)$  is of the same sign as  $G$ . From (2) we obtain an explicit expression for  $\lambda\phi$  and substituting it into (4) we obtain

$$G = (1+i) [1-\lambda(1+i)] qu'[W-N+X+S] \{R_A[W+S] - R_A[W-N+X+S]\}$$

Since  $\lambda(1+i) < 1$ , and  $u'(Y) > 0$ ,  $G$  is of the same sign as the expression in  $\{ \}$ . From theorem 1 we know that  $X < N$ , thus

$$[W+S] > [W-N+X+S]$$

and it follows that  $G < 0$  since  $R'_A(Y) < 0$ .

Therefore

$$\frac{dX}{dQ} < 0.$$

Now suppose that

$$\frac{dX}{dQ} \leq \frac{(1+i)}{[\lambda(1+i) - 1]}$$

From (3) we can write

$$\begin{aligned} \frac{\lambda(1+i)^2 pu''[W+S] - (1+i)[1 - \lambda(1+i)]qu''[W-N+X+S]}{\lambda^2(1+i)^2 pu''[W+S] + [1 - \lambda(1+i)]^2 q''[W-N+X+S]} &\leq \\ &\leq \frac{(1+i)}{[\lambda(1+i) - 1]} \end{aligned}$$

which becomes

$$-\lambda(1+i)^3 pu''[W+S] \leq 0$$

and finally

$$u''[W+S] \geq 0,$$

which is impossible by hypothesis.

Therefore we must have

$$\frac{dX}{dQ} > \frac{(1+i)}{[\lambda(1+i) - 1]}$$

*Corollary 1*

$$0 < \frac{d(S+X)}{dQ} < (1+i)$$

since

$$\frac{d(S+X)}{dQ} = (1+i) + [1 - \lambda(1+i)] \frac{dX}{dQ}$$

The amount  $(S+X)$  can be interpreted as the effective risk coverage since the "raison d'être" of  $S$  is the risky situation only.

*Corollary 2*

$$(1 + i) < \frac{dS}{dQ} < \frac{(1 + i)}{[1 - \lambda(1 + i)]}$$

since

$$\frac{dS}{dQ} = (1 + i) - \lambda(1 + i) \frac{dX}{dQ}$$

*Comment*

This theorem and its second corollary illustrate clearly that saving can be considered as a substitute to insurance coverage since  $(dX/dQ)$  and  $(dS/dQ)$  have opposite signs.

The first corollary is quite illuminating, as it shows that the effective risk coverage increases when the risk-bearing budget increases. By looking only at the insurance coverage, we would be left under the misleading impression that an increase in the risk-bearing budget implies a willingness to accept the eventuality of a larger decline of the terminal wealth.

*Theorem 3*

If  $u'(Y) > 0$ ,  $u''(Y) < 0$  and  $R'_A(Y) < 0$ , then

$$\frac{dZ}{dQ} = \frac{(1 + i)}{[1 - \lambda(1 + i)]} pu'[W + S] > 0$$

and

$$\frac{d^2Z}{dQ^2} = \frac{(1 + i)^2}{[1 - \lambda(1 + i)]} \left[ 1 - \lambda \frac{dX}{dQ} \right] pu''[W + S] < 0$$

*Proof*

$$\begin{aligned} \frac{dZ}{dQ} &= (1 + i) \left[ 1 - \lambda \frac{dX}{dQ} \right] pu'[W + S] \\ &+ \left\{ \frac{dX}{dQ} + (1 + i) \left[ 1 - \lambda \frac{dX}{dQ} \right] \right\} qu'[W - N + X + S] \quad (5) \end{aligned}$$

From (2) we have

$$u'[W - N + X + S] = \frac{\lambda(1+i)p}{[1 - \lambda(1+i)]q} u'[W + S]$$

substituting in (5) we obtain

$$\frac{dZ}{dQ} = \frac{(1+i)}{[1 - \lambda(1+i)]} pu'[W + S]$$

which is obviously of the sign as  $u'[W + S]$ , i.e. strictly positive.

Now

$$\begin{aligned} \frac{d^2Z}{dQ^2} &= \frac{d}{dQ} \left[ \frac{dZ}{dQ} \right] \\ &= \frac{(1+i)^2}{[1 - \lambda(1+i)]} \left[ 1 - \lambda \frac{dX}{dQ} \right] pu''[W + S] \end{aligned}$$

From theorem 2 we have

$$1 < \left[ 1 - \lambda \frac{dX}{dQ} \right] < \frac{1}{[1 - \lambda(1+i)]}$$

Therefore,  $\frac{d^2Z}{dQ^2}$  is of the same sign as  $u''[W + S]$  i.e. strictly negative.

*Comment*

This theorem is primarily of operational significance. It is a prerequisite for the following theorems, that will study the effect of a variation in  $A$  and  $W$  on the optimal solution.

*Theorem 4*

If  $g''(C) < 0$  and  $Z''(Q) < 0$ , then  $\frac{\partial Q}{\partial A} > 0$ .

*Proof*

By implicit differentiation of (1) we obtain

$$\frac{\partial Q}{\partial A} = \frac{g''(A - Q)}{g''(A - Q) + Z''(Q)}$$

Since  $g''(C) < 0$ , and  $Z''(Q) < 0$ ,  $\frac{\partial Q}{\partial A}$  must be of opposite sign as of  $g''(A - Q)$ , i.e. strictly positive.

*Corollary*

Since  $\frac{\partial X}{\partial Q} < 0$ , and  $\frac{\partial(S + X)}{\partial Q} > 0$ , we now have

$$\frac{\partial X}{\partial A} < 0, \quad \text{and} \quad \frac{\partial(S + X)}{\partial A} > 0$$

*Comment*

This theorem and its corollary add a new perspective to the theory of risk-bearing. We can see that current consumption affects directly the optimal risk-bearing strategy. Our approach prescribes that an individual with a higher consumption level should put aside more money for risk-bearing purposes, and in doing so—should rely increasingly on self-insurance rather than on outside insurance. When we make such a statement, we must keep in mind that it is assumed that all other variables are kept constant. However, there is strong empirical evidence that the consumption level is related to the wealth. Therefore, it might be less hazardous to draw conclusions in the light of the interaction between current consumption and wealth, at the condition that one can be expressed explicitly in terms of the other.

*Theorem 5*

If  $g''(C), u''(Y) < 0$  and  $Z''(Q) < 0$ , then

$$\frac{\partial Q}{\partial W} < 0 \quad \text{and} \quad \frac{\partial X}{\partial W} < 0$$

*Proof*

Differentiating (1) and (2) with respect to  $W$  we obtain

$$\begin{aligned} \left[ g''(A - Q) + \frac{\partial^2 Z}{\partial Q^2} \right] \frac{\partial Q}{\partial W} + \frac{\partial^2 Z}{\partial Q \partial X} \frac{\partial X}{\partial W} + \frac{\partial^2 Z}{\partial Q \partial W} &= 0 \\ \frac{\partial^2 Z}{\partial Q \partial X} \frac{\partial Q}{\partial W} + \frac{\partial^2 Z}{\partial X^2} \frac{\partial X}{\partial W} + \frac{\partial^2 Z}{\partial X \partial W} &= 0 \end{aligned}$$

Solving these two equations we obtain:

$$\frac{\partial Q}{\partial W} = \frac{\frac{\partial^2 Z}{\partial Q \partial W} \cdot \frac{\partial^2 Z}{\partial X^2} - \frac{\partial^2 Z}{\partial X \partial W} \cdot \frac{\partial^2 Z}{\partial Q \partial X}}{\left[ \frac{\partial^2 Z}{\partial Q \partial X} \right]^2 - \left[ g''(A-Q) + \frac{\partial^2 Z}{\partial Q^2} \right] \frac{\partial^2 Z}{\partial X^2}}$$

and

$$\frac{\partial X}{\partial W} = \frac{g''(A-Q) \cdot \frac{\partial^2 Z}{\partial Q \partial W}}{\left[ \frac{\partial^2 Z}{\partial Q \partial X} \right]^2 - \left[ g''(A-Q) + \frac{\partial^2 Z}{\partial Q^2} \right] \frac{\partial^2 Z}{\partial X^2}}$$

Knowing that

$$\frac{\partial Z}{\partial W} = \frac{1}{(1+i)} \frac{\partial Z}{\partial Q}, \quad \frac{\partial X}{\partial Q} = - \left[ \frac{\partial^2 Z}{\partial Q \partial X} \right] \Big| \frac{\partial^2 Z}{\partial X^2}$$

and

$$Z''(Q) = \frac{\partial^2 Z}{\partial Q^2} + \frac{\partial^2 Z}{\partial Q \partial X} \cdot \frac{\partial X}{\partial Q}$$

we can write in a simpler form

$$\frac{\partial Q}{\partial W} = \frac{1}{(1+i)} \left\{ \frac{-Z''(Q)}{g''(A-Q) + Z''(Q)} \right\}$$

and

$$\frac{\partial X}{\partial W} = \frac{1}{(1+i)} \frac{\partial X}{\partial Q} \left\{ \frac{g''(A-Q)}{g''(A-Q) + Z''(Q)} \right\}$$

Recalling from theorem 2 that  $\frac{\partial X}{\partial Q} < 0$ , it follows that

both  $\frac{\partial Q}{\partial W}$  and  $\frac{\partial X}{\partial W}$  are strictly negative.

*Corollary*

$$\frac{\partial(S+X)}{\partial W} < 0$$

*Proof*

$$\frac{\partial(S+X)}{\partial W} = (1+i) \frac{\partial Q}{\partial W} + [1 - \lambda(1+i)] \frac{\partial X}{\partial W}$$

which is strictly negative since

$$\frac{\partial Q}{\partial W} < 0, \quad \frac{\partial X}{\partial W} < 0 \quad \text{and} \quad [1 - \lambda(1 + i)] > 0$$

*Comment*

The current view in the theory of risk-bearing is that an individual with decreasing risk aversion should pay less for insurance the greater his assets are, in dealing with a given risky situation. Theorem 5 and its corollary reinforce this view point. It is in fact seen that both the insurance coverage ( $X$ ) and the effective risk coverage ( $S + X$ ) should decrease when wealth (marketable assets) increases. Again, the interdependence of current consumption and wealth should be kept in mind and incite us to a certain reserve in drawing conclusions.

### 5. GENERAL DISCUSSION

One might wonder if our model might prescribe buying no insurance at all. A necessary condition for such an optimal solution is to have

$$\left. \frac{dZ}{dX} \right|_{X=0} \leq 0$$

which implies

$$\frac{[1 - \lambda(1 + i)]q}{\lambda(1 + i)p} \leq \frac{u'[W + Q(1 + i)]}{u'[W - N + Q(1 + i)]}$$

If  $N$  is sufficiently small with respect to  $W$ , there might exist a value of  $Q$  for which the condition is satisfied. Unfortunately, it seems impossible to be more conclusive about that.

By looking at figures 1 and 2, one can have a global picture of the optimisation process. It is noticeable that  $Z(Q)$  is somewhat flatter than  $g(A - Q)$ , it is not merely a drawing fantasy. Let us recall that

$$Z'(Q) = \left\{ \frac{(1 + i)p}{[1 - \lambda(1 + i)]} \right\} u'[W + S]$$

Since  $X < N$ , the smallest value that  $S$  can take is strictly greater than  $-\lambda N(1 + i)$ , where  $\lambda N$  is the premium for a full coverage. In practice, we can expect  $\lambda N$  to be a fairly small fraction

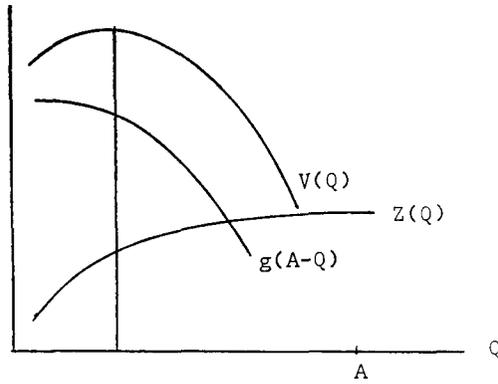


Fig. 1. Determination of  $Q$ .

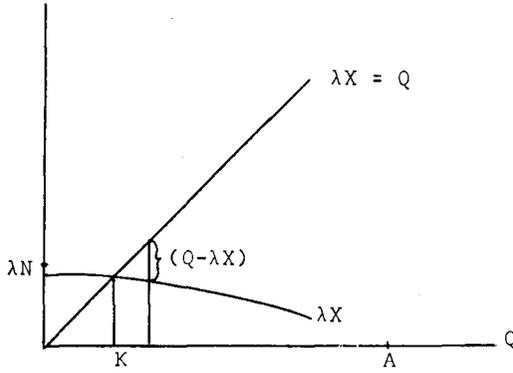


Fig. 2. Allocation of  $Q$ .

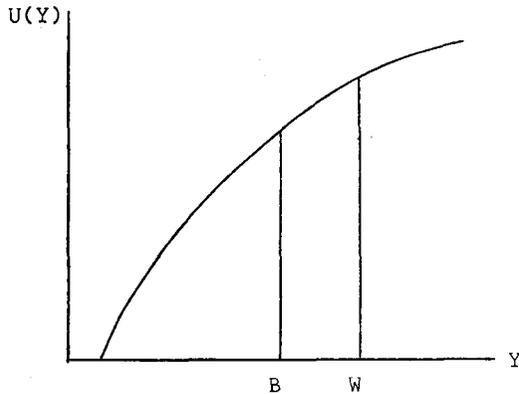


Fig. 3. Utility of terminal wealth,  $B = [W - \lambda N(1 + i)]$ .

of  $N$ . Since  $W \geq N$ , it follows that  $u'(Y)$  will be relatively small for any  $Y > [W - \lambda N(1 + i)]$ , as illustrated by figure 3. At the same time,  $\left\{ \frac{(1 + i)p}{[1 - \lambda(1 + i)]} \right\}$  should not be much greater than  $(1 + i)$ .<sup>2)</sup>

Our last argumentation has a far-reaching implication. It suggests that a risk-averse insurance buyer should be more sensitive to his level of current consumption than to the value of his assets in making a decision in this given risky situation. There might be there at least a partial explanation to the frustrating efforts of those who have tried to test the expected-utility hypothesis against objective data, while concentrating mainly on the effect of the decision upon terminal wealth.

We might still think that quite a few of our best friends would anyway buy a full coverage in dealing with the risky situation we have described—we should not forget about them. Inspection of figure 2 reveals that there is a unique value of the risk-bearing budget for which insurance only is involved, it is on the graph at  $Q = K$ . For  $Q < K$ , there should be some borrowing at rate  $i$  in order to buy insurance and for  $Q > K$ , there should be a combination of saving and insurance coverage.

Most of the people will agree on the difficulty inherent to an optimal allocation of the risk-bearing budget between saving and insurance buying. It is much easier to buy a full coverage and forget about the eventuality of a decrease of the assets, particularly if the level of current consumption is not much affected by the outlay of the premium for a full coverage. On the other hand, can one expect an insurance salesman to recommend to his client to buy less insurance and save more?

## 6. A SPECIAL CASE: RISK-BEARING WITHOUT SAVING

Since it might be realistic to assume that many of our friends take care of their risk-bearing problems only through insurance coverage,

<sup>2)</sup> In fact  $\left\{ \frac{(1 + i)p}{[1 - \lambda(1 + i)]} \right\} > (1 + i)$  for any value of  $q$ .

If  $\lambda = \alpha q$  with  $\frac{1}{1 + i} < \alpha < \frac{1}{q(1 + i)}$ , it is easy to verify that

$$\frac{d}{dq} \left\{ \frac{(1 + i)p}{[1 - \lambda(1 + i)]} \right\} > 0.$$

even if it is theoretically non optimal, we shall now modify our model as to make it possible to understand the behaviour of this class of people. The problem becomes:

Max  $V(\lambda X) = g[A - \lambda X] + pu[W] + qu[W - N + X]$   
subject to

$$0 \leq X \leq N$$

Let  $Z(\lambda X) = pu[W] + qu[W - N + X]$  and let us still assume that the decision-maker is risk averter in both current consumption and terminal wealth. Then  $Z(\lambda X)$  is concave since

$$Z'(\lambda X) = qu'[W - N + X] > 0$$

and

$$Z''(\lambda X) = qu''[W - N + X] < 0$$

The first-order condition for the existence of an optimal solution to our new problem is:

$$qu'[W - N + X] = \lambda g'[A - \lambda X] \quad (6)$$

For a full coverage to be optimal, we must have

$$V'(\lambda X) |_{X=N} \geq 0$$

which implies

$$\frac{q}{\lambda} \geq \frac{g'[A - \lambda N]}{u'[W]}$$

It is certainly possible to find some people for which this condition is respected; it merely implies that they are more concerned about avoiding a decrease in the value of their assets than about reducing their current consumption by an amount equal to the premium for a full coverage.

At the other extreme, no insurance at all should be taken if

$$V'(\lambda X) |_{X=0} \leq 0$$

which implies

$$\frac{q}{\lambda} \leq \frac{g'[A]}{u'[W - N]}$$

This is the case of an individual particularly sensitive to a decrease of his current consumption while being little affected by a complete loss of his asset at risk.

The new decision process is fully portrayed by figure 4. It can be observed that  $g[A - \lambda X]$  is much flatter than in figure 1; it is so because, by excluding the saving element, the current consumption level cannot get any lower than  $[A - \lambda N]$ . On the other hand, the slope of  $Z(\lambda X)$  is greatly influenced by the size of  $N$  with respect to  $W$ , since  $Z'(\lambda X) = qu'[W - N + X]$ , where  $W - N \leq W - N + X \leq W$ ; however, this influence will be less perceptible the smaller  $q$  will be. The subjective evaluation of  $q$  might turn out to be a key element of the decision process.

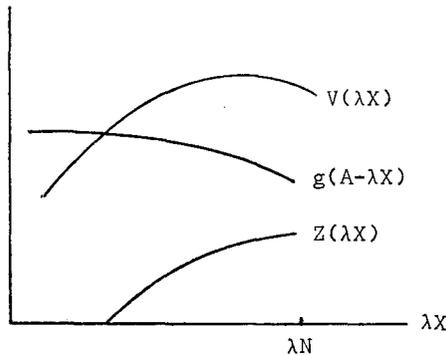


Fig. 4. Decision process without saving.

By implicit differentiation of (6) we obtain

$$\frac{\partial X}{\partial W} = \frac{-qu''[W - N + X]}{\lambda^2 g''[A - \lambda X] + qu''[W - N + X]} < 0$$

and

$$\frac{\partial X}{\partial A} = \frac{\lambda g''[A - \lambda X]}{\lambda^2 g''[A - \lambda X] + qu''[W - N + X]} > 0$$

If we recall that, in this special case, the effective risk coverage is equivalent to the insurance coverage, then these last results are in accord with the general risk-bearing theory we have developed before.

### 7. CONCLUSION

The obvious simplicity of the risky situation we have studied should restrain us from moving hastily toward a drastic generalisa-

tion of our findings. However, the results we have obtained should hopefully contribute to clarifying the relationship existing between the risk aversion concept and the insurance buying practices.

From a normative standpoint, it has unequivocally been shown that an individual with decreasing risk aversion should always buy less than a full insurance coverage, while—at the same time—complementing his risk coverage with saving (either negative or positive). Furthermore, it was shown that there is a substitution effect between insurance coverage and saving.

From a descriptive standpoint, one might suspect that the saving element is completely left out in actual decision-making. If such is the case, it has been shown that risk aversion can be compatible with buying full insurance coverage, and the condition for realisation of such a case has been explicitly brought out.

In all cases, our analysis brings forth evidence that the risk-bearing decision involves both current consumption and terminal wealth. It might be a new starting point for any further research on the theory of individual risk-bearing.

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