Notation

Numbers refer to pages where definitions are given

$\equiv$ definition $\Rightarrow$ implies
$\exists$ there exists Σ summation sign
$\square$ end of a proof

Sets

$\cup$ $A \cup B$, union of A and B
$\cap$ $A \cap B$, intersection of A and B
$\supset$ $A \subset B$, $B \supset A$, A is contained in B
$-$ $A - B$, B subtracted from A
$\in$ $x \in A$, is a member of A
$\varnothing$ the empty set

Maps

$\phi: \mathscr{U} \to \mathscr{V}$, ϕ maps $p \in \mathscr{U}$ to $\phi(p) \in \mathscr{V}$
$\phi(\mathscr{U})$ image of $\mathscr{U}$ under ϕ
ϕ^{-1} inverse map to ϕ
$f \circ g$ composition, g followed by f
ϕ_*, ϕ^* mappings of tensors induced by map ϕ, 22–4

Topology

$\bar{A}$ closure of A
$\dot{A}$ boundary of A, 183
$\operatorname{int} A$ interior of A, 209

Differentiability

$C^0, C^r, C^{r-}, C^\infty$ differentiability conditions, 11

Manifolds

$\mathscr{M}$ n-dimensional manifold, 11
$(\mathscr{U}_\alpha, \phi_\alpha)$ local chart determining local coordinates x^a, 12

$\partial\mathscr{M}$ boundary of $\mathscr{M}$, 12
R^n Euclidean n-dimensional space, 11
$\frac{1}{2}R^n$ lower half $x^1 \leqslant 0$ of R^n, 11
S^n n-sphere, 13
$\times$ Cartesian product, 15

Tensors

$(\partial/\partial t)_\lambda$, $\mathbf{X}$ vectors, 15
$\boldsymbol{\omega}$, $\mathrm{d}f$ one-forms, 16, 17
$\langle\boldsymbol{\omega}, \mathbf{X}\rangle$ scalar product of vector and one-form, 16
$\{\mathbf{E}_a\}, \{\mathbf{E}^a\}$ dual bases of vectors and one-forms, 16, 17
$T^{a_1 \dots a_r}{}_{b_1 \dots b_s}$, components of tensor $\mathbf{T}$ of type (r, s), 17–19
$\otimes$ tensor product, 18
$\wedge$ skew product, 21
$()$ symmetrization (e.g. $T_{(ab)}$), 20
$[]$ skew symmetrization (e.g. $T_{[ab]}$), 20
$\delta^a{}_b$ Kronecker delta ($+1$ if $a = b$, 0 if $a \neq b$)
$T_p, T^*{}_p$ tangent space at p and dual space at p, 16
$T^r_s(p)$ space of tensors of type (r, s) at p, 18
$T^r_s(\mathscr{M})$ bundle of tensors of type (r, s) on $\mathscr{M}$, 51
$T(\mathscr{M})$ tangent bundle to $\mathscr{M}$, 51
$L(\mathscr{M})$ bundle of linear frames on $\mathscr{M}$, 51

Derivatives and connection

$\partial/\partial x^i$ partial derivatives with respect to coordinate x^i
$(\partial/\partial t)_\lambda$ derivative along curve $\lambda(t)$, 15
d exterior derivative, 17, 25
$L_\mathbf{X}\mathbf{Y}$, $[\mathbf{X}, \mathbf{Y}]$ Lie derivative of $\mathbf{Y}$ with respect to $\mathbf{X}$, 27–8
∇, $\nabla_\mathbf{X}$, $T_{ab;c}$ covariant derivative, 30–2
$\mathrm{D}/\partial t$ covariant derivative along curve, 32
$\Gamma^i{}_{jk}$ connection components, 31
exp exponential map, 33

Riemannian spaces

$(\mathscr{M}, \mathbf{g})$ manifold $\mathscr{M}$ with metric $\mathbf{g}$ and Christoffel connection
$\boldsymbol{\eta}$ volume element, 48
R_{abcd} Riemann tensor, 35
R_{ab} Ricci tensor, 36

R curvature scalar, 41
C_{abcd} Weyl tensor, 41
$O(p, q)$ orthogonal group leaving metric G_{ab} invariant, 52
G_{ab} diagonal metric diag $(\underbrace{+1, +1, \ldots, +1}_{p \text{ terms}}, \underbrace{-1, \ldots, -1}_{q \text{ terms}})$
$O(\mathscr{M})$ bundle of orthonormal frames, 52

Space–time

Space–time is a 4-dimensional Riemannian space $(\mathscr{M}, \mathbf{g})$ with metric normal form diag $(+1, +1, +1, -1)$. Local coordinates are chosen to be (x^1, x^2, x^3, x^4).
T_{ab} energy momentum tensor of matter, 61
$\Psi_{(i)}{}^{a \ldots b}{}_{c \ldots d}$ matter fields, 60
L Lagrangian, 64
Einstein's field equations take the form

$$R_{ab} - \tfrac{1}{2} R g_{ab} + \Lambda g_{ab} = 8\pi T_{ab},$$

where Λ is the cosmological constant.
$(\mathscr{S}, \boldsymbol{\omega})$ is an initial data set, 233

Timelike curves

$\perp$ perpendicular projection, 79
$\mathrm{D}_F/\partial s$ Fermi derivative, 80–1
θ expansion, 83
$\omega^a, \omega_{ab}, \omega$ vorticity, 82–4
σ_{ab}, σ shear, 83–4

Null geodesics

$\hat{\theta}$ expansion, 88
$\hat{\omega}_{ab}, \hat{\omega}$ vorticity, 88
$\hat{\sigma}_{ab}, \hat{\sigma}$ shear, 88

Causal structure

I^+, I^- chronological future, past, 182
J^+, J^- causal future, past, 183
E^+, E^- future, past horismos, 184
D^+, D^- future, past Cauchy developments, 201
H^+, H^- future, past Cauchy horizons, 202

Boundary of space–time

$\mathscr{M}^* = \mathscr{M} \cup \Delta$ where Δ is the c-boundary, 220

$\mathscr{I}^+, \mathscr{I}^-, i^+, i^-$ c-boundary of asymptotically simple and empty spaces, 122, 225

$\bar{\mathscr{M}} = \mathscr{M} \cup \partial\mathscr{M}$ when $\mathscr{M}$ is weakly asymptotically simple; the boundary $\partial\mathscr{M}$ of $\mathscr{M}$ consists of $\mathscr{I}^+$ and $\mathscr{I}^-$, 221, 225

$\mathscr{M}^+ = \mathscr{M} \cup \partial$ where ∂ is the b-boundary, 283