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Limits on a stochastic background of gravitational waves

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1. Introduction

A gravitational wave background (GWB) of sufficient strength, characterized by Ω, the energy density per logarithmic frequency interval in units of the closure density, would introduce timing residuals in the most stable millisecond pulsars. For a description pertaining to the observations of PSR's 1937+21 and 1855+09 see Kaspi, Taylor and Ryba (1994), hereafter KTR, and references therein. Thorsett and Dewey (1996, see also this volume) present a method for placing a statistical upper limit on Ω. Their method however, cannot correctly account for the presence of a known level of white measurement noise in the timing residuals. We use a Bayesian approach which can best account for this white noise along with our lack of previous knowledge on the parameter Ω (McHugh, Zalamansky, Vernotte and Lantz, submitted).

2. Power spectrum of the timing residuals

Application of a multiparameter model to pulsar timing observations by means of a least-squares fit generates a set of timing residuals. Any description of the noise spectrum of these residuals must include a 'white' term due to measurement uncertainties. In addition, it can be shown that for many GWB scenarios a significant Ω would produce a red spectrum of the form

$$S(f) = 1.34 \times 10^4 \cdot \Omega h^2 f^{-5} \mu\text{s}^2\text{yr} \tag{1}$$

where $h = H_0/(100 \text{ km s}^{-1}\text{Mpc}^{-1})$ and f is given in units of yr^{-1} (KTR). Such a noise level would be isotropic and is expected to produce a correlation in the timing residuals between pulsars close together in the sky.

Estimates of the power spectrum of the residuals for 1937+21 and 1855+09 at four, octave-spaced frequencies, the lowest being the inverse of the total observation time, are presented in KTR. The expectation value due to a GWB with $\Omega h^2 = 10^{-7}$ is $< S_m >_g$, and $< S_m >_w$ is the expectation value due to the known level of measurement noise ($m = 1, 2, 4, 8$ indicating different octaves). The measured spectrum of 1937+21 has a strong red component whereas 1855+09 has a relatively flat spectrum. Because of the expected uniformity of a GWB, the red noise in 1937+21 can be assumed to have its origin in some other source (e.g. intrinsic pulsar timing noise) and the quieter (in terms of red noise) 1855+09 can be used to place an upper limit on Ωh^2.

3. A Bayesian approach

The probability distribution for the power spectrum measurements, denoted by σ is conditional on Ω and the level of measurement noise. The overall distribution is a product of the χ^2 distributions for individual spectral estimates (KTR). Bayes' theorem gives the distribution for Ω conditional on the measurements

$$p(\Omega|\vec{\sigma}) = \lambda \cdot \pi(\Omega) \cdot p(\vec{\sigma}|\Omega) \quad (2)$$

where $\pi(\Omega)$ is the prior distribution and λ is a normalization constant. *Any* statistical method that places an upper limit on Ω introduces an effective prior distribution. The advantage of the Bayesian method is that the optimum choice can be made. In this case the prior should best describe our lack of knowledge about Ω and account for the known level of white noise. The best choice, described in many advanced statistic references such as Schervish (1995), is known as Jefferys' prior. This prior has the property that the maximum information (defined mathematically as Fischer's Information) comes from the measurements, and a minimum comes from the prior itself. In this case the prior has the form

$$\pi_J(\Omega) = \sqrt{\frac{\gamma_1^2}{2(1+\gamma_1\Omega)^2} + \frac{\gamma_2^2}{(1+\gamma_2\Omega)^2} + \frac{2\gamma_4^2}{(1+\gamma_4\Omega)^2} + \frac{4\gamma_8^2}{(1+\gamma_8\Omega)^2}} \quad (3)$$

where $\gamma_m = \frac{<S_m>_g}{<S_m>_w}$ and Ω is in units of 10^{-7}. This prior has the property of equal probability per logarithmic interval for an Ω that is significant with respect to the white noise. A uniform prior strongly favors large values of Ω and is thus unacceptable. Jeffrys' prior is also invariant under suitable reparameterization, thus our choice of parameterization does not change the prior (Tarantola, 1987).

Calculating Eq. (2) for the measurements of 1855+09 and normalizing by integrating over all Ω allows us to calculate the upper limit at any confidence level. The upper limits at the confidence levels of 95% and 90%, are $\Omega h^2 \leq 9.3 \times 10^{-8}$ and $\Omega h^2 \leq 5 \times 10^{-8}$ respectively. These limits are for a frequency range of a GWB from around 4×10^{-9} Hz to 4×10^{-8} Hz. The limits are about 10 times less stringent than those placed by Thorsett and Dewey.

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