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REPRESENTING TATE COHOMOLOGY OF G-SPACES

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0. Introduction

Tate cohomology of finite groups [5] is very good at emphasising periodic cohomological behaviour and hence at the study of free actions on spheres [8]. Tate cohomology of spaces was introduced by Swan [10] for finite dimensional spaces to systematically ignore free actions, and hence to simplify various arguments in fixed point theory.

There is also a geometric way of ignoring free actions, and we show (Theorem 2) that it can be used to represent Tate cohomology in any reasonable category of equivariant spectra (see for instance [1, 6 or 7]), and hence extend the definition to infinite dimensional spaces and spectra. There are several important consequences of this. Firstly that Tate cohomology admits suspension isomorphisms for arbitrary representations of G, and hence that there is a transfer for finite coverings. Secondly it points out a natural generalisation of the condition that G has periodic cohomology (Corollary 6). Furthermore the methods make clear how one could define cohomology theories at the chain level which ignore actions more general than free ones.

From the other direction the represented theory arises naturally in an attempt to understand Borel homology and cohomology [6]. Theorem 2 is then seen as identifying this represented theory in more familiar terms.

The proof is a byproduct of the construction of a geometric realisation of the Tate (complete) resolution.

1. Cohomology and filtrations of G-spaces

Throughout, G will be a fixed finite group.

In this section we will work in the G-equivariant Spanier–Whitehead category [1]; it is only for the representation of cohomology theories that we need more general G-spectra.

The most effective technique in the equivariant world is escape to a simpler world. Thus if EG is a contractible G–CW complex on which G acts freely, we construct EG_+ by adding a disjoint basepoint, and $\widetilde{EG}$ as the unreduced suspension of it. We then have the enormously useful cofibering exploited by Carlsson:

$$EG_+ \rightarrow S^0 \rightarrow \widetilde{EG}. \tag{*}$$

Thus $\widetilde{EG}$ is a nonequivariantly contractible space with fixed point sets $\widetilde{EG}^H = S^0$ for all $H \mathrel{\underline{\triangleright}} 1$. Hence it provides the geometric means for ignoring free G-spaces.

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To motivate our realisation of Tate resolutions we first recall the method for ordinary resolutions. We start in the algebraic world.

To calculate ordinary cohomology, $H^*(G; M)$, we take a resolution of $\mathbb{Z}$ by projective $\mathbb{Z}[G]$-modules,

$$0 \leftarrow \mathbb{Z} \leftarrow P_0 \leftarrow P_1 \leftarrow P_2 \leftarrow \cdots \tag{$\uparrow$}$$

apply $\mathrm{Hom}_{\mathbb{Z}[G]}(\ , M)$ and take cohomology of the resulting complex.

To calculate Tate cohomology, $\hat{H}^*(G; M)$, we take a projective resolution as above, and also a backwards resolution of $\mathbb{Z}$ by $\mathbb{Z}[G]$-projectives,

$$\cdots \leftarrow P_{-3} \leftarrow P_{-2} \leftarrow P_{-1} \leftarrow \mathbb{Z} \leftarrow 0. \tag{$\downarrow$}$$

We splice ($\uparrow$) and ($\downarrow$) together to form the Tate resolution,

$$\begin{tikzcd}[column sep=small] \cdots & P_{-3} \arrow[l] & P_{-2} \arrow[l] & P_{-1} \arrow[l] & & P_0 \arrow[ll] \arrow[dl] & P_1 \arrow[l] & P_2 \arrow[l] & \cdots \arrow[l] \\ & & & & \mathbb{Z} \arrow[ul] & & & & \end{tikzcd} \tag{$\updownarrow$}$$

apply $\mathrm{Hom}_{\mathbb{Z}[G]}(\ , M)$ and take cohomology.

The geometric analogue of ($\uparrow$) comes by taking the skeletal filtration of EG_+,

$$\begin{tikzcd} * \arrow[r] & EG_+^{(0)} \arrow[r] \arrow[d] & EG_+^{(1)} \arrow[r] \arrow[d] & EG_+^{(2)} \arrow[r] \arrow[d] & \cdots \\ & R_0 & R_1 & R_2 & \end{tikzcd} \tag{G$\uparrow$}$$

(where $R_i := EG_+^{(i)}/EG_+^{(i-1)}$) and applying ordinary nonequivariant homology $u_*(\ )$ (which is defined by $u_*(X) := H_*(UX)$, where UX is the underlying space of X, with G-action forgotten).

Thus we consider the spectral sequence with $E^1_{s,t} = u_{s+t}(R_s)$; since R_s is a wedge of G-free cells $S^s \wedge G_+$ we find that the spectral sequence collapses at the E_2-level. Since it is also convergent, we find that the homology of the chain complex

$$u_*(R_0) \leftarrow u_*(S^{-1}R_1) \leftarrow u_*(S^{-2}R_2) \leftarrow u_*(S^{-3}R_3) \leftarrow \cdots$$

is $u_*(EG_+) = \mathbb{Z}$, and hence the complex provides a resolution of $\mathbb{Z}$ by free $\mathbb{Z}[G]$-modules.

Just as the backwards resolution ($\downarrow$) can be constructed by dualising a (finitely generated) resolution ($\uparrow$), even so we may obtain a backwards geometric resolution by Spanier–Whitehead duality.

Now, since the cofibering $S^0 \to \widetilde{EG} \to SEG_+$ is filtration preserving, the skeletal filtration of $\widetilde{EG}$ has subquotients as follows:

$$\begin{tikzcd} * \arrow[r] & \widetilde{EG}^{(0)} \arrow[r] \arrow[d] & \widetilde{EG}^{(1)} \arrow[r] \arrow[d] & \widetilde{EG}^{(2)} \arrow[r] \arrow[d] & \widetilde{EG}^{(3)} \arrow[r] \arrow[d] & \cdots \\ & S^0 & SR_0 & SR_1 & SR_2 & \end{tikzcd}$$

It is therefore appropriate to consider the filtration

$$\begin{array}{ccccccccc} S^0 = D\widetilde{EG}^{(0)} & \leftarrow & D\widetilde{EG}^{(1)} & \leftarrow & D\widetilde{EG}^{(2)} & \leftarrow & D\widetilde{EG}^{(3)} & \leftarrow & \cdots \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\ DR_0 & & DR_1 & & DR_2 & & DR_3 & & \end{array} \qquad (\mathrm{G}\!\downarrow)$$

(In the category of spectra we can take the horizontals to be inclusions of subspectra and DR_i to be the appropriate quotients; for the moment it suffices that DR_i is the cofibre of the map $D\widetilde{EG}^{(i+1)} \to D\widetilde{EG}^{(i)}$).

In fact $\widetilde{EG}$, which is $S^0 \cup CEG_+$ by the cofibering (*), provides the correct framework for splicing the resolutions. Thus we define a filtration,

$$\begin{array}{ccccccccccc} \cdots \to & F_{-2} & \to & F_{-1} & \to & F_0 & \to & F_1 & \to & F_2 & \to \cdots \text{ of } \widetilde{EG} \\ & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & \\ & Q_{-2} & & Q_{-1} & & Q_0 & & Q_1 & & Q_2 & \end{array}$$

where Q_i is the cofiber of $F_{i-1} \to F_i$, by taking

$$F_k = \begin{cases} S^0 \cup CEG_+^{(k)} & k \geqq 0 \\ DEG^{(-k-1)} & k < 0. \end{cases}$$

Thus we have the picture

$$\begin{array}{ccccccccc} \cdots \to D\widetilde{EG}^{(1)} & \to & D\widetilde{EG}^{(0)} = S^0 & \to & S^0 \cup CEG_+^{(0)} & \to & S^0 \cup CEG_+^{(1)} & \to \cdots \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & \\ DR_1 & & DR_0 & & SR_0 & & SR_1 & \end{array} \qquad (\mathrm{G}\!\updownarrow)$$

We apply $u_*(\ )$ to obtain a spectral sequence with

$$E^1_{s,t} = u_{s+t}(Q_s),$$

and which collapses at the E_2 stage since Q_s is equivalent to a wedge of G-free cells $S^{s+1} \wedge G_+$ (for $s \geqq 0$ this is clear and for $s < 0$ we use the identification $DG/H_+ \simeq G/H_+$).

Lemma 1. *The chain complex*

$$\begin{tikzcd}[column sep=small] \cdots \arrow[r] & u_*(SQ_{-1}) \arrow[rr] \arrow[dr] & & u_*(Q_0) \arrow[r] & u_*(S^{-1}Q_1) \arrow[r] & u_*(S^{-2}Q_2) \arrow[r] & \cdots \\ & & u_*(S^1) \arrow[ur] & & & & \end{tikzcd}$$

is a Tate resolution of $\mathbb{Z} = u_*(S^1)$ *by* $\mathbb{Z}[G]$ *projectives.*

Proof. We could argue that the spectral sequence converged, and hence, since $\widetilde{EG}$ is contractible, that the above complex was exact. It seems more elementary to say that exactness in positive dimensions follows from convergence of the SEG_+ filtration spectral sequence above, and in negative dimensions from the SEG_+ filtration spectral sequence in cohomology.

Exactness in the middle comes since $u_*(Q_0) \to u_*(S^1)$ and $u_*(S^1) \to u_*(SQ_{-1})$ are the edge homomorphisms for these spectral sequences. □

3. Borel and Tate cohomology

We must now introduce the two cohomology theories we are going to show to be equal.

Definition ([4, VII.7]). If X is a finite dimensional G–CW complex, M a G-module, then we define $\hat{H}^*_G(X;M)$, the cellular Tate cohomology of X, to be the cohomology of the total complex of $\mathrm{Hom}_{Z[G]}(P_\bullet, C^*(X;M))$ (where $P_\bullet$ is a Tate resolution for G, and $C^*(X;M)$ is the complex of reduced cellular cochains).

For the representable definition we must first define Borel cohomology [3] as a functor on the G Spanier–Whitehead category: that is we must show that it admits suspension isomorphisms by arbitrary real representations of G. If M is any $\mathbb{Z}$-module and X a based G–CW complex then we may define

$$b^r(X;M) := H^r(EG_+ \wedge_G X; M) \qquad (r \in \mathbb{Z}),$$

and this agrees with ordinary (reduced) equivariant cohomology theory $H^*_G(X;M)$ ([4, VII.7, Exercise 3]).

It is a consequence of work of Waner [11] and May [9] that this cohomology theory extends to one defined on the category of G-spaces and spectra stable for suspension by arbitrary representations of G. Under the additional assumption that if G is of even order M is an $\mathbb{F}_2$ vector space, there is the following more elementary description known to many. We define

$$b^\alpha(X;M) := H^{|\alpha|}(EG_+ \wedge_G X;\ M) \quad \text{for} \quad \alpha \in RO(G),$$

and obtain a theory which has the property that $b^\alpha(X;M)$ depends only on the virtual dimension $|\alpha|$ of α. Henceforth we will omit the coefficient module M.

Suspensions come from Thom isomorphisms:

$$\begin{array}{ccc} b^\alpha(X) & \longrightarrow & b^{\alpha+[V]}(S^V \wedge X) \\ = \downarrow & & = \downarrow \\ H^{|\alpha|}(EG \times_G X, EG \times_G pt) & \xrightarrow{\cong} & H^{|\alpha|+|V|}(EG \times_G (S^V \times X), EG \times_G (S^V \times pt \cup pt \times X)) \end{array}$$

By Brown representability or direct construction, Borel cohomology is represented by a G-spectrum b.

Definition. The Tate spectrum t is defined by $t := b \wedge \widetilde{EG}$, and hence representable Tate cohomology is defined on integer gradings by

$$t^n(X) := [X, S^n t]^G.$$

We may now state our main theorem:

Theorem 2. *If X is a finite dimensional G–CW complex and M is a G-trivial module, then there is a natural isomorphism,*

$$t^n(X; M) = \hat{H}^n_G(X; M).$$

Proof. The proof is now a single spectral sequence, namely the one obtained by doubly filtering $\widetilde{EG} \wedge X$ (by skeleta of X and by the (G$\updownarrow$) filtration of $\widetilde{EG}$), and applying Borel cohomology.

Now that we are unavoidably in the category of spectra we take the filtration (G$\updownarrow$) to be by strict subspectra.

Consider the double filtration:

$$\begin{tikzcd}
{} \arrow[r] & F_{-1} \wedge X^{(d)} \arrow[r] & F_0 \wedge X^{(d)} \arrow[r] & F_1 \wedge X^{(d)} \arrow[r] & F_2 \wedge X^{(d)} \arrow[r] & {} \\
{} \arrow[r] & F_{-1} \wedge X^{(d-1)} \arrow[r] \arrow[u] & F_1 \wedge X^{(d-1)} \arrow[r] \arrow[u] & F_1 \wedge X^{(d-1)} \arrow[r] \arrow[u] & F_2 \wedge X^{(d-1)} \arrow[r] \arrow[u] & {} \\
& \vdots \arrow[u] & \vdots \arrow[u] & \vdots \arrow[u] & \vdots \arrow[u] & \\
{} \arrow[r] & F_{-1} \wedge X^{(1)} \arrow[r] \arrow[u] & F_0 \wedge X^{(1)} \arrow[r] \arrow[u] & F_1 \wedge X^{(1)} \arrow[r] \arrow[u] & F_2 \wedge X^{(1)} \arrow[r] \arrow[u] & {} \\
{} \arrow[r] & F_{-1} \wedge X^{(0)} \arrow[r] \arrow[u] & F_0 \wedge X^{(0)} \arrow[r] \arrow[u] & F_1 \wedge X^{(0)} \arrow[r] \arrow[u] & F_2 \wedge X^{(0)} \arrow[r] \arrow[u] & {}
\end{tikzcd}$$

From it we may form the single filtration

$$\begin{tikzcd}
{} \arrow[r] & \Delta_s \arrow[r] \arrow[d] & \Delta_{s+1} \arrow[r] \arrow[d] & \Delta_{s+2} \arrow[r] \arrow[d] & {} \\
& \Delta_s/\Delta_{s-1} & \Delta_{s+1}/\Delta_s & \Delta_{s+2}/\Delta_{s+1} &
\end{tikzcd}$$

by diagonals.

Thus $\Delta_s := \bigcup_{i+j=s} F_i \wedge X^{(j)}$; and, by considering ziggurats,

$$\Delta_s/\Delta_{s-1} = \bigvee_{i+j=s} F_i/F_{i-1} \wedge X^{(j)}/X^{(j-1)}.$$

Lemma 3. $b^*(R \wedge X^{(j)}/X^{(j-1)}) = \mathrm{Hom}_{Z[G]}{}^*(u_*R, C^j(X))$ *naturally in R, where R is any wedge of cells $S^n \wedge G_+$.*

Proof. It suffices to check the result for $R = G_+$ and $X = S^j \wedge (G/H)_+$. □

Now we may identify the E_1-term:

$$E_1^{s,t} = b^{s+t}(\Delta_s/\Delta_{s-1}) = \bigoplus_{i+j=s} \mathrm{Hom}^{s+t}_{Z[G]}(u_*F_i/F_{i-1}, C^j(X)) \quad \text{if} \quad s+t = s+1$$

and zero if $s+t \neq s+1$ (since Δ_s/Δ_{s-1} consists entirely of cells $S^{i+1} \wedge G_+ \wedge S^j \wedge (G/H)_+$ for $i+j=s$).

A check of differentials shows that

$$E_2^{s,t} = \hat{H}_G^s(X) \text{ if } t=1, \text{ and zero if } t \neq 1.$$

It remains to see what this has to do with $t^*(\ )$.
In fact

$$\begin{aligned} t^r(X) &= [X, b \wedge \widetilde{EG}]_G^r \\ &= \lim_{\to k} [X, b \wedge \widetilde{EG}^{(k)}]_G^r \quad \text{(since } X \text{ is finite dimensional and } b \text{ is nonequivalently bounded below)} \\ &= \lim_{\to k} [D\widetilde{EG}^{(k)} \wedge X, b]_G^r \quad \text{(since } EG^{(k)} \text{ is finite)} \\ &= \lim_{\to k} b^r(D\widetilde{EG}^{(k)} \wedge X), \end{aligned}$$

and we can form another filtration of $\widetilde{EG} \wedge X$ whose spectral sequence has clearer convergence to this.

Thus we use the filtration by using the filtration of $\widetilde{EG}$ only, and apply $b^*(\ )$ to get the spectral sequence with unravelled exact couple [2]:

$$\begin{tikzcd}[column sep=small] \cdots & b^*(F_s \wedge X) \arrow[l] \arrow[dr] & & b^*(F_{s+1} \wedge X) \arrow[ll] \arrow[dl] \arrow[dr] & & b^*(F_{s+2} \wedge X) \arrow[ll] & \cdots \arrow[l] \\ & & b^*(Q_{s+1} \wedge X) \arrow[ul] & & b^*(Q_{s+2} \wedge X) \arrow[ul] \arrow[ur] & & \end{tikzcd} \qquad (^+)$$

We observe:

(1) the associated filtration is Hausdorff and complete;

and

(2) $\lim_{\to s} b^*(F_s \wedge X) = \lim_{\to s} b^*(D\widetilde{EG}^{(s)} \wedge X) = t^*(X)$.

Proof (of (1)). Indeed we have the Milnor exact sequence

$$0 \longrightarrow R\lim_{\leftarrow s} b^*(SF_s \wedge X) \longrightarrow b^*\left(\operatorname{holim}_{\to s} F_s \wedge X\right) \longrightarrow \lim_{\leftarrow s} b^*(F_s \wedge X) \longrightarrow 0$$

On the other hand $b^*\left(\operatorname{holim}_{\to s} F_s \wedge X\right) = b^*(\widetilde{EG} \wedge X)$, which is zero since $\widetilde{EG} \wedge EG_+ \simeq *$.

$\square$

Thus the spectral sequence of $(^+)$ is conditionally convergent [2] to $t^*(X)$; by finite dimensionality of X the spectral sequence collapses at some stage and is therefore strongly convergent.

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Finally (also since X is finite dimensional) the spectral sequence of the double filtration has the analogous properties to (1) and (2) for its filtered chain complex and is therefore also conditionally convergent. So the theorem is proved. □

Remark. To see $t^*(\ )$ does ignore G-free actions we use the fact that $[G_+, b \wedge \widetilde{EG}]^G = [S^0, Ub \wedge U\widetilde{EG}]^1$ (a slight extension of Adams' result [1, 5.1]) and the fact that $U\widetilde{EG} \simeq *$; then argue by cofiberings and limits to see $t^*(X) = [X, b \wedge \widetilde{EG}]^*_G = 0$ for a general G-free spectrum X.

There is also a homological version of the theorem, but since its proof is very similar, and its usefulness less certain, we just state it.

Thus if

$$t_*(X) := [S^0, t \wedge X]^G_*,$$

and if $\hat{H}^G_*(X; M)$ is the homology of the total complex of $P_\bullet \otimes_{Z[G]} C_*(X; M)$ then we have

Theorem 2$_*$. *If X is any G-spectrum and M is any G-trivial module, then*

$$\hat{H}^G_*(SX; M) = t_*(X; M).$$

The proof is by applying Borel homology to the same two filtrations as in Theorem 2; it is convenient to deal with the case that X is finite and then pass to limits. Remember that if X is finite then $b_r(X) = b^{-r}(DX)$.

4. Applications

We have already commented that Theorem 2 extends the definition of Tate cohomology to arbitrary spaces and spectra and that it shows Tate cohomology admits transfer for finite coverings.

Let us next see what Theorem 2 has to say about periodic cohomology. Suppose first that G acts freely on the unit sphere $S(V)$ of the representation V. It follows that $S(\infty V) = \bigcup_{k \geq 0} S(kV)$ provides a model for EG, and hence that we have

$$\widetilde{EG} \simeq S^{\infty V}.$$

From this we immediately deduce the familiar fact that the Tate cohomology of any finite X is a localisation of ordinary cohomology:

$$\begin{aligned} t^*(X) &= [X, b \wedge S^{\infty V}]^*_G \\ &= \lim_{\to k} [X, b \wedge S^{kV}]^*_G \\ &= b^*(X)[e(V)^{-1}] \text{ (where } e(V) \in b^{|V|} \text{ is the Euler class of } V). \end{aligned}$$

For instance, taking $X=S^0$, and working with mod 2 coefficients if G is of even order, $t^*=H^*(G)[e(V)^{-1}]$ (where $e(V)\in H^{|V|}(G)$ is the ordinary Euler class). Considering the fact that Tate cohomology is ordinary cohomology in positive codegrees and homology in negative codegrees we see that we have proved the familiar fact that G has periodic cohomology.

Now suppose that G acts freely on $S(V)\times S(W)$. This means that for any subgroup H except 1, either $V^H=0$ or $W^H=0$. We thus have the following result.

Proposition 5. *If G is a finite group acting freely on $S(V)\times S(W)$ then there is a stable cofibering*

$$\widetilde{EG}\longrightarrow S^{\infty V}\vee S^{\infty W}\xrightarrow{i_V-i_W}S^{\infty V\oplus W}.$$

Proof. We have the inclusion of V in $V\oplus W$, and hence the map $i_V:S^{\infty V}\to S^{\infty V\oplus W}$, and similarly for W. After a single suspension we may form the difference i_V-i_W, and use it to start a cofiber sequence in which the following occurs:

$$C\longrightarrow S^2(S^{\infty V}\vee S^{\infty W})\longrightarrow S^2S^{\infty V\oplus W}.$$

Even if we stabilise with respect to G-trivial suspensions only, this has the exactness properties of a fibration.

Now, since $S^{\infty V}$ is nonequivariantly contractible, we may extend the inclusion $S^0\to S^{\infty V}$ over the rest of $\widetilde{EG}$ (which consists of G-free cells), and similarly for W; furthermore these extensions are unique by the same argument.

Hence we may pose the problem:

$$\begin{tikzcd} & \widetilde{EG} \arrow[dl, dashed] \arrow[d] & \\ S^{-2}C \arrow[r] & S^{\infty V}\vee S^{\infty W} \arrow[r] & S^{\infty V\oplus W}. \end{tikzcd}$$

This is soluble since the restriction of $\widetilde{EG}\to S^{\infty V\oplus W}$ to S^0 is null (again we extend the nullhomotopy by obstruction theory). Now, by the equivariant Whitehead Theorem stabilised with respect to G-trivial representations, it follows that the solution provides an equivalence $\widetilde{EG}\simeq S^{-2}C$. □

By smashing with b and mapping in the finite complex X we obtain the following.

Corollary 6. *If G is a finite group which acts freely on $S(V)\times S(W)$ then for any finite X we obtain the following natural long exact sequence connecting Tate cohomology with localised forms of ordinary cohomology:*

$$\cdots\to b^*(SX)[e(V)^{-1},e(W)^{-1}]\to t^*(X)\to b^*(X)[e(V)^{-1}]\oplus b^*(X)[e(W)^{-1}]$$

$$\to b^*(X)[e(V)^{-1},e(W)^{-1}]\to\cdots$$

Remarks. (1) As before we can consider the case $X=S^0$ and use mod 2 coefficients if G is of even order, and obtain conditions generalising those of periodic cohomology. These will take the form of restrictions on $H^*(G)$ as a module over $\mathbb{Z}[e(V), e(W)]$.

(2) Evidently a similar construction will apply to connect Tate and localised ordinary cohomology whenever G acts freely and linearly on a product of spheres; for instance this applies to arbitrary p-groups. However in general there will be several long exact sequences required.

Finally, the construction of a Tate resolution clearly generalises. Thus if $E\mathscr{F}$ is the classifying space for a family $\mathscr{F}$ of subgroups of G so that the fixed points of $\widetilde{E\mathscr{F}}$ are nonequivariantly as follows

$$\widetilde{E\mathscr{F}}^H \simeq \begin{cases} S^0 & \text{if} \quad H \notin \mathscr{F} \\ * & \text{if} \quad H \in \mathscr{F} \end{cases}.$$

It follows by obstruction theory that the theory represented by $b \wedge \widetilde{E\mathscr{F}}$ vanishes on any space which may be constructed from cells $G/H_+ \wedge S^n$ for $H \in \mathscr{F}$. The associated geometric Tate resolution of $\widetilde{E\mathscr{F}} = S^0 \cup CE\mathscr{F}_+$, and the proof of Theorem 2 show how to define the theory by cellular methods.

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