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VERTEX-PRIMITIVE s-ARC-TRANSITIVE DIGRAPHS OF ALMOST SIMPLE GROUPS

LEI CHEN

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The investigation of s-arc-transitivity can be dated back to 1947. Tutte [7] studied cubic graphs and showed that a cubic graph can be at most 5-arc-transitive. A more general result for s-arc-transitivity of graphs was obtained by Weiss [8] and it turns out that finite undirected graphs of valency at least 3 that are not cycles can be at most 7-arc-transitive. In stark contrast with the situation in undirected graphs, Praeger [6] showed that for each s and d, there are infinitely many finite s-arc-transitive digraphs of valency d that are not $(s+1)$-arc-transitive.

However, once we add the condition of primitivity, the situation is quite different. Given the lack of evidence of the existence of vertex-primitive 2-arc-transitive digraphs, Praeger [6] asked if there exists any vertex-primitive 2-arc-transitive digraph. This question was answered in [2, 4] by constructing infinite families of G-vertex-primitive $(G,2)$-arc-transitive digraphs such that G has *AS* and *SD* type, respectively. In [4], Giudici and Xia then asked for the upper bound on s for a G-vertex-primitive (G,s)-arc-transitive digraph that is not a directed cycle. A reasonable conjecture is that $s \leqslant 2$. At the same time, Giudici and Xia [4] showed that to answer that question, it suffices for us to consider the case when G is almost simple.

Various attempts have been made to analyse the s-arc-transitivity of different almost simple groups. For instance, Giudici *et al.* [3] showed that $s \leqslant 2$ when the socle of G is a projective special linear group, Pan *et al.* [5] proved that $s \leqslant 2$ when the socle of G is an alternating group except for one subcase and Chen *et al.* [1] addressed the case when the socle of G is a Suzuki group or a small Ree group, when it turns out that the upper bound on s is 1. The result from [1] is part of Chapter 4.

In this thesis, we investigate the upper bound on s for G-vertex-primitive (G,s)-arc-transitive digraphs for almost simple groups G with $\mathrm{Soc}(G) = \mathrm{PSp}_{2n}(q)'$,

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$\mathrm{PSU}_n(q)$ (for certain cases), $\mathrm{Sz}(q)$, $\mathrm{Ree}(q)$, ${}^2\mathrm{F}_4(q)$, ${}^3\mathrm{D}_4(q)$ and $\mathrm{G}_2(q)$. It turns out that such an upper bound is $s \leqslant 2$ for all the groups mentioned above, giving some evidence to the conjecture that $s \leqslant 2$.

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LEI CHEN, Department of Mathematics and Statistics,
University of Western Australia, Perth, Western Australia 6009, Australia
e-mail: lei.chen@uwa.edu.au

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