ONE OF THE PROBLEMS OF LONG-TERM INTEGRATIONS OF COMETARY ORBITS

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ABSTRACT. One of the problems of the numerical integrations of cometary orbits is that of their numerical stability. For those bodies which undergo close encounters with the giant planets the problem has a specific feature. Apart from the numerical instability of integrations in the mathematical sense, there is an additional source of instability due to the inaccuracy of initial data, i.e. the orbital elements. An example of this case is presented in this paper by the numerical integration of the motion of comet P/Shajn-Schaldach over an interval of 368.4 years, within which six close encounters with Jupiter occurred. The inaccuracy of the starting orbital elements of this comet is modelled by changes in the last digit of each element of the central orbit, determined by the set of the best orbital elements. In the process of integration, eight model orbits experience differential perturbations with respect to the central orbit. The numerical instability, caused by the inaccuracy of starting orbital elements and represented by the dispersion of these orbits, tends to increase abruptly beyond each encounter with Jupiter. It is shown that, with the attainable accuracy of the osculating elements representing the observations, one or two approaches to within 1 AU from Jupiter can make the orbit entirely indeterminate.

1. INTRODUCTION

The study of cometary evolution is not possible without the knowledge of individual cometary motions during longer time spans. Comets moving in the solar system undergo perturbations by the gravitational attraction of the planets, and in some cases by nongravitational forces, too. Their motion in full range corresponds to the motion in a n-body system. There is generally only one way for the solution of such a dynamical

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problem: the numerical integration of differential equations of motion.

The solution of these equations by means of difference approximations or polynomial function representations is subject to numerical truncating errors. Several authors have compared the relative efficiencies of integrating differential equations by means of the classical single-step or multi-step methods and the recurrent power series methods. Generally they have investigated these methods from the point of view of cumulation of local truncating error versus total computer time. They analyzed the accuracy of the integration in difficult problems, such as close encounters (Bettis and Szebehely, 1972; Everhart, 1974a,b; Roy et al., 1972; Schubart and Stumpff, 1966; Sitarski, 1979; Szebehely and Bettis, 1972). For a detailed analysis of the effects of infinitesimal changes in starting elements and of integrator's characteristics the reader may refer to Oikawa and Everhart (1979).

In the dynamical evolution of some comets there is a series of close encounters with giant planets. These events are the sources of other inaccuracies in the determination of cometary orbits. Close encounters not only require convenient integration methods, but they can cause indeterminacy of the orbit. This is a consequence of the inaccuracy of the initial orbital elements from which the numerical integration begins.

2. SELECTION OF THE MODEL ORBITS

The number of close planetary ecounters after which the effect of the inaccuracy of starting orbital elements becomes serious is generally unknown, and to investigate this problem we must start from a convenient example. For this purpose, it is necessary to select a comet undergoing repeated close encounters - one whose motion is also not affected by nongravitational forces - and follow it over a relatively short time span, in which the cumulation of local truncating errors can be neglected. Otherwise, it is not possible to distinguish the contribution of the numerical effects due to the integration and those caused by the inaccuracy of the initial orbital data.

One of the most suitable comets for our purpose is P/Shajn-Schaldach. The comet has a well determined orbit (Marsden, 1977, 1978b), the nongravitational effects on it prior to 1945 should have been negligible, since its perihelion distance at every return was larger than 4.1 AU, and there were repeated close encounters with Jupiter at relatively short time intervals (Pittich, 1981). The high value of the Tisserand invariant, 2.93, makes these encounters very effective.

The inaccuracy of the starting orbital elements for the numerical

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integration can be modelled by small changes of the known elements of the comet. To this purpose, the tenth digit of each element of P/Shajn-Schaldach (Marsden, 1977) was modified by an increment d = 0, ±0.25, ±0.50, ±0.75, ±1.00 (see Table I). The set of the nine potential orbits of P/Shajn-Schaldach is defined by these modified elements. The orbits are designated by their corresponding d-value: 000, ±025, ±050, ±075, ±100; for example, in the +025 case, a q-value of 2.233905338 becomes 2.23390533825, and a similar increment is made to all the elements.

TABLE I. Starting orbital elements

Epoch	1949 Nov. 20.0 ET	
T	1949 Nov. 26.9192662 + d ET	
ω	215°3052345 + d	
Ω	167°3928552 + d	1950.0
i	6°1520408 + d	
q	2.233905338 + d AU	
e	0.404977568 + d	

d = 0.00, ±0.25, ±0.50, ±0.75, ±1.00 in the last significant digit

The evolution of these orbits was calculated from 1949 November 20.0 (JD 2433240.5) back in time until 1585 February 1.0 (JD 2433240.5), with the same procedure used in Carusi et al. (1984). The main characteristics of the integrations are: the use of an equatorial reference frame centred in the barycentre of the solar system; the integration of the barycentric equations of motion with the subroutine RADAU (Everhart, 1974b; for a listing and general comments on the subroutine see also Everhart, this volume) to the 19th order; the positions of the Sun and planets taken from the JPL DE-102 Long Ephemeris (Newhall et al., 1983). The evolution data of P/Shajn-Schaldach model orbits used in this paper will be published elsewhere (Carusi et al., 1985).

3. ORBITAL INSTABILITY AFTER CLOSE ENCOUNTERS

During the considered interval of 368.4 years, P/Shajn-Schaldach passed very close to Jupiter six times in the orbits 000 and -025, and five times in the other model orbits (see Table II). The minimum jovicentric distance was in three cases (orbit 000) and in two cases (all other model orbits) not greater than 0.22 AU.

The differences between the individual elements of the central orbit

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and those of each other model orbit within the considered interval of time are shown in Figures 1 and 2.

TABLE II. Jovicentric distances of P/Shajn-Schaldach (AU) at close encounters with Jupiter for various model orbits.

Date	-100	-075	-050	-025	orbit 000	+025	+050	+075	+100
1590 02 15				.7142					
1592 02 05					.1606				
1651 09 03	.7268	.7098	.6930	.6763	.6597	.6432	.6269	.6108	.5948
1780 09 06	.5490	.5489	.5489	.5488	.5487	.5486	.5486	.5485	.5484
1875 09 29	.2125	.2125	.2125	.2125	.2125	.2125	.2125	.2125	.2125
1941 06 15	.7374	.7473	.7473	.7473	.7473	.7473	.7473	.7473	.7473
1946 04 10	.1828	.1828	.1828	.1828	.1828	.1828	.1828	.1828	.1828

Since a good starting orbit was available, the small changes of the elements did not practically affect the dynamical evolution of this comet during the close encounters from 1946 until 1780 in a backward integration, but later the situation has become quite different. There are increasing differences in the model orbits prior to the next encounter in 1651. However, it is still possible to define the orbit of this comet in the limited space occupied by all the model orbits. This bundle of orbits is quite stable during the interval without other close planetary encounters. The cross-section of the bundle in its steady state is strongly dependent upon the accuracy of the starting elements; the sharper the actual orbit is defined, the smaller is the cross-section of the bundle at any moment.

There is one interesting feature of the evolutions of these fictitious objects: after the encounter of 1651 the orbits 000 and -025 are much more sensitive than the others. This fact follows from the position of the comet relative to Jupiter at the time of next encounter. Similar cases are described by Carusi et al. (1981).

The encounters of the 000 orbit in 1592 and the -025 orbit in 1590 indicate how much the model orbits diverge beyond that point; the spread of the bundle after (backward in time) these encounters is no longer compensated by some convergece at next encounter.

4. INACCURACY OF ACTUAL ELEMENTS

The maximum change of one unit in the tenth significant digit (10^{-7} day

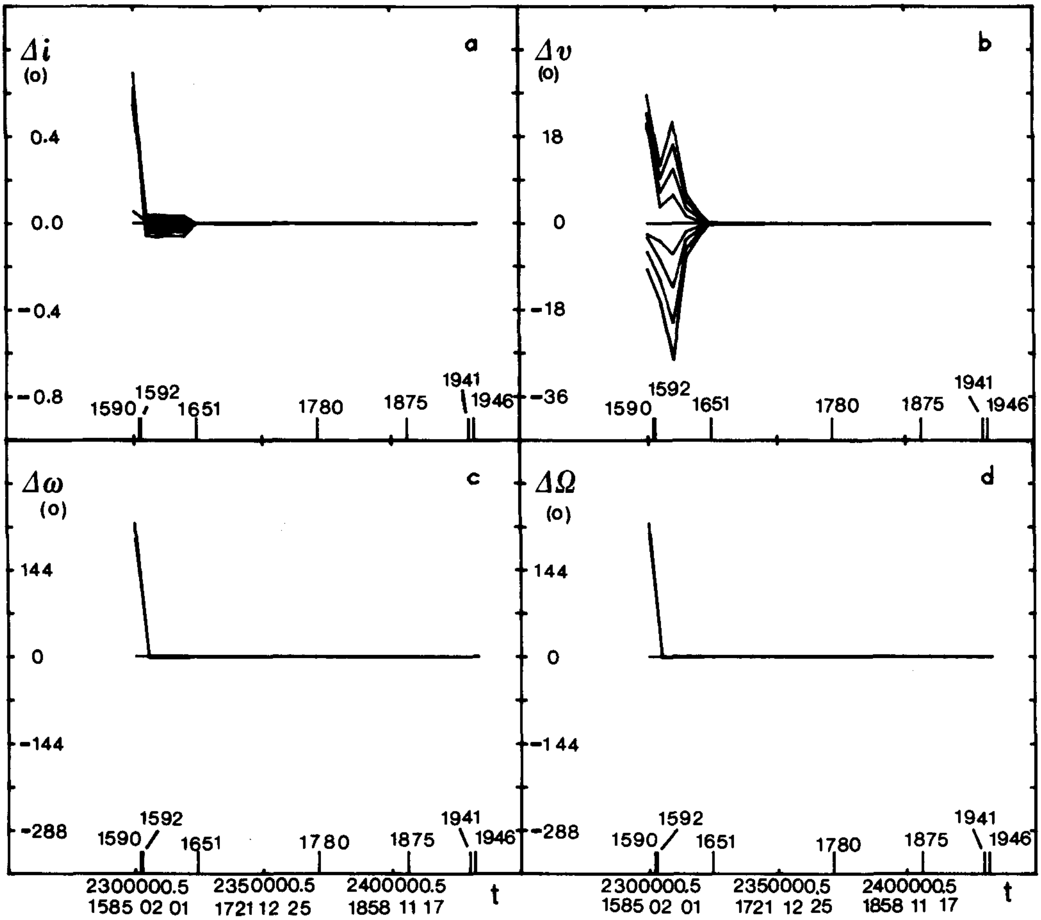

Figure 1. The differences between the evolved orbital elements and the elements of the standard 000 orbit are plotted versus time; a: inclination; b: true anomaly; c: argument of perihelion; d: longitude of node.

in T, 10^{-7} deg in angular elements, 10^{-9} AU in q and 10^{-9} in e) was intentionally set unrealistically low. Even though the starting elements are based on linking up of two apparitions spaced by three revolutions, their probable errors are four to five orders of magnitude greater than d, and they are mutually strongly correlated (in particular, $\Delta\omega \simeq -\Delta\Omega$ and $\Delta q \sim \Delta(1-e)$). Our choice is equivalent to rounding-off all elements upwards and downwards; proceeding in time, the differences tend to increase as the orbits diverge by differential perturbations. Only after some time they reach realistic values. At that time they already bear signatures of the sensitivity of individual elements to differential perturbations; from there on, the divergence can be compared with that due to the actual uncertainty of the orbit.

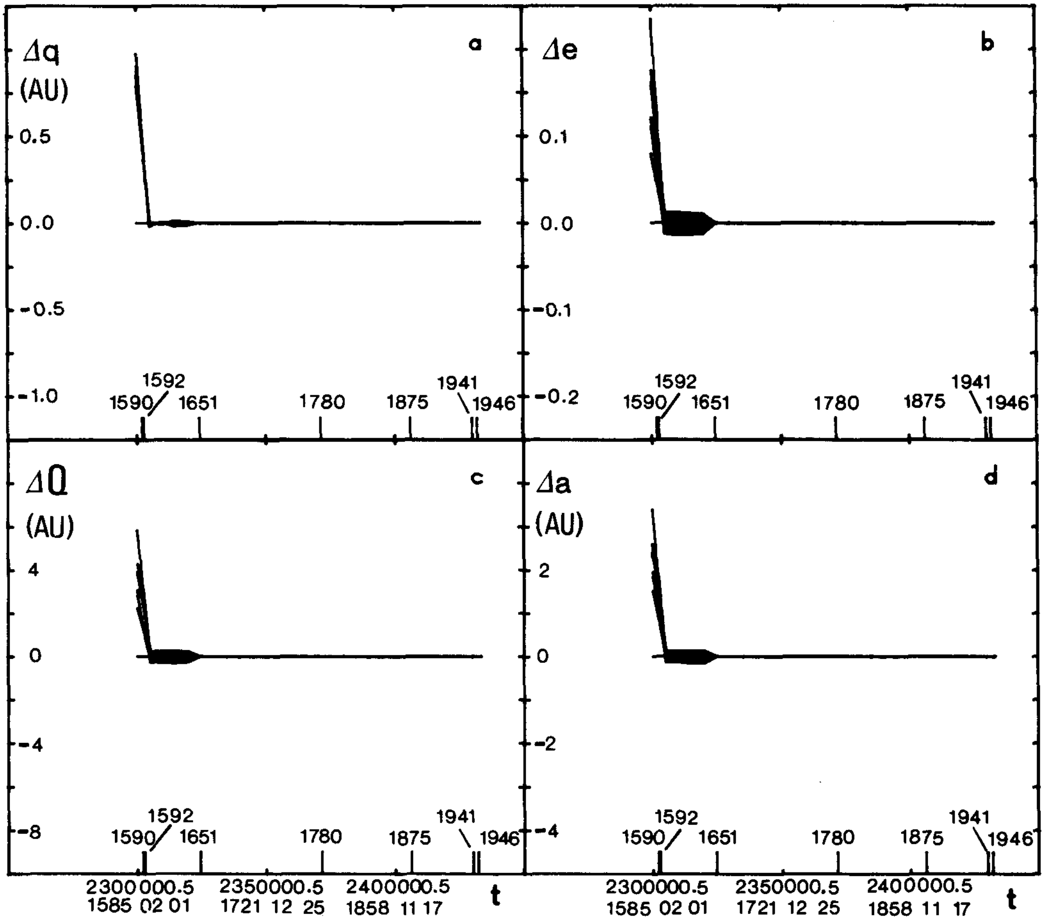

Figure 2. The same as Fig. 1; a: perihelion distance; b: eccentricity; c: aphelion distance; d: semimajor axis.

The differences between the central orbit and the others in all osculating elements at four epochs placed approximately half-way between the close encounters with Jupiter are listed in Table III. They are expressed as the means and standard deviations of the quantities obtained dividing the final differences with respect to orbit 000 by the arbitrary starting change in the corresponding element and adding the base 10 logarithms of the ratios so obtained.

The values of Table III can be compared with the corresponding values for three available element sets (Table IV). They are:

a) Marsden's (1978a) orbit for 1949 November 20, based on 68 observations between 1949 August 28 and 1972 January 20, and Marsden's (1971) orbit for 1949 November 20, based on 64 observations between 1949 September 18 and 1949 December 20. Here the differences should be characteristic for the improvement by linking up two apparitions against a final one-apparition orbit of average accuracy;

b) Marsden's (1978b) orbit for 1971 October 16, based on 68 observations between 1949 August 28 and 1972 January 20, and Nakano's (1984) orbit for 1971 October 16, based on 37 observations between 1971 September 15 and 1978 October 31. Here the differences should be characteristic for two independent linkages of pairs of apparitions, one proceeding forwards and one backwards, and meeting at the same osculating date;

c) Nakano's (1984) mean errors for the 1979 orbit (mean residual ±1.05" in the apparent position). Here the differences should account for a high-quality least-square solution, as internal errors not taking into account the correlations between individual elements.

TABLE III. Means and standard deviations of the differences in the osculating elements for each model orbit

Orbit	2420000.5 m s	2390000.5 m s	2350000.5 m s	2310000.5 m s
-100	2.12±1.56	3.27±1.13	5.56±1.13	7.29±1.36
-075	1.59±0.86	3.32±1.33	5.55±1.13	7.17±1.36
-050	2.85±1.13	3.35±1.37	5.26±1.13	7.00±1.36
-025	2.83±1.14	3.18±0.91	4.96±1.13	6.70±1.36
+025	2.99±1.01	3.42±1.11	4.96±1.13	6.72±1.36
+050	2.38±1.11	3.00±0.74	5.26±1.13	7.02±1.36
+075	2.11±1.26	3.61±1.03	5.44±1.13	7.19±1.36
+100	2.93±1.12	3.61±1.06	5.56±1.13	7.32±1.36

TABLE IV. Mean values and standard deviations from differences of osculating elements of observed orbits

Orbit	mean	st. deviation
a	4.76	0.68
b	4.55	0.66
c	4.42	0.64

5. CONCLUSIONS

The analysis of the model and actual orbits of P/Shajn-Schaldach allows some general conclusions. Determination of orbits of periodic comets experiencing close encounters with the giant planets is practically impossible in a long time interval. The orbits become poorly determined and have to be replaced by bundles of changing cross-section within the time intervals between successive encounters. The inaccuracy of orbital elements tends to increase abruptly after each encounter with a giant planet.

The actual uncertainty of the orbit of P/Shajn-Schaldach is comparable to the dispersion of our model orbits before the encounter with Jupiter of 1780, as can be deduced by the values given in Tables III and IV. This means that even a well determined orbit like that of P/Shajn-Schaldach becomes entirely indeterminate after one or two passages within 1 AU from Jupiter. Even for an unrealistic accuracy of the starting orbital elements - by four to five orders of magnitude higher than their observational uncertainty - a considerable divergence takes place after five or six encounters with Jupiter.

Of course, these results apply to a particular comet, with a high value of the Tisserand invariant, and therefore a low relative speed at encounters with Jupiter. Nevertheless, the results seem to indicate that in other cases the complete indeterminacy of the orbits can simply take place in somewhat longer time spans.

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