Canad. Math. Bull. Vol. **14** (2), 1971

RANK OF THE SUM OF CERTAIN MATRICES

BY
P. M. GIBSON

In this note we give a very elementary proof of a result of Meyer's [1]. Let $r(A)$ be the rank of the matrix A. By using generalized inverses, Meyer proved the following.

THEOREM. *If A and B are $m\times n$ complex matrices such that $AB^*=0$ and $B^*A=0$ then $r(A+B)=r(A)+r(B)$.*

Proof. Let $C=[A, B]$, $D=A+B$. Since $AB^*=BA^*=0$ *and* $B^*A=A^*B=0$,

$$DD^*=CC^*,\ C^*C = \begin{bmatrix} A^*A & 0 \\ 0 & B^*B \end{bmatrix}.$$

Hence,

$$r(D) = r(DD^*) = r(CC^*) = r(C^*C) = r(A^*A)+r(B^*B) = r(A)+r(B)$$

REFERENCE

1. C. D. Meyer, *On the rank of the sum of two rectangular matrices*, Canad. Math. Bull. **12** (1969), p. 508.

UNIVERSITY OF ALABAMA,
HUNTSVILLE, ALABAMA

https://doi.org/10.4153/CMB-1971-046-6 Published online by Cambridge University Press