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RECURSIVE CHARACTERISATIONS OF RANDOM MATRIX ENSEMBLES AND ASSOCIATED COMBINATORIAL OBJECTS

ANAS A. RAHMAN

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Mixed moments and cumulants of random matrices have been studied extensively over the last half-century with applications in a large variety of fields ranging from enumerative geometry to quantum mechanics. It is particularly interesting to expand the cumulants in $1/N$, where N denotes matrix size, and study the coefficients of these expansions along with their generating functions, which we call correlator expansion coefficients. We give an overview of the recursive characterisations of random matrix ensembles that are currently at the forefront of random matrix theory by way of studying two classes of ensembles using two different types of recursive schemes. Established theory on Selberg correlation integrals is used to derive linear differential equations on the eigenvalue densities and resolvents of the classical matrix ensembles, which lead to 1-point recursions, understood to be analogues of the Harer–Zagier recursion, for the expansion coefficients of the associated 1-point cumulants, while loop equation analysis is used to recursively compute some leading order correlator expansion coefficients pertaining to certain products of random matrices that have recently come into interest due to their connections to Muttalib–Borodin ensembles and integrals of Harish-Chandra–Itzykson–Zuber type. We also show how the aforementioned differential equations can be used to characterise the large N limiting statistics of the eigenvalue densities of the classical matrix ensembles in the global and edge scaling regimes.

A common theme between the two classes of ensembles studied in this thesis is that their representative random matrices can, for the most part, be constructed from Ginibre matrices (matrices whose entries are independently and identically distributed normal variables), which allows for their mixed moments and cumulants to be interpreted as enumerations of particular ribbon graphs, topological and combinatorial

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maps, and/or topological and combinatorial hypermaps. A major part of this thesis is devoted to a comprehensive review of how the Isserlis–Wick theorem implies these interpretations for the mixed moments and cumulants of the Gaussian and Laguerre ensembles, which leads on to original discussion on how the relevant theory extends to the matrix products mentioned above. Thus, the loop equations derived in this thesis have the added value of solving certain problems in enumerative combinatorics.

To make this thesis self-contained and to properly motivate our original contributions, a decent portion of our development constitutes a pedagogically detailed survey of the contiguous literature. It is therefore expected that this thesis will serve as a valuable resource for readers wanting a well-rounded introduction to classical matrix ensembles, matrix product ensembles, 1-point recursions, loop equations, Selberg correlation integrals and ribbon graphs.

Some of this research has been published in [1–3].

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ANAS A. RAHMAN, School of Mathematics and Statistics,
University of Melbourne, Parkville, Victoria 3010, Australia
e-mail: anas.rahman@live.com.au

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