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ERRATUM

Magnetohydrodynamic stability and the effects of shaping: a near-axis view for tokamaks and quasisymmetric stellarators – ERRATUM

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In the recently published paper Rodríguez (2023), a few typographical errors were identified in the equations. This erratum is issued to rectify those errors, although we note that these corrections have no impact on the conclusions and discussions presented in the original work. For the sake of thoroughness and precision, we provide supplementary computational and numerical materials alongside this erratum. These additional resources can be accessed in a Zenodo repository (10.5281/zenodo.10442682).

The first error in the paper may be found when examples of quasisymmetric configurations are presented in Figure 5 and Table 1. Specifically, the case labelled as '2022 QA' was misrepresented in those, as its most vertically elongated up-down symmetric cross-section was not chosen to describe it. This contravenes the criterion considered for constructing Figure 5, in which the configurations are meant to be represented by their most vertically elongated up-down cross-section. The root cause of this inaccuracy lies in the fact that '2022 QA' was the sole configuration with its more horizontally elongated cross-section at $\varphi = 0$. To rectify this, we have addressed the error by rotating the configuration and recalculating its pertinent features. The corrected features, along with the updated Figure and Table, are presented herein for clarity and accuracy.

Table 1 presented a typo in the triangularity of 'N3B', which had a missing negative sign. The triangularity, δ, of the vertically elongated cross-section of this configuration is negative. We remind the reader that this measure of triangularity, as indicated in Appendix C and (C2), is different from the total triangularity of the cross-section (as may be seen in the Figure as well). Although this configuration presents a negative value of δ, which is beneficial for stability, it is not enough to make the configuration Mercier unstable, emphasising the point stressed throughout the paper about the triangular shaping of the cross-section being but one part of the whole contribution to stability.

	PQA	PQH	NQH	22QA	N3V	N4LA	N4W	N4M	N7	N3B
$\bar{F}$	−3.1	−1.4	−1.6	−2.6	−1.5	−1.3	−1.7	−1.6	−0.8	−1.6
δ	4.9	4.6	−1.7	9.8	0.9	1.2	11.8	−9.1	1.3	−0.9
$\mathcal{T}_\delta$	−0.7	−0.5	−1.9	−0.5	−0.8	−0.6	−0.5	−3.9	−1.0	−0.6
$V''/8\pi^2G_0$	1.1	1.1	−1.4	−0.2	1.9	2.0	−0.5	−11.9	7.3	−1.7*

TABLE 1. Details of the configurations in Figure 5. The table includes the values of $\bar{F}$, the triangularity δ, the effect of triangularity $\mathcal{T}_\delta$, and the magnetic well V'' for the configurations represented in Figure 5. The short labels on top refer to PQA - precise QA, PQH - precise QH (from Landreman & Paul 2022), NQH - new QH (from Rodríguez *et al.* 2022*c*), 22QA - 2022 Qa, N3V - N3 vacuum, N4LA - N4 long axis, N4W - N4 well, N4M - N4 Mercier, N7 and N3B - N3 beta (all these from Landreman 2022). For the latter instead of the magnetic well we show the $\epsilon^2 D_{\mathrm{Merc}}$, which shows that this finite β configuration is unstable.

In § 4.3 of the paper, there is a dimensional typo in the expression in (4.4), where the left hand side should be normalised to $(\ell')^2$ where $\ell' = \mathrm{d}\ell/\mathrm{d}\varphi$ along the magnetic axis,

$$\frac{\bar{\iota}_0^2\mathcal{T}_{|p|}}{4(\ell')^2\kappa^2} = \frac{2\alpha}{(3+\alpha)-\bar{F}(1+\alpha)} - \sqrt{\alpha}\mathcal{I}[\sqrt{\alpha},\sigma] \tag{4.4a}$$

$$= -\frac{\alpha[(1-\sqrt{\alpha})^2-(1+\alpha)\bar{F}]}{(1+\sqrt{\alpha})[(3+\alpha)-(1+\alpha)\bar{F}]} - \sqrt{\alpha}\left(\mathcal{I}[\sqrt{\alpha},\sigma]-\frac{\sqrt{\alpha}}{1+\sqrt{\alpha}}\right). \tag{4.4b}$$

Recall that the expression is written for $B_0 = 1$. This typographic error has no consequence on the paper.

In Appendix E, the equations from the second order near-axis expansion required to assess the contribution of shaping to stability are presented. We rewrite expressions (E6)–(E8) in a slightly more elegant way, and correcting for some typos that the original form presented. Their derivation may be found in the Zenodo repository. We rewrite (E6) as,

$$\mathcal{C}B_{20} + \mathcal{D} = 0, \tag{E6}$$

where,

$$\mathcal{C} = -\frac{8\ell'\tilde{\tau}}{\eta} + \frac{2\iota_0\kappa^2}{\eta^3}\left[1+\sigma^2-3\left(\frac{\eta}{\kappa}\right)^4\right]. \tag{E7a}$$

The expression for the $\mathcal{D}$ components must then read,

$$\mathcal{D}_{-1} = -\frac{4\kappa^2}{\eta}\frac{\mathrm{d}}{\mathrm{d}\phi}\left\{\frac{1}{\kappa^2}\frac{\mathrm{d}}{\mathrm{d}\phi}\left[\tilde{\tau}^2+\frac{1}{(l')^2}\left(\frac{\kappa'}{\kappa}\right)^2+\frac{\kappa^2}{4}\right]\right\} - 4\frac{B_{\alpha 1}}{B_0}\eta\ell', \tag{E8a}$$

$$\mathcal{D}_0 = -\frac{24\ell'\tilde{\tau}}{B_0\eta}B_{22}^C + \frac{8\tilde{\tau}}{B_0\eta}B_{\alpha 1} + 24B_{22}^S\left(\frac{\ell'\tilde{\tau}\sigma}{\eta B_0}+\frac{\eta\kappa'}{B_0\kappa^3}\right)$$
$$+\frac{4I_2^2}{B_0^2}\left(\frac{\sigma\kappa'}{\eta\kappa}-\frac{\eta\ell'\tilde{\tau}}{\kappa^2}\right)+\frac{4\eta I_2}{B_0}\left(\frac{2\ell'\tilde{\tau}^2}{\kappa^2}-\ell'+2\frac{(\kappa')^2}{\kappa^4\ell'}\right)$$

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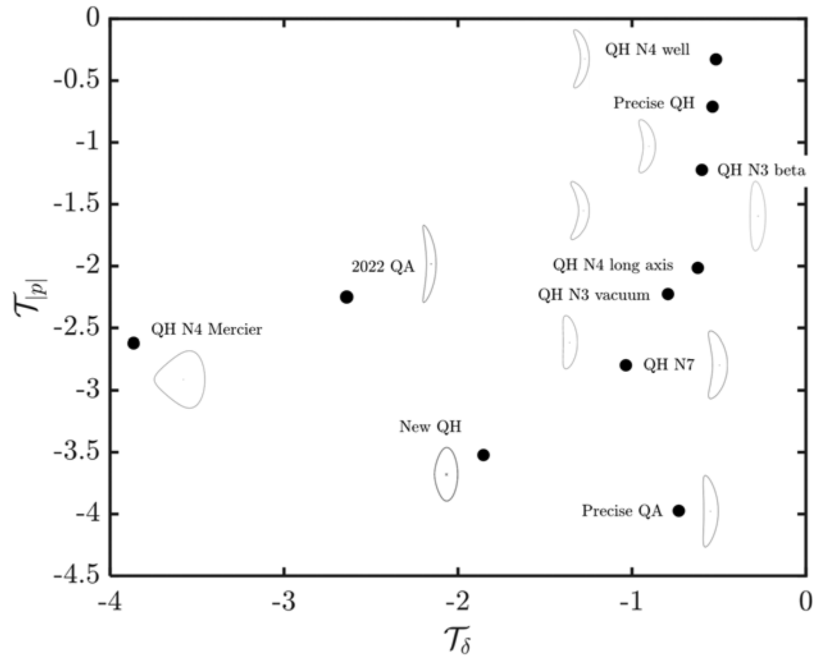

FIGURE 1. Effect of triangularity and pressure on MHD stability for some quasisymmetric stellarators. The plot shows as scatter points the factors regulating the effect of the triangularity ($\mathcal{T}_\delta$) and pressure gradient ($\mathcal{T}_{|p|}$) for several optimised quasisymmetric near-axis stellarators. The 'precise' QA and QH are from Landreman & Paul (2022), the new QH corresponds to the new optimised stellarator example from Rodríguez *et al.* (2022*c*), while all others are from a recent publication (Landreman 2022). We chose those configurations with reduced B_{20} variation so that the magnetic well computation, using a constant B_{20}, showed good agreement with the full V''. The cross-sections shown correspond to the $\phi = 0$ cross-sections in each configuration.

$$
\begin{aligned}
&+\frac{4\eta}{\ell'}\left[2(\ell')^2\tilde{\tau}+8\tilde{\tau}\frac{(\kappa')^2}{\kappa^4}+\frac{1}{\kappa^3}(9\kappa'\tau'-5\tilde{\tau}\kappa'')+\frac{1}{\kappa^2}(3(\ell')^2\tilde{\tau}^3-2\tau'')\right]\\
&-\frac{2\sigma}{\eta}\left[3\kappa\kappa'-24\frac{(\kappa')^3}{(\ell')^2\kappa^3}+10\tilde{\tau}\tau'+30\frac{\kappa'\kappa''}{(\ell')^2\kappa^2}+\frac{2}{\kappa}\left(\tilde{\tau}^2\kappa'-2\frac{\kappa'''}{(\ell')^2}\right)\right] \qquad \text{(E8}b\text{)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{D}_1 =\ &\frac{12\kappa^2\sigma}{B_0\eta^3}B_{22}^S+\frac{6\kappa^2B_{22}^C}{B_0\eta^3}\left(\sigma^2-1-3\left(\frac{\eta}{\kappa}\right)^4\right)\\
&+\frac{2I_2^2}{\eta B_0^2}\left(1-2\left(\frac{\eta}{\kappa}\right)^4-2\sigma^2\right)+\frac{16\eta I_2}{B_0\kappa^2}\left(\frac{\tilde{\tau}\eta^2}{\kappa^2}-\frac{\sigma\kappa'}{\ell'\kappa}\right)\\
&+\frac{2\kappa^2B_{\alpha1}}{B_0\ell'\eta^3}\left(3\left(\frac{\eta}{\kappa}\right)^4-1-\sigma^2\right)-\frac{8\eta\sigma}{\kappa^3\ell'}(\kappa\tau'+8\tilde{\tau}\kappa')\\
&-\frac{1}{\eta\kappa^6}\left[2\tilde{\tau}^2\kappa^6\left(1+6\left(\frac{\eta}{\kappa}\right)^4\right)+\kappa^8\left(1-8\left(\frac{\eta}{\kappa}\right)^4\right)+(48\eta^4-30\kappa^4)\left(\frac{\kappa'}{\ell'}\right)^2\right.\\
&\left.+\frac{4\kappa^5\kappa''}{(\ell')^2}\left(4+\left(\frac{\eta}{\kappa}\right)^4\right)\right]+\frac{2\sigma^2}{\eta\kappa^2}\left[12\left(\frac{\kappa'}{\ell'}\right)^2+\kappa^2(2\tilde{\tau}^2+\kappa^2)+\frac{2\kappa\kappa''}{(\ell')^2}\right] \qquad \text{(E8}c\text{)}
\end{aligned}
$$

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$$\mathcal{D}_2 = \frac{8\eta I_2}{B_0\ell'\kappa^2}\left(\left(\frac{\eta}{\kappa}\right)^4+\sigma^2\right) - \frac{8\tilde{\tau}\eta\sigma^2}{\ell'\kappa^2} - \frac{32\sigma^3\kappa'}{\eta(\ell')^2\kappa} - \frac{16\sigma\kappa'}{\eta(\ell')^2\kappa}\left(1+\left(\frac{\eta}{\kappa}\right)^4\right) - \frac{8\tilde{\tau}\eta}{\ell'\kappa^2}\left(3\left(\frac{\eta}{\kappa}\right)^4+5\right) \tag{E8d}$$

$$\mathcal{D}_3 = -\frac{2}{\eta(\ell')^2}\left[4\left(\frac{\eta}{\kappa}\right)^8 + 11\left(\frac{\eta}{\kappa}\right)^4 + 4\right] - \frac{2\sigma^2}{\eta(\ell')^2}\left[4\left(\frac{\eta}{\kappa}\right)^4+7\right] \tag{E8e}$$

where $B_{\alpha 1} = G_2 + \iota_0 I_2 = -G_0 p_2/B_0^2$.

In Appendix F the paper presents the dependence of the Mercier criterion D_{Merc} on the lower order near-axis quantities and the Shafranov shift. The paper also presents a few typos in there. The dependence on lower order quantities, that is the expression for $\varLambda$, should read

$$\begin{aligned}\varLambda_2 = {} & \frac{1}{2\eta^2\iota_0\kappa^6(3\eta^4+5\kappa^4) - 8\eta^4\kappa^8(I_2-\tau)}[\iota_0^3(\eta^4+\kappa^4)(32\eta^8+11\eta^4\kappa^4-3\kappa^8) \\ & + 4\eta^2\kappa^6\tau(I_2-\tau)(20\eta^4 I_2 - 3(5\eta^4+\kappa^4)\tau) - 2\eta^2\iota_0^2\kappa^2(4(16\eta^8+8\eta^4\kappa^4-3\kappa^8)I_2 \\ & - (75\eta^8+50\eta^4\kappa^4-9\kappa^8)\tau) + \iota_0\kappa^4((201\eta^8+20\eta^4\kappa^4+3\kappa^8)\tau^2 \\ & + 20\eta^4(-17\eta^4+\kappa^4)\tau I_2 + 16\eta^4(8\eta^4-3\kappa^4)I_2^2)],\end{aligned} \tag{F1}$$

$$\begin{aligned}\varLambda_0 = {} & -\frac{\eta^4\iota_0^2\kappa^3(5\eta^4-\kappa^4) - 4\eta^6\iota_0\kappa^5(3I_2+\tau)}{2\eta^2\iota_0\kappa^3(\iota_0(3\eta^4+5\kappa^4)+4\eta^2\kappa^2(\tau-I_2))} \\ & - \frac{\iota_0(11\eta^8-46\eta^4\kappa^4+3\kappa^8)+4\eta^2\kappa^2(11\eta^4-3\kappa^4)(I_2-\tau)}{2\eta^2\kappa^3(\iota_0(3\eta^4+5\kappa^4)+4\eta^2\kappa^2(\tau-I_2))}\kappa'' \\ & + \frac{16\eta^2[\eta^2\iota_0+\kappa^2(\tau-I_2)]}{\iota_0(\iota_0(3\eta^4+5\kappa^4)+4\eta^2\kappa^2(\tau-I_2))}\tau'',\end{aligned} \tag{F2}$$

$$\varLambda_{-2} = \frac{4\eta^2\kappa''(\kappa^3+4\kappa'')}{\iota_0(\iota_0(3\eta^4+5\kappa^4)+4\eta^2\kappa^2(\tau-I_2))}. \tag{F3}$$

These expressions, which were presented for completeness, are not used in the paper explicitly and thus have no further consequence. Concerning the expression for the dependence of D_{Merc} on the Shafranov shift, $\mathcal{T}_\Delta$, there is a minus sign typo in the numerator of (F4),

$$\mathcal{T}_\Delta = 3\kappa\frac{(\alpha-1)-\bar{F}(1+\alpha)}{\alpha-\bar{F}(1+\alpha)}. \tag{F4}$$

Finally, in Appendix C, the expression presented in (C2) for the total triangularity of the up-down cross section in the 'lab-frame' is not correct. The original derivation

actual expression is significantly more complicated than that presented. The correct expression describing the geometry at the stellarator symmetric point is,

$$\begin{aligned}\delta_{\text{lab}} = \delta_{\text{tok}} + \frac{\epsilon}{R_0}\kappa^R\frac{\eta}{\kappa}\Bigg\{ & \sin\nu\left[\frac{1}{2}\left(\frac{\kappa}{\eta}\right)^4\left(1+\frac{R_0''}{R_0}\right) - 1 + 2\left[1-\left(\frac{\kappa}{\eta}\right)^4\right]\frac{R_0''}{R_0-R_0''}\right] \\ & + \left(\frac{\kappa}{\eta}\right)^2 \text{sign}[\cos\nu]\sigma' + \left[1-\left(\frac{\kappa}{\eta}\right)^4\right]\frac{Z_0'''}{R_0-R_0''}\cos\nu\Bigg\}\sin\nu,\end{aligned} \tag{C2}$$

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where ϕ represents the cylindrical coordinate (and primes derivatives with respect to it), $R_0(\phi)$ and $Z_0(\phi)$ are the radial and vertical position of the magnetic axis, κ^R is the projection of the axis normal along the radial direction ($\kappa^R = -\text{sign}[R_0 - R_0'']$), ν the angle denoting the deviation of the axis binormal from $\hat{z}$, and all quantities are being evaluated at the origin $\phi = 0$. Of course, in the limit of $\nu = 0$, the triangularity is precisely δ_{tok}, which is the same conclusion that was reached originally in the paper. It might appear surprising that the expression presents potentially divergent expressions with $R_0 - R_0''$ in the denominator. This factor is however directly related to the curvature at $\phi = 0$, $\kappa(0) = |R_0 - R_0''|/\sqrt{R_0^2 + (Z_0')^2}$, and thus so long as κ is non-vanishing, as it must be for a QS field, there is no divergence.

REFERENCE

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