

AESTHETIC FACTORS IN GEOMETRIC MODELLING

BINH PHAM¹

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Abstract

CAD systems have traditionally catered for architectural and mechanical engineering designs which are somewhat constrained in scope. Problems concerning other types of design such as art design, industrial design or sculpting, where creativity and aesthetic factors play an important role, are not adequately addressed. These types of design require much more flexibility in both geometric modelling capability and user-machine interaction. This paper first gives a brief overview of recent work which deals with creative activities, and analyses important issues that need to be addressed in object representations for 3D creative activities. We then discuss a scheme to categorise and represent aesthetic factors in geometric modelling.

1. Introduction

The purpose of drafting is to produce polished and professional artwork which has been designed by an artist or designer. Drafting does not require great drawing skills or imagination. It requires instead the ability to pay attention to precise details and produce them meticulously once they have been defined. These precise characteristics of drafting match well with what is provided by current CAD systems in terms of their capabilities for geometric modelling and computerised facilities for editing. These systems are becoming increasingly more sophisticated and powerful. They provide tools for constructing and manipulating 2D and 3D objects in a variety of ways as well as for producing high quality and realistic colour images of these objects. Sophisticated modes of display and presentation (for example, animation and fly-through) facilitate the communication between designers, users and manufacturers. Interactions between users and CAD systems have also shifted from being text-based to being graphical user-interfaces with hierarchical menus. Furthermore, data can also be entered or selected interactively to avoid the tedium of constructing input files in

¹Faculty of Information Technology, Queensland University of Technology, GPO Box 2434 Brisbane Qld 4001, Australia.

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advance. These efforts aim at allowing a designer to work in a more intuitive fashion. However, there is an increasing demand to produce objects that are more artistically pleasing. People are not merely satisfied with the effects of photographic exactness, but demand the style and expressiveness produced by artistic flair. Furthermore, in design areas such as art design, industrial design and sculpture, it is imperative not to neglect aesthetic issues. In this paper, we use the term 'creativity' in connection with any activity relating to 2D and 3D drawing, sketching, painting, design or sculpting that requires imagination and aims to create aesthetically pleasing products.

We first examine how 2D creative activities such as font design, calligraphy and brush painting have been catered for, then analyse important issues that need to be addressed in object representations for 3D creative activities. A framework for categorising and representing aesthetic factors in geometric modelling will also be discussed.

2. 2D creative activities

2.1. Geometric models One of the early works which made a critical comparison of the quality of art work in ink to that in digital form is carried out by Knuth [13]. He noted that the physical properties of ink made it impossible to distinguish fine details at a resolution of more than 500 dots per inch, and thus it is possible to consider using discrete raster-based printing devices for high-quality mathematical typography and fonts which retain their artistic characteristics. Other previous attempts had obtained digital fonts by first digitising existing fonts using a camera and then refining them laboriously by manual editing. Knuth improved this process by using piecewise cubic splines for constructing the characters for his METAFONT system for type design. The shapes of these characters are manipulated by specifying a number of critical points and the tangent directions at these points. He also generated most digits by imitating the process a calligrapher uses with pen and ink. Different shapes of pen (circular, elliptical, horizontal and vertical) are also modelled using cubic splines. Other parameters used for manipulation included cap size, height, width and thickness of pen strokes. This method produced pleasing-looking fonts in a systematic manner. Knuth also extended the method to approximate handwriting characters and introduced a certain amount of randomness to vary slightly the position of each written letter in order to make the letters less regular and more natural-looking.

Another area of aesthetic design is in the modelling of brush strokes in calligraphy and in Chinese or Japanese brush painting. In these activities, every brush stroke has a definite trajectory along which a brush is moved. The amount of ink absorbed by the paper on either side of the trajectory varies mainly with the pressure and the amount of ink put on the brush by the artist. There are two main approaches: to model painting as

a physical process or to simulate the effects of the painting process. The modelling in most cases has been carried out for both the geometric shape and the shade (or colour) of a brush stroke. Ghosh and Mudur [11] derive algebraic solutions for describing the outlines of brushstrokes for some specific brushes and trajectories. A fourth-order parametric equation is used for describing a closed, smooth and convex brush. The trajectories considered are of linear, circular, elliptical and cubic type. Although their techniques are efficient, they are not intuitive and not suitable for interactive use as it is not easy to specify required parametric values for desired effects. The Chinese painting system developed by Pang *et al.* [14] is based on a set of parameterised primitives of brush strokes. Procedures for drawing a number of primitives such as leaves, branches and waves are built into the system.

Strassmann [21] simulates the physical process of brush painting in terms of four objects: brush as composed of bristles, stroke as a trajectory of position and pressure, dip as the initial amount of ink on a brush and paper as a mapping onto a display device. Cubic splines are used to represent the strokes, and the width of a stroke at each node of the cubic spline is calculated as a function of pressure. The dip is used to calculate the amount of ink on a bristle at a given time and hence the colour of each pixel on the stroke. This technique manages to capture the expressiveness of Chinese painting brush strokes, but it is computationally expensive and not intuitive. In particular, it is not easy to simulate the pressure required to produce a desired stroke.

There are two methods (see [4, 19]) which simulate the effect of a brush stroke without imitating the exact details involved in the physical process. Chua [4] uses piecewise cubic Bezier curves to model the outline of brush strokes for Chinese calligraphy. The variation of shades (or colours), moisture, texture and absorbency of the paper are also represented by cubic Bezier functions. The area between each pair of brush segments is divided into a specified number of regions which are painted with an appropriate shade determined by the shade function. This method, although producing pleasing results, does not explore the user-interface issues completely.

Pham's method [19] aims to rely mainly on human visual perception and intuition to provide a simple and efficient technique for producing interactively expressive brush strokes. The representation consists of three major components: the trajectory, thickness and shade. The trajectory is modelled as a cubic B-spline whose knots may be inputted by using a mouse. In general, only a small number of knots are needed for a fairly complicated trajectory. Variable offset B-splines are used to represent the brush strokes, where the thickness of each stroke is defined by the offset distances from the trajectory at the knots. Each bristle is represented as an offset B-spline curve whose knots can be automatically calculated from the specified number of bristles and the offset distances at the knots of the trajectory. Both the thickness and the number of bristles may differ on each side of the trajectory, hence the effects of both the variable pressure and of the density of the brush are simulated. Further irregular

effects, such as scratchiness due to a drier brush and spreading bristles due to stronger pressure, could also be simulated by varying the offset distance of each bristle from the trajectory.

Realistic shading is achieved by modelling each bristle as a 3D offset B-spline curve whose first two coordinates are the spatial coordinates, and whose third coordinate represents the shade value (or intensity) of the bristle. To facilitate the inputting of shade values at the knots on the trajectory, a bar containing the available range of shades is displayed on the screen, from which shades are selected using a mouse. The main advantage of modelling bristles as 3D B-splines is that the shape and shades of brush strokes can be manipulated locally in a flexible and intuitive manner. This representation has a further advantage in that it facilitates the animation of brush strokes. It is shown that basic transformations such as scaling, rotation and translation, which are essential for producing in-between frames, can be performed on the control vertices instead of on the curves themselves [19].

Although the above-mentioned techniques are developed as independent systems for specific purposes, it would be quite easy to integrate them into a CAD system as they are based on different types of spline curve and offset curve generation already available in common CAD systems. This would expand the CAD capability to allow the creation of more artistic drawings with a user-specified range of dynamics and variability.

2.2. Intensity and colour Most methods described in the previous subsection treat shading and intensity in a similar manner to spatial coordinates. Different types of spline functions are used to describe intensity. This allows both smooth and abrupt changes in intensity to be simulated. Control nodes for the Bezier or B-spline functions also provide users with a tool to specify and manipulate different shading effects. A bar displaying all the available shades on the screen would assist users in selecting desired intensities in a more intuitive fashion.

These techniques can be readily extended to produce different shades of colour by using the same types of splines with three variables, one for each colour component. HSB colour space is used in preference to RGB space because it is more intuitive to perceive a colour in terms of hue, saturation and brightness, whereas predicting a colour resulting from a specified amount of red, green and blue components is a very difficult task.

Random functions may also be used to achieve special effects such as scratchiness or smudging. For example, to produce a soft and smudging effect around the boundary region between two different colour regions, pixels of one colour are moved randomly to the other colour region. To produce scratchy brush strokes, Pham [19] varies the amount of ink on a brush in a pseudo-random fashion.

3. 3D creative design

Creativity in design occurs at the conceptualizing level when ideas are vague, tentative and often expressed by a series of rough sketches (for example, in industrial design) or rough prototypes (for example, sculpture) of different alternatives. These alternatives may be variations of the same theme, or distinct products resulting from mental leaps. The designer's mind, moving quickly back and forth between these alternatives, mentally evaluates their aesthetic values or suitability. Detailed information is not a concern at this stage, and is left to be worked out much later. In [16], we evaluate the progress in a number of research areas towards the facilitation of the creative design process. These areas include geometric techniques for flexible object manipulation, data input techniques to CAD systems, parametric and feature-based design and artificial and knowledge-based techniques. This section discusses essential characteristics of geometric representations for 3D objects which are suitable for use in the creative design stage.

3.1. Requirements of object representation for creative design The most commonly-used object representations in CAD systems are wireframes, polyhedra, CSG, boundary representation and volumetric. These representations are exact, precise and consistent. They offer no scope for fuzziness. The designer needs to know the precise details of each object before it can be constructed. In the early stages, the most important feature that a designer wishes to capture is the global form of an object, hence a desirable object representation must be able to capture the qualitative characteristics which distinguish one form from another (for example, whether it is linear, curved, round, elongated or twisted). Such a representation must be invariant with respect to translation, scaling and rotation. It should be able to smooth out any extraneous noise and capture the intrinsic properties of the surface such as surface orientation and curvature. In other words, the representation must be able to handle both smooth and sharp curvature changes without blurring sharp corners. It should also provide a mechanism for adding detailed information at a specified accuracy to the object model, because although detailed information is not important at this stage, it may be required at later stages. In addition, the designer must be able to control the degree of coarseness of the model at any stage. Hence the representation must cater for this *via* the use of parameters for models or the use of multi-resolution schemes. Efficiency and flexible manipulation are also essential to get real-time responses and to cater for a large domain of objects in a variety of forms. The representation must also be sufficiently compact and easily converted to an object representation commonly used by CAD systems.

Both computer vision and computer graphics require representation schemes for 3D objects, for object recognition in the former case, and for image generation and

manipulation in the latter case. In some cases, these representations are identical. For example, in CAD-based vision, objects are represented in terms of a CSG tree or boundary representations. However, there are some distinct differences between these two cases. For computer vision, features of an object model are explicit because it is necessary to be able to index into them in order to recognise the object, whereas a CAD model only contains explicit geometric information in terms of primitives. Information about object features are only implicit and need to be derived if required. Furthermore, in computer graphics, it is important to have a compact representation with a small set of control nodes, which can provide a highly accurate rendering. An object in computer vision, on the other hand, is often constructed from sparse, noisy or inadequate data which is extracted from digitised images by performing edge detection and segmentation on them. Accurate local information is not essential in this case. For this reason, an object model suitable for creative design shares more common characteristics with this latter model than with the usual model for CAD.

3.2. Deformable object representations Some work in the field of computer vision has analysed how humans perceive and recognise shapes (see for example [2, 15]) in an attempt to find a good object representation for object recognition systems. This is the basis for the approach of recognising an object by its components. Biederman [2] represents an object as a hierarchy of geons (geometrical ions). This model, which gives a qualitative description of the whole object and its topology, is based on the fact that our visual system uses collinearity, curvature, symmetry and cotermination to discriminate between and recognise shapes. Geons form a restricted class of generalised cylinders whose cross-sections, axes and sweep properties are arbitrary functions. To construct a set of 36 primitives, Biederman considers a number of variations in cross-section shape (straight or curved), cross-section symmetry (rotationally symmetric, reflectively symmetric or asymmetric), axis shape (straight or curved) and cross-section sweeping property (keeping the cross-section constant, expanding or shrinking).

To extract component geons from an object, Dickinson *et al.* [6] make the assumption that any junction of two primitives involves exactly one attachment surface from each primitive. They then label each primitive attachment surface and the position of the join. A hierarchy of primitives, face structures, faces and face features are used for indexing the model and for constructing strategies for matching image features to the model. As the geon representation describes the larger structures of the object, it avoids some restrictions imposed by other models that are based on local information. However, the geon model suffers the drawback of not being able to provide adequate quantitative information about the object. This type of information is required at later stages for object refinement and manipulation.

Other attempts to improve flexibility in shape manipulation use more complex but

flexible primitives such as superquadrics [1, 10, 15], hyperquadrics [12] and cyclides [7, 8]. One approach that attempts to combine both local and global information uses deformable superquadrics [23]. This approach is motivated by the fact that shape reconstruction is carried out based on available low-level information such as edge points, while more abstract and high-level information are required to make object recognition possible. The global shape characteristics are governed by the global parameters of the superquadrics while the local characteristics are modelled by splines. The model is deformed by applying simulated forces on data points and deriving the equations for free rigid motion. Two common types of forces that are used to force the model to conform to the data shape are the gradients of potential function and to the distances between data points and the model surface.

Cyclides are quartic surfaces whose spine curves are formed from the loci of the centers of two families of variable spheres. The shapes constructed from cyclides are somewhat more restricted than those from superquadrics and hyperquadrics. To obtain hyperquadrics, Hanson [12] extends the class of superquadrics to retain their elegant properties, but expand their geometric limits from cubes to convex polyhedra. The basic equation for hyperquadrics allows a hierarchical representation which provides a gradual fit to a shape by adding further terms. Although a wide range of imaginative 3D shapes could be created using these primitives, it is neither easy nor intuitive to exert control over appropriate parameters to obtain a desired shape.

There exist a number of dynamic (or deformable) models which are designed to capture the dynamic nature of a malleable surface and yet retain surface discontinuities (see for example [3, 5, 22]). These techniques obtain a surface description in terms of triangular meshes from a set of 3D data which may come from various sources such as range data or contours. These techniques differ mainly in their object representation schemes and in the approaches to how model fitting is carried out. Some aim to minimise the approximation errors globally, while others pay more attention to local information. All these models may be used for object representation for 3D creative design, if the intention is to control shape *via* a set of 3D points on the surface. However, one pertinent question is whether the level of accuracy offered by these models is necessary for this purpose. As only global form is required at this stage, it is sufficient to look for any model that can capture global form and yet has the additional capability to include further detail when required later on. It is expected that such a model would also require less computational effort and processing time.

A model which appears to satisfy the requirements for creative design is a set of parameterised geons proposed by Wu and Levine [24] for the purpose of qualitative recognition of object components. These parametric geons are based on superellipsoids which are a parameterised family of closed surfaces. The size and degree of squareness, bending and tapering of these geons are controlled by various parameters and parameterised equations. This model is more intuitive than those proposed by

Hanson and Terzopoulos which are mentioned earlier. Another property that makes this model more attractive for creative design purpose is that it is not difficult to extend the parameterised functions for other types of shape deformation. Because of these reasons, we elect to use these parametric geon primitives to provide global shapes for our current work to investigate generic techniques for specifying and realising aesthetic factors. To obtain local shape manipulation and accuracy, we use the butterfly subdivision scheme [9] which allows the adjustment of local tension.

4. Representation schemes for aesthetic factors

In [17], we categorise aesthetic factors into four main types: form, physical attributes, emotion-evoking attributes and specific styles.

Form relates to the shape of an object or its components. In simple cases, it is concerned with general geometric shape characteristics (such as being squarish or oblong), fairing or styling of curves and surfaces (as for a car body shape). In more complex situations, form may involve procedures to perform complicated transformations of the object shape (such as morphing or deformation of axes).

Physical attributes are concerned with aesthetic factors such as shading, light reflection and texture. These factors may depend on physical properties (such as material and paint) or geometric properties (for example, the way light reflects on a surface depends on its shape). In the latter case, the last two types of factors are interdependent. Both of them may be quantitative or qualitative.

Emotion-evoking attributes refer to those characteristics of object appearance that induce emotional responses. Such attributes may be typically soft, aggressive, expressive, sensual, rich or desolate. Although these attributes are hard to define exactly, one can identify a number of form or physical attributes that would provoke such emotional responses. For example, curvy and rounded objects tend to provoke more sensuality while objects containing sharp lines or strong colour contrast tend to appear more aggressive.

Specific styles refer to attributes that are common to a certain period or fashion, for example, the high-tech look, the style of the 60s, or art deco style. Again, it is possible to identify a number of geometric or physical attributes that are typical for each style. Thus, although there is no one-to-one correspondence between the last and the first two categories of attributes, it is possible to establish a relationship between them in the form of a many-to-one or one-to-many mapping. This would enable the last two categories of attributes to be achieved *via* geometric and physical methods which are implementable.

In addition to this categorisation of aesthetic factors, we have also proposed a general hybrid representation for these factors in design, which consists of three parts:

visual, symbolic and quantitative [18]. To deal with aesthetic factors effectively, we need not only a combination of qualitative and quantitative representations, but different types of reasoning. It is also crucial to connect such reasoning with visual perception as well as with mathematical and geometric concepts that allow the model to be realised. Visual representations which include diagrams, sketches, pictures, graphics and animation use both visual and spatial properties for conveying information. Symbolic representations describe and convey information *via* the semantics of symbols and how those symbols are combined to form a more complex structure. A common example of a symbolic representation is a natural language where words and sentences are used. In some cases, a visual representation can be constructed using symbols. Quantitative representations include numerical values, sets, intervals, fuzzy sets, functions, mathematical and geometric techniques and algorithms.

In addition to a hybrid representation, a mixture of reasoning processes may also be used. The choice of which representation and reasoning process is to be used for each aesthetic factor depends very much on how it is viewed intuitively by designers, and how it can be realised by mapping to appropriate numerical values or mathematical and geometric techniques. We call such relations ‘quantitative mappings’. Thus, each aesthetic factor to be introduced to the system needs to be analysed individually to obtain the optimal representation. This process involves finding answers to the following questions:

- What terminology do designers use to express this particular aesthetic factor?
- Which of the above-mentioned four categories does this aesthetic factor belong to?
- Which quantitative mappings should be used to relate this aesthetic factor to the characteristics of shape or other properties?
- How does this aesthetic factor affect the shape and other properties of the object?

For example, the ‘sleekness’ factor that is often used to describe the characteristic of a curve (as in car or furniture design) may be represented in a number of ways:

- a set of finite scales expressed in a natural language (such as not sleek, fairly sleek, moderately sleek, very sleek, extremely sleek);
- a finite set of curves with different degrees of sleekness;
- a sliding bar to represent different values of the curve tangent or tension that designers can specify.

The first two representations are more intuitive to designers than is the last representation, although they require an extra step to map these symbolic and visual representations to corresponding values for curve tangent or tension in order to generate these curves. Furthermore, the first two representations provide a finite set of alternatives while the last representation provides a continuous and infinite set of al-

ternatives. Thus, a better representation which can take advantage of the good features of all three schemes is to represent the degree of sleekness by a sliding bar with values from 0 to 1. Specified values are then mapped to corresponding values for curve tangent or tension in a manner which is transparent to users. This scheme would provide a continuous and infinite set of alternatives, yet does not require designers to think in terms of tangent and curve tension, which requires a familiarity with mathematical knowledge.

In [20], we propose a systematic scheme to enable the realisation of aesthetic factors in geometric modelling using fuzzy logic. The scheme is composed of five main components: fuzzy specification, fuzzy database, fuzzy mappings, precise graphical algorithms and fuzzy or crisp output.

Fuzzy specification may be done *via* symbolic descriptors in a natural language for expressing aesthetic intents. The mapping from these symbols to numerical values, intervals or mathematical functions must be transparent to users. These symbolic descriptors allow designers to convey a fuzzy concept. For example, a shape may be specified as very round, roundish or slightly round; a greyscale intensity may be very dark, medium dark, medium, medium light and very light. The number of descriptors, which may be increased to cater for a finer distinction as required, is only restricted by the number of suitable words available in a particular language for communicating different meanings. In addition to this type of fuzzy descriptors, a fuzzy factor which depicts the level of fuzziness of the specification, may also be introduced to enrich each specification. For example, a sphere specified with a fuzzy factor of 0.01 implies that the radius and centre could be anywhere within 0.01 of the distance from the specified radius and centre. The multiple outputs resulting from such a specification would correspond very well with the multiple circles often sketched by a designer during the 'doodling' process before the circle that appears to be optimal is finally selected.

A fuzzy database must be constructed to contain data that can convey fuzzy meanings about aesthetic intents. This database stores fuzzy data types (such as numerical intervals and categories) as well as the hierarchies of these types. Each hierarchy of a fuzzy data type provides a finer description of the variable, hence allowing a designer's intents to be better matched. For example, a designer may wish to deal with the sleekness of a curve. In the first level of hierarchy, a number of degrees of sleekness may be specified. In the second level, a number of geometric techniques which can achieve a specified degree of sleekness may be selected. These techniques may require the specification of a range of the tangential angles at each end point of curves, or the specification of a range of the curvature of the curve. The decision on which parameter is to be made fuzzy and with what degree of fuzziness depends very much on the graphical representations and techniques used. These representations and techniques may also be either geometrical (that is, concerned with shape and

topology) or non-geometrical (such as shading, colour and texture).

The fuzzy mapping refers to how each fuzzy aesthetic intent may be realised. It may be achieved by applying a particular graphical technique while varying appropriate parameters, or by applying a number of different graphical techniques. However, for the scheme to be feasible, it is essential that each fuzzy mapping eventually leads to a precise graphical function with appropriate parameters, or to a number of precise graphical functions with appropriate parameters. In other words, by using fuzzy specifications, fuzzy data and fuzzy mappings, we allow the designer greater freedom during the cognitive process and permit final decisions on design objects to be delayed. This whole process produces fuzzy output in terms of design alternatives from which the designer can select the optimal ones after some comparative analysis.

5. Conclusion

A review of recent work which deals with creative aspects of geometric modelling is presented. In particular, methods for categorising aesthetic factors and representing them are covered, with the aim of removing existing technical barriers that discourage designers and disrupt their flow of thought during the creative design process. We are currently designing a fuzzy logic engine to allow designers to specify and manipulate the shape and appearance of 3D objects, based on this hybrid model for aesthetic factor representation. The parametric geons with extended functions for deformation together with the butterfly subdivision scheme have been chosen as our geometric models due to their intuitive properties and flexibility for shape manipulation.

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