

The book has been expertly produced, misprints and mathematical errors being very rare, and the notation and layout commendably well chosen.

In summary, anyone new to operator algebras might be advised to use some other text as an introduction. However as a work of reference for the specialist, this book will be invaluable.

C. J. K. BATTY

HUTSON V. and PYM, J. S. *Applications of Functional Analysis and Operator Theory* (Mathematics in Science and Engineering, Volume 146, Academic Press, London, 1980), xii + 390 pp.

The title of this book could mislead the potential reader. It might suggest that a working knowledge of the basics of functional analysis was assumed and that the text was intended to give reasonably extensive introductions to a number of modern applications of the standard theory. This, however, is not the case as the authors present a considerable amount of theory along with more modest introductions to the modern applications. Indeed, of the fourteen chapters in the book, the first nine could be said to be theoretical in nature, even if there is a plentiful supply of illustrative examples.

In their introduction, the authors claim that the abstract functional analysis required in a wide range of applications is not too great. Their aim is to substantiate this claim by making a careful choice of what they regard as the essential abstract techniques and by then putting these techniques into practice, using "solution of equations" as a unifying theme. The book is intended as the basis for a postgraduate course for applied mathematicians, physicists and engineers or others wishing to become acquainted with the powerful techniques of functional analysis. The prerequisites mentioned in the book are real analysis and linear algebra but others could be added to the list, such as a knowledge of Green's functions which appear regularly in the examples.

The book begins with a chapter on Banach and Hilbert spaces with many of the standard function spaces used as illustrations. Next comes a brief survey of those results in Lebesgue integration required for the L^p spaces, proofs usually being omitted. Chapter 3 introduces the foundations of linear operator theory including the Open Mapping Theorem, the Principle of Uniform Boundedness, elementary spectral theory and closed operators. Chapter 4 provides an introduction to non-linear operators covering the Contraction Mapping Theorem, Fréchet derivatives, the Implicit Function Theorem and Newton's Method, with examples involving non-linear integral equations. After a short chapter on compactness in Banach spaces, Chapter 6 introduces the concept of an adjoint operator, first in a Banach space and then in a Hilbert space. The spectral theory of bounded self-adjoint operators in Hilbert space is given and the adjoint of an unbounded operator defined. Chapter 7 deals with the spectral theory of linear compact operators and the theory is illustrated in the context of the numerical solution of linear integral equations. In Chapter 8, non-linear compact operators are discussed and the Schauder Fixed Point Theorem proved. Positive and monotone operators in partially ordered Banach spaces are examined and applications given to some non-linear differential and integral equations. Chapter 9 is devoted to the Spectral Theorem for both bounded and unbounded self-adjoint operators in Hilbert space.

Chapter 10 makes the transition from the theory to the applications with a discussion of generalised eigenfunction expansions associated with ordinary differential equations, including a treatment of self-adjoint extensions of symmetric operators and deficiency indices. In Chapter 11, we enter the world of partial differential equations with weak derivatives, Sobolev spaces and the generalised Dirichlet problem for linear elliptic equations. The next application is a brief discussion of the finite element method in the context of the generalised Dirichlet problem for the operator $-\nabla^2 + p$. Chapter 13 contains an introduction to the Leray-Schauder degree with frequent use of homotopy invariance and an application to a problem in radiative transfer. Degree theory also appears in the last chapter which discusses bifurcation theory, both local and global, and includes examples involving buckling of a compressed rod and periodic wave trains in deep water.

Throughout, the authors aim to supply as much motivation as possible and each chapter begins with a chatty introduction. The background to the Spectral Theorem (§9.3) and the preamble to the definition of degree (§13.1) are two particularly successful instances. Thus, although such material is available elsewhere, the authors produce, for the most part, a text which is more lucid and less indigestible than others. As already indicated, there is a good supply of worked examples and each chapter also contains a good selection of exercises. As regards the applications in the later chapters, it is perhaps a pity that they are not taken further, while space might have been found for a chapter on semigroups of operators and evolution equations, which receive only very brief mention. Thus the reader's appetite is whetted but not sated and those who are big eaters must look elsewhere. Fortunately, there is a list of over 100 references including a number of recent papers as well as a lot of old faithfuls, such as the "voluminous" Dunford and Schwartz, which contain some of the more complicated proofs (for instance, the proof of the Open Mapping Theorem) which the present authors purposely omit.

No text is perfect and the present is no exception. There are a number of mathematical slips, many of which are obvious and trivial but occasionally things go slightly haywire, the worst instance being Example 13.2.14 where there are several slips in quick succession. Grammatical pedants might have cause to complain. There seems to be inconsistency in the use of hyphens in phrases such as "infinite-dimensional" and "real-valued" and, likewise, in the use of commas, while there is at least one rather unfortunate spelling error at an early stage. There are also typographical errors and the quality of reproduction is sometimes imperfect. There are places where the type is broken, while the process used is such that corrections to the original typescript are easily visible. In a book costing so much, one perhaps expects better.

In summary, the authors succeed in their aim of presenting the essential tools of the trade and the book could be used for an introduction to operator theory. Indications of several areas of application are given although these do not cover the whole spectrum. The book can thus be recommended to someone who knows the areas of application but does not know the functional analysis, whereas a functional analyst wishing to learn in depth about applications should regard this book merely as a tasty *apéritif*.

ADAM C. MCBRIDE

JONES, W. B. and THRON, W. J. *Continued Fractions: Analytic Theory and Applications* (Encyclopedia of Mathematics Vol. 11, Addison-Wesley, 1980), 428 pp. £20.65.

As the General Editor of the Section on Analysis of the Encyclopedia of Mathematics and its Applications states in his Foreword, the volume under review is the first systematic treatment of the theory of continued fractions for over two decades, and forms a worthy successor to the well-known treatise by Oskar Perron whose first edition appeared in 1929. Continued fractions have for long played an important role in number theory. In the nineteenth century, however, the theory began to develop in a new direction, with important applications in analysis, and it is this aspect of the subject that is the concern of the present volume. The central theme is the expansion and convergence theory of continued fractions whose terms are linear functions of a complex variable.

In several applications of the analytical theory continued fractions can be used to give more computational accuracy than other methods. This arises in control theory where it is often necessary to decide whether a given polynomial with real coefficients is stable, i.e., whether all its zeros have negative real parts, and continued fractions may also, in certain cases, increase the range of use of asymptotic series. They can also provide representations for transcendental functions that are more widely valid than those using power series.

The longest and most basic chapter in the book is Chapter 4, which deals with the convergence of continued fractions and presents the most useful convergence criteria. This is followed by chapters on the representation of analytic functions, including hypergeometric and confluent hypergeometric functions. Various classes of continued fractions, such as *C*-, *J*- and *T*-fractions are introduced and the concluding chapters contain applications to a variety of topics, such as