BOOK REVIEWS

PITTS, C. G. C., *Introduction to Metric Spaces* (Oliver & Boyd, 1972), £1·50.

This is a comprehensive and readable treatise on metric spaces and their applications. The first chapter is a resumé of the basic results in analysis that are assumed, such as the properties of the real numbers, Riemann integration, convergence and uniform convergence. The inequalities of Hölder and Minkowski are also proved. Metric spaces are introduced in Chapter 2 and numerous examples are given. The basic topological concepts associated with a metric are described. Continuous functions from one metric space to another are discussed in Chapter 3 and the topological characterization of continuity in terms of open sets is given; equivalent metrics are also discussed. In the following chapter complete metric spaces are defined. Cantor's intersection theorem is proved and the completion of incomplete metric spaces is discussed and questions of uniqueness are considered. The contraction mapping theorem is proved and several applications are given, including a form of the implicit function theorem.

In Chapter 5 compactness is defined and several different equivalent characterizations for metric spaces are given. The properties of continuous functions over compact sets are discussed. The chapter also contains the Arzelà-Ascoli theorem and Peano's theorem concerning the differential equation $y' = f(x, y)$. Chapter 6 contains an account of connectedness including pathwise connectedness. The final chapter contains a number of further topics such as the Tietze extension theorem, Baire's category theorem, Weierstrass's approximation theorem and Stone's generalization. There are numerous exercises for the reader and the book is well set out and clearly printed.

R. A. RANKIN

STEEN, S. W. P., *Mathematical Logic with special reference to the natural numbers* (Cambridge University Press, 1972) xvi+638 pp., £15.

This book is an erudite and useful contribution to the study of Foundations. The author sets out to investigate the arithmetic of natural numbers on a systematic and rigorous basis. With this in mind he develops in the early chapters the theory of formal systems and of propositional and predicate calculi. The treatment is thorough and the chapter on predicate logic particularly good.

Having set up the necessary logical apparatus the author describes a straightforward formal arithmetic introducing many of the basic ideas which permeate the book. The theory of recursive functions is described in detail, the notions of Turing machines and unsolvability being carefully expounded.

It is then shown how the introduction of the universal quantifier to the arithmetical system previously described leads to Gödel's incompleteness theorems. An important chapter is devoted to induction, the treatment being akin to that developed by Goodstein. A further extension to incorporate the real numbers is then outlined.

The final section of the book deals with models and recent independence results. Cohen's forcing methods are briefly described. As the author readily admits a rather different approach would have been taken to some of the topics covered in the book if Cohen's results had been known earlier.

This work is lucid, rigorous and comprehensive. Although no previous knowledge