BELLINI-MORANTE, A. and MCBRIDE, A. C. *Applied nonlinear semigroups: an introduction* (Wiley Series in Mathematical Methods in Practice, vol. 3, John Wiley & Sons, 1998), ix + 273 pp., 0 471 97867 1, £50.

Mathematical models of systems which are evolving in time often produce an equation, or a system of equations, which involves only first-order derivatives in the time variable, with more complicated structure in other variables. A fundamental example is the heat equation, but this is untypical in that it can be solved exactly in terms of convolution with the Gaussian kernel. An effective approach to a more complicated evolution equation or system is to consider it as a first-order ordinary differential equation with values in a suitably chosen Banach space X of functions. Then one often arrives at a first-order Cauchy problem of the form:

$$u'(t) = A(u(t)) + f(t, u(t)) \quad (t \geqslant 0), \qquad u(0) = u_0.$$

Here A is an operator (unbounded in all important examples, often nonlinear, but independent of time for our purposes) on X which typically encapsulates the behaviour of the system itself, while the inhomogeneity f may represent some input and/or feedback.

Semigroups of operators arise from solutions of such Cauchy problems in the homogeneous case (when $f = 0$). The transition operators $T(t) : u_0 \to u(t)$ on X satisfy the semigroup relations $T(s+t) = T(s)T(t)$ and $T(0) = I$, as well as a continuity property. In the linear case, such semigroups are known as C_0-semigroups, and their theory has developed over the last 50 years into a fairly complete form. There is a precise correspondence between C_0-semigroups and well-posed, linear, homogeneous Cauchy problems and the operators A which generate C_0-semigroups are neatly characterized by the Hille–Yosida theory. Many inhomogeneous and ill-posed, but linear, problems can also be handled by extensions of the theory.

In the nonlinear case, the correspondences between semigroups, well-posed homogeneous problems, and generators are only partial. Nevertheless, semigroup theory does provide an effective method of establishing existence and uniqueness of solutions of some problems, and of studying their behaviour and obtaining approximations to the solutions. The linear theory is relevant to the nonlinear case both because it serves as a guide, and also because one approach to nonlinear problems is to make linear approximations and use fixed point theorems.

Each of the authors has previously written a book on C_0-semigroups, and now they have collaborated in providing a basic introduction to applications of semigroups in nonlinear evolution equations. The first chapter of 60 pages summarizes the abstract functional analysis needed—much of this would be available in more discursive form in the fourth year of a typical British undergraduate degree, but students who had specialized in applied mathematics might have missed it—while Lebesgue integration is summarized in an appendix. The rest of the book is suitable for a graduate student setting out to undertake research in applied analysis. The reader is led through linear semigroups, semilinear problems, nonlinear dissipative operators, and nonlinear semigroups to a final chapter which analyses examples arising from particle transport, contaminant diffusion, and combustion of solid fuel. The theoretical parts of the book are kept fairly brief by referring the reader to other texts for some proofs.

The book is 'applied' in the sense that it contains very many examples which are simplified forms of evolution equations arising in physical and other problems. However, these examples are not typical mathematically, because they all involve only one space variable. This suits the authors' purpose by eliminating any need for Sobolev spaces, avoiding difficulties with domains of operators and boundary conditions, and enabling Hille–Yosida conditions to be verified by solving ordinary differential equations. However, it conceals a lot of mathematical depth, and innocent readers are given no clue about the further effort required, not even in the preface or in a concluding section. With this reservation, I would recommend the book to students starting research in applied analysis as an excellent source of the basic ideas about applications of semigroups.

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RAMAKRISHNAN, D. and VALENZA, R. J. *Fourier analysis on number fields* (Graduate Texts in Mathematics, vol. 186, Springer, 1999), xxi + 350 pp., 0 387 98436 4, £30.50.

This book is addressed to students with a basic US graduate-level knowledge of algebra, analysis and topology, possibly intending to do research in modern number theory, harmonic analysis or the representation theory of Lie groups. It is divided into seven chapters, covering (1) Topological groups, (2) (classical) Representation theory, (3) Pontryagin duality for locally-compact abelian groups, (4) Structure of 'arithmetic fields', (5) Adeles, ideles and class-groups, (6) Quick tour of class field theory, (7) Tate's thesis and applications. Each chapter is supplemented by a set of instructive exercises, some broken into convenient 'bite-sized' sub-exercises. Those for Chapter 7 are particularly copious.

The material of Chapters 1–3, together with related appendices at the end of the book, is very carefully written, in a reader-friendly format, and constitutes a valuable introduction to the basic functional-analytic aspects of the theory. Chapters 4 and 5 apply the foregoing theory to derive the structure of 'arithmetic fields' (i.e. local or global fields) and of the related adele-rings, idele-groups and idele-class-groups; again, these are carefully written and enjoyable to read.

On the other hand, as a practising number-theorist, I am less satisfied by some aspects of Chapters 6 and 7. Although the exercises here are quite good, I am not convinced that a budding Ph.D. student in contemporary number theory would gain from this book enough of the important insights into the subject that he/she would need for work on some of the research topics alluded to in the Preface. The treatments of L-functions attached to Galois representations, and of p-adic L-functions, are rather scanty. For instance, a short account of the highlights of appropriate parts of p-adic analysis, placed, say, at the end of Chapter 5, might have served to whet the appetite for research into a topic of great current interest, but this opportunity has been missed. A similar criticism applies to the treatment of Galois representations and the associated L-functions.

Overall, it seems to the reviewer that this book is rather lop-sided, the first few chapters being admirably suited to the needs of the intended readership, but the last two failing to achieve the same high standard.

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