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On Some Stochastic Perturbations of Semilinear Evolution Equations

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Abstract. We consider semilinear evolution equations with some locally Lipschitz nonlinearities, perturbed by Banach space valued, continuous, and adapted stochastic process. We show that under some assumptions there exists a solution to the equation. Using the result we show that there exists a mild, continuous, global solution to a semilinear Itô equation with locally Lipschitz nonlinearites. An example of the equation is given.

1 Introduction

Let $(\Omega, \mathcal{F}, P)$ be a probability space together with the normal filtration $\mathbb{F} = \{\mathcal{F}_t, t \geq 0\}$. Let $\mathcal{P}_\infty$ denote a predictable σ-field on $\Omega_\infty = [0, \infty) \times \Omega$ and the restriction of $\mathcal{P}_\infty$ to $\Omega_T = [0, T] \times \Omega$ will be denoted by $\mathcal{P}_T$. Let P_∞ be the product of the Lebesgue measure in $[0, \infty)$ and the measure P. Let P_T be the product of the Lebesgue measure in $[0, T]$ and the measure P.

Let E be a separable Banach space and let $\mathcal{B}(E)$ be the σ-field of its Borel subsets. Given a C_0-semigroup $S(\cdot)$ of linear operators in E, a mapping $f \colon \mathbb{R}_+ \times \Omega \times E \to E$, a stochastic process β on $\mathbb{R}_+$ and $x_0 \in E$, we are interested in finding a stochastic process X on $\mathbb{R}_+$ such that

$$X(t, \omega) = S(t)x_0 + \int_0^t S(t-s) f(s, \omega, X(s, \omega)) ds + \beta(t, \omega), \quad t \geq 0, \tag{1.1}$$

for P-almost all $\omega \in \Omega$. We fix $T > 0$ and make the following assumptions:

(i) $f \colon [0, T] \times \Omega \times E \to E$ is measurable from $(\Omega_T \times E, \mathcal{P}_T \times \mathcal{B}(E))$ into $(E, \mathcal{B}(E))$.

(ii) There exists an increasing mapping $\varphi \colon \mathbb{R}_+ \to \mathbb{R}_+$ such that for some $K > 0$, for all $\omega \in \Omega, 0 \leq s \leq T$ and $x, y \in E$ such that $\|x\| \leq r, \|y\| \leq r$, the following conditions hold:

(a) $\|f(s, \omega, x)\| \leq K + \varphi(r) \cdot \|x\|$,
(b) $\|f(s, \omega, x) - f(s, \omega, y)\| \leq \varphi(r) \cdot \|x - y\|$.

(iii) $\beta \colon [0, T] \times \Omega \to E$ is an adapted, continuous stochastic process.

The main result of this paper is the following.

Theorem 1.1 *If* (i)–(iii) *hold,* $S(\cdot)$ *is a contraction* $\mathcal{C}_0$*-semigroup, and*

$$\int_0^\infty \frac{dx}{x\varphi(x) + 1} = \infty,$$

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then there exists a continuous adapted process X determined on $[0, T]$, satisfying (1.1). *For each $\omega \in \Omega$ the following estimation of X holds:*

$$\sup_{0\le t\le T} \|X_t(\omega)\| \le \Phi^{-1}\big[\Phi\big(\sup_{0\le t\le T} \|\beta_t(\omega)\| + 1\big) + \big(2\sup_{0\le t\le T} \|\beta_t(\omega)\| + 1\big)\, T\big],$$

where

$$\Phi(x) = \int_0^x \frac{dt}{t\varphi(t) + K}.$$

If (i)–(iii) *hold for every $T > 0$, then the solution is determined on $[0, \infty)$.*

2 Proof of Theorem 1.1

We have divided the proof into a sequence of lemmas. For abbreviation we write β_t instead of $\beta(t, \omega)$, $f(s, x)$ instead of $f(s, \omega, x)$, and X_s instead of $X(s, \omega)$.

Lemma 2.1 *If $\{X_t^{(1)}\}, \{X_t^{(2)}\}, t \in [0, T]$ are continuous processes,* (i)–(iii) *hold, and P-a.s.,*

$$X_t^{(i)} = \beta_t + \int_0^t S(t-s) f(s, X_s^{(i)})\, ds, \quad t \in [0, T], i = 1, 2,$$

then $X_t^{(1)}(\omega) = X_t^{(2)}(\omega)$ and for all $t \in [0, T]$, for P-almost all $\omega \in \Omega$.

Proof Fix $\omega \in \Omega$ and denote $\sup\{\|X_t^{(i)}(\omega)\|, t \in [0, T], i = 1, 2\}$ by r. Then for some $K(r)$ we have

$$\|X_t^{(1)}(\omega) - X_t^{(2)}(\omega)\| \le K(r) \int_0^t \|X_s^{(1)}(\omega) - X_s^{(2)}(\omega)\|\, ds, \quad t \in [0, T].$$

Gronwall's lemma leads to the desired conclusion. ■

Lemma 2.2 *If* (i)–(iii) *hold, X is a continuous process such that P-a.s.*

$$X_t = \beta_t + \int_0^t S(t-s) f(s, X_s) ds, \quad t \in [0, T],$$

then X is adapted.

Proof We begin with the additional assumption that f is Lipschitzean. Define

$$X_t^{(0)} = \beta_0 \text{ and } X_t^{(n+1)} = \beta_t + \int_0^t S(t-s) f(s, X_s^{(n)}) ds, \text{ for } \omega \in \Omega, t \in [0, T], n \in \mathbb{N}.$$

$X_t^{(n)}$ are $\mathcal{F}_t$-measurable for $t \in [0, T]$ and $n \in \mathbb{N}$. Moreover

$$\sup_{0\le t\le\tau} \|X_t^{(n+1)} - X_t^{(n)}\| \le C \cdot \tau \cdot \sup_{0\le t\le\tau} \|X_t^{(n)} - X_t^{(n-1)}\| \quad P\text{-a.s., for some } C > 0.$$

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By Lemma 2.1 $\lim_{n\to\infty}\sup_{0\le t\le\tau}\|X_t^{(n)}-X_t\|=0$ P-a.s., where $\tau<\min\{T,\frac{1}{C}\}$. We conclude that X is adapted to $\{\mathcal{F}_t, 0\le t\le\tau\}$. Now consider equation (1.1) on the interval $[\tau,2\tau]$:

$$X_{\tau+t}=\beta_{\tau+t}+\int_0^\tau S(\tau+t-s)f(s,X_s)\,ds+\int_0^t S(t-u)f(\tau+u,X_{\tau+u})\,du,\quad t\in[0,\tau].$$

Denoting $\tilde{X}_t=X_{\tau+t}$ and $\tilde{\beta}_t=\beta_{\tau+t}+\int_0^\tau S(\tau+t-s)f(s,X_s)\,ds$, we have

$$\tilde{X}_t=\tilde{\beta}_t+\int_0^t S(t-u)f(\tau+u,\tilde{X}_u)\,du,$$

where $\tilde{\beta}_t$ is $\tilde{\mathcal{F}}_t=\mathcal{F}_{\tau+t}$-measurable for $0\le t\le\tau$. From what has already been proved, $\tilde{X}$ is adapted to $\{\tilde{\mathcal{F}}_t, 0\le t\le\tau\}$. After a finite number of steps we conclude that X is adapted to $\{\mathcal{F}_t, 0\le t\le T\}$. We now turn to assumption (ii). Let us take a bounded, lipschitzean mapping $h^{(n)}\colon E\to E$ such that $h^{(n)}(x)=x$ for $x\in E$ such that $\|x\|\le n$. Let $X^{(n)}$ denote a solution to (1.1) with f replaced by $f^{(n)}$, where $f^{(n)}(s,\omega,x)=f(s,\omega,h^{(n)}(x))$. Let us regard $\omega\in\Omega$ as fixed and let $r=\sup\{\|X_s(\omega)\|, 0\le s\le T\}$.

For $n\ge r$ we have $\|X_s(\omega)\|\le n$ and consequently

$$X_t(\omega)=\beta_t(\omega)+\int_0^t S(t-s)f(s,\omega,X_s(\omega))\,ds=\beta_t(\omega)+\int_0^t S(t-s)f^{(n)}(s,\omega,X_s(\omega))\,ds.$$

But $X_t^{(n)}(\omega)=\beta_t(\omega)+\int_0^t S(t-s)f^{(n)}(s,\omega,X_s^{(n)}(\omega))\,ds$. By Lemma 2.1, $X_t(\omega)=X_t^{(n)}(\omega)$ for each $n\in\mathbb{N}, n\ge r$. Since $X_t^{(n)}$ are $\mathcal{F}_t$-measurable, for $n\in\mathbb{N}$ and $X_t^{(n)}\to X_t, n\to\infty P$-a.s., it follows that X_t is $\mathcal{F}_t$-measurable. ■

Lemma 2.3 *If β is a continuous, adapted process such that*

$$P\Big\{\sup_{0\le t\le T}\|\beta(t)\|\le L\Big\}=1$$

for some $L>0$, and moreover (ii) *and* (iii) *hold, then there exists an adapted and continuous stochastic process X, determined on $[0,\Delta]$ for some $0<\Delta\le T$, satisfying* (1.1).

Proof Here we apply the idea of the proof of Theorem 1.4 in [1]. Let $K>0$ be such that $\|f(t,\omega,0)\|\le K$ for $t\in[0,T],\omega\in\Omega$, moreover let $\|S(t)\|\le M$ for $t\in[0,T]$. Let us regard $\omega\in\Omega$ as fixed. Consider the transformation

$$(\mathcal{T}x)(t)=\beta_t+\int_0^t S(t-s)f(s,x_s)\,ds,\quad \text{for } x\in\mathcal{C}([0,T],E).$$

Define

$$\varrho=L+1,\quad \Delta=\frac{1}{M[(L+1)\varphi(L+1)+K]}.$$

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The mapping $\mathcal{T}$ maps the closed ball $B(0,\varrho)$ of radius ϱ centered at 0 of $\mathcal{C}([0,\Delta],E)$ into itself, because

$$\begin{aligned}\|\mathcal{T}x\| &= \sup_{0\le t\le\Delta}\|\beta(t)+\int_0^t S(t-s)f(s,x_s)\,ds\|\\ &\le L+\Delta M\sup_{0\le s\le\Delta}\|f(s,x_s)\|\\ &\le L+M\Delta\sup_{0\le s\le\Delta}(\|f(s,X_s)-f(s,0)\|+\|f(s,0)\|)\\ &\le L+M\Delta(\varrho\varphi(\varrho)+K)=\varrho.\end{aligned}$$

Moreover,

$$\begin{aligned}\|\mathcal{T}x-\mathcal{T}y\| &= \sup_{0\le t\le\Delta}\|\int_0^t S(t-s)(f(s,x_s)-f(s,y_s))\,ds\|\le M\Delta\varphi(\varrho)\cdot\|x-y\|\\ &\le\frac{1}{L+1}\|x-y\|,\quad\text{for } x,y\in B(0,\varrho).\end{aligned}$$

Thus $\mathcal{T}$ possesses a unique fixed point x in the ball, being the desired solution to (1.1) on the interval $[0,\Delta]$. ■

Proof of Theorem 1.1 We begin analogously to the proof of Lemma 2.3. For fixed ω we have a mapping $x\in B(0,\varrho)\subset\mathcal{C}([0,\Delta],E)$, satisfying (1.1) on the interval $[0,\Delta]$. Let

$$L=\sup_{0\le t\le T}\|\beta_t(\omega)\|,\quad \varrho=\varrho_1=L+1,\quad \Delta=\Delta_1=\frac{1}{(L+1)\varphi(L+1)+K}.$$

Proceeding by induction, we assume that there exists $x\in\mathcal{C}([0,\Delta_i],E)$ satisfying (1.1) on $[0,\Delta_i]$. Moreover, we assume that x considered as a function on $[\Delta_{i-1},\Delta_i]$ is a unique fixed point of the transformation

$$(\mathcal{T}u)(t)=\beta_t+S(t-\Delta_{i-1})(X_{\Delta_{i-1}}-\beta_{\Delta_{i-1}})+\int_{\Delta_{i-1}}^t S(t-s)f(s,u_s)\,ds,$$

$\Delta_{i-1}\le t\le\Delta_i$, in the ball $B(0,\varrho_i)\subset\mathcal{C}([\Delta_{i-1},\Delta_i],E)$.

We proceed to show that x can be extended to the interval $[0,\Delta_{i+1}]$ with $\Delta_{i+1}>\Delta_i$, by defining x on $[\Delta_i,\Delta_{i+1}]$, as a solution to the equation

$$X_t=\beta_t+S(t-\Delta_i)(X_{\Delta_i}-\beta_{\Delta_i})+\int_{\Delta_i}^t S(t-s)f(s,X_s)\,ds,$$

$\Delta_i\le t\le\Delta_{i+1}$. For this purpose, we set

$$\varrho_{i+1}=2L+1+\varrho_i,\quad \delta_{i+1}=\frac{1}{\varrho_{i+1}\varphi(\varrho_{i+1})+K},\quad \Delta_{i+1}=\Delta_i+\delta_{i+1},\quad \Delta_1=\delta_1.$$

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We consider the mapping

$$(\mathcal{T}u)(t) = \beta_t + S(t-\Delta_i)(X_{\Delta_i} - \beta_{\Delta_i}) + \int_{\Delta_i}^{t} S(t-s) f(s, u_s)\, ds, \quad \Delta_i \le t \le \Delta_{i+1},$$

acting in the space $\mathcal{C}([\Delta_i, \Delta_{i+1}], E)$. If $u \in B(0, \varrho_{i+1}) \subset \mathcal{C}([\Delta_i, \Delta_{i+1}], E)$, then

$$\|\mathcal{T}u\| \le L + \varrho_i + L + \delta_{i+1} \sup_{\Delta_i \le s \le \Delta_{i+1}} \|f(s, u_s)\| \le 2L + \varrho_i + 1 = \varrho_{i+1}.$$

Hence $\mathcal{T}$ maps the ball $B(0, \varrho_{i+1})$ of $\mathcal{C}([\Delta_i, \Delta_{i+1}], E)$ into itself. Moreover, in the ball we have the following estimation:

$$\begin{aligned}\|\mathcal{T}u - \mathcal{T}v\| &\le \delta_{i+1}\varphi(\varrho_{i+1}) \cdot \|u - v\| = \frac{\varphi(\varrho_{i+1})}{\varrho_{i+1}\varphi(\varrho_{i+1}) + K}\|u - v\| \\ &\le \frac{1}{\varrho_{i+1}}\|u - v\| \le \frac{1}{L+1}\|u - v\|.\end{aligned}$$

Hence $\mathcal{T}$ has a unique fixed point $\tilde{x} \in B(0, \varrho_{i+1}) \subset \mathcal{C}([\Delta_i, \Delta_{i+1}], E)$:

$$\tilde{x}_t = \beta_t + S(t-\Delta_i)(X_{\Delta_i} - \beta_{\Delta_i}) + \int_{\Delta_i}^{t} S(t-s) f(s, \tilde{x}_s)\, ds, \quad \Delta_i \le t \le \Delta_{i+1}.$$

The function $\tilde{x}$ is a continuous extension of x, being a solution to (1.1) on $[0, \Delta_i]$, to a solution to (1.1) on $[0, \Delta_{i+1}]$, because

$$\begin{aligned}\tilde{x}_t &= \beta_t + S(t-\Delta_i)\Big(\beta_{\Delta_i} + \int_0^{\Delta_i} S(\Delta_i - s) f(s, x_s)\, ds - \beta_{\Delta_i}\Big) + \int_{\Delta_i}^{t} S(t-s) f(s, \tilde{x}_s)\, ds \\ &= \beta_t + S(t-\Delta_i)\int_0^{\Delta_i} S(\Delta_i - s) f(s, x_s)\, ds + \int_{\Delta_i}^{t} S(t-s) f(s, \tilde{x}_s)\, ds \\ &= \beta_t + \int_0^{\Delta_i} S(t-s) f(s, x_s)\, ds + \int_{\Delta_i}^{t} S(t-s) f(s, \tilde{x}_s)\, ds,\end{aligned}$$

$\Delta_i \le t \le \Delta_{i+1}$ and $\tilde{x}_{\Delta_i} = x_{\Delta_i}$. Thus we obtain a solution to (1.1) on the interval $[0, \Delta)$, where $\Delta = \sum_{i=1}^{\infty} \delta_i$. Theorem 1.1 will be proved once we show $\sum_{i=1}^{\infty} \delta_i = \infty$. It is easy to see that $\varrho_i = L + 1 + (i-1)(2L+1)$, $i \in \mathbb{N}$. Since φ is increasing and $\varrho_i < i(2L+1)$, so

$$\begin{aligned}\sum_{i=1}^{\infty} \delta_i &= \sum_{i=1}^{\infty} \frac{1}{\varrho_i \varphi(\varrho_i) + K} > \sum_{i=1}^{\infty} \frac{1}{i(2L+1)\varphi[i(2L+1)] + K} \\ &= \sum_{i=1}^{\infty} \int_i^{i+1} \frac{dx}{i(2L+1)\varphi[i(2L+1)] + K} \\ &\ge \sum_{i=1}^{\infty} \int_i^{i+1} \frac{dx}{x(2L+1)\varphi[x(2L+1)] + K} \\ &= \int_1^{\infty} \frac{dx}{x(2L+1)\varphi[x(2L+1)] + K} = \int_{2L+1}^{\infty} \frac{dt}{[(t\varphi(t) + K](2L+1)} = \infty.\end{aligned}$$

Denoting

$$\Phi(x) = \int_0^x \frac{dt}{t\varphi(t) + K},$$

it is easy to see that

$$\sum_{i=1}^{N} \delta_i \geq \int_1^{N+1} \frac{dx}{[L+1+(x-1)(2L+1)]\varphi[L+1+(x-1)(2L+1)]+K}$$
$$= \frac{1}{2L+1}\int_{\varrho_1}^{\varrho_{N+1}} \frac{dt}{[(t\varphi(t)+K]} = \frac{1}{2L+1}(\Phi(\varrho_{N+1}) - \Phi(\varrho_1)).$$

We have

$$\frac{1}{2L+1}(\Phi(\varrho_{N+1}) - \Phi(\varrho_1)) = T \Longleftrightarrow \varrho_{N+1} = \Phi^{-1}(\Phi(\varrho_1) + (2L+1)T).$$

Since $L = \sup_{0\leq t\leq T} \|\beta_t(\omega)|$, we obtain the following estimation of the solution:

$$\sup_{0\leq t\leq T} \|X_t(\omega)\| \leq \Phi^{-1}\big[\Phi\big(\sup_{0\leq t\leq T}\|\beta_t(\omega)\|+1\big) + \big(2\sup_{0\leq t\leq T}\|\beta_t(\omega)\|+1\big)T\big],$$

$\omega \in \Omega$. ■

3 An Application

Let $E = H$ and U be separable Hilbert spaces, Q be a bounded, self-adjoint, strictly positive operator on U such that $\operatorname{Tr} Q \leq \infty$. Denote by U_0 the subspace $Q^{1/2}(U)$ of U equipped with the inner product $\langle u, v\rangle = \langle Q^{-1/2}u, Q^{-1/2}v\rangle$.

Let W be a cylindrical Q-Wiener process with respect to $\mathbb{F}$ on an arbitrary Hilbert space U_1 such that U is embedded continuously into U_1 and the embedding of U_0 into U_1 is Hilbert–Schmidt. Let $L_2^0 = L_2(U_0, H)$ be the Hilbert space of all Hilbert–Schmidt operators acting from U_0 into H, with the norm $\|\Phi\|_{L_2^0} = \operatorname{Tr}[\Phi Q \Phi]$. Let $N_W^2(0, T, L_2^0)$ denote a Hilbert space of all L_2^0 predictable processes Φ such that $\mathbb{E}(\int_0^T \|\Phi(s)\|_{L_2^0}^{2r}\, ds) < \infty$. If $S(\cdot)$ is a contraction semigroup and $\Phi \in N_W^2(0, T, L_2^0)$, then the process

$$\beta_t = \int_0^t S(t-s)\Phi(s)\, dW_s, \quad t \in [0, T]$$

is adapted and has a continuous modification [2, Theorem 6.10]. Hence by Theorem 1.1 we obtain the following.

Corollary 3.1 *If* (i)–(iii) *hold, $S(\cdot)$ is a contraction C_0-semigroup, and*

$$\int_0^\infty \frac{dx}{x\varphi(x)+1} = \infty,$$

then there exists a continuous and $\mathbb{F}$-adapted process $\{X_t, 0 \leq t \leq T\}$ such that P-a.s.

$$X_t = S(t)x_0 + \int_0^t S(t-s) f(s, X_s)\, ds + \int_0^t (S(t-s)\Phi(s)\, dW_s, \quad 0 \leq t \leq T. \tag{3.1}$$

The process is unique, up to indistinguishability.

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To obtain global existence of a unique solution to (3.1) in $\mathcal{C}([0,\infty], H)$ it is sufficient to assume that conditions (i)–(iii) hold for every $T > 0$.

Example

Let D be an open subset of $\mathbb{R}^d$ and let $H = L^2(D)$. We will show that $F\colon H \to H$, given by the formula

$$(F(x))(\xi) = x(\xi) \cdot \ln(1 + \|x\|^2), \quad x \in H, \xi \in D,$$

satisfies (ii). Let e_j, $j \in \mathbb{N}$ be a complete orthonormal system in H and let $x_j = \langle x, e_j\rangle$, $x \in H$, $j \in \mathbb{N}$. Then

$$\|F(x) - F(y)\|^2 = \sum_{i=1}^{\infty} (x_j \ln(1 + \|x\|) - y_j \ln(1 + \|y\|))^2, x, y \in H. \tag{3.2}$$

Let us fix $n \in \mathbb{N}$ and consider the mapping $\Phi(x) = x \cdot \ln(1 + \|x\|^2)$, for $x \in \mathbb{R}^n$. It is easy to see that

$$\Phi'(x) = \ln(1 + \|x\|^2) \cdot I + \frac{2}{1 + \|x\|^2} \cdot A, \quad x \in \mathbb{R}^n,$$

where $I = [\delta_{ij}]_{n\times n}$ and $A = [a_{ij}]_{n\times n}$ with $a_{ij} = x_i x_j$.

Since $\|Au\| = \|x\| \times |\langle x, u\rangle|$ for $u \in \mathbb{R}^n$, it follows that $\|A\| \le \|x\|^2$ and

$$\Phi'(x) \le \ln(1 + \|x\|^2) + 2, \quad \text{for } x \in \mathbb{R}^n.$$

Hence for $x, y \in \mathbb{R}^n$ such that $\|x\| \le r$ and $\|y\| \le r$ we have

$$\|\Phi(x) - \Phi(y)\| = \Big[\sum_{j=1}^{n} \Big(x_j \ln\Big(1 + \sum_{j=1}^{n} x_j^2\Big) - y_j \ln\Big(1 + \sum_{j=1}^{n} y_j^2\Big)\Big)^2\Big]^{1/2}$$
$$\le \Big(\sum_{j=1}^{n} (x_j - y_j)^2\Big)^{1/2} \cdot (2 + \ln(1 + r^2)).$$

Consequently, for $x, y \in H$ such that $\|x\| \le r$ and $\|y\| \le r$ and for each $n \in \mathbb{N}$ we have

$$\Big[\sum_{j=1}^{n} \Big(x_j \ln\Big(1 + \sum_{j=1}^{n} x_j^2\Big) - y_j \ln\Big(1 + \sum_{j=1}^{n} y_j^2\Big)\Big)^2\Big]^{1/2}$$
$$\le \Big(\sum_{j=1}^{\infty} (x_j - y_j)^2\Big)^{1/2} \cdot (2 + \ln(1 + r^2)).$$

and (ii) is shown, letting n tend to infinity. Moreover, $\int_1^{\infty} \frac{dr}{r(2+\ln(1+r^2))} = \infty$. Hence there exists a global, continuous, mild solution to the equation

$$dX = (AX + F(X)))dt + \Phi(t)dW_t$$

for A and Φ satisfying assumptions of Corollary 3.1, and F given by (3.2).

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