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A QUANTITATIVE EVALUATION OF LAGRANGIAN COHERENT STRUCTURE DETECTION METHODS BASED ON COMPUTATIONAL AND EXPERIMENTAL LIMITATIONS

ALEKSANDAR BADZA

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Lagrangian coherent structures (LCSs) are used in fluid mechanics and the analysis of dynamic systems to visualise the most influential flow structures present within a velocity system over a finite period of time. Over the last two decades, a wide variety of methods have been conceptualised for the numerical detection of various forms of these structures within different flows. These include continuous curves of maximal particle repulsion which act as flow barriers, two-dimensional objects such as jets or eddies formed from more robust flow behaviour, or larger partitions which remain separated from the rest of the domain over an entire flow interval. While some studies which focus on comparing the basic functionality of groups of these methods have been undertaken, the impact of certain computational factors such as the uncertainty of velocity data or the available resolution of such data on the resultant structures generated from these methods has seldom been investigated. We address both of these issues by performing a systematic analysis of eight of these Lagrangian coherent structure detection methods using a variety of velocity systems including analytically defined flows (such as the Double Gyre, a nonautonomous Stuart vortex system and the Bickley jet), computational fluid dynamics velocity data (corresponding to flows which each contain two layers of Kelvin–Helmholtz instability) and an oceanographic velocity dataset representing the Gulf Stream.

The methods we consider here are the finite time Lyapunov exponent (a measure of the exponential stretching rate of flow trajectories), variational Lagrangian coherent structures (geodesic solutions of variational problems related to flow stretching),

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Lagrangian averaged vorticity deviation (an objective measure of the vorticity of a flow trajectory against that of the entire domain), stochastic sensitivity (the expected uncertainty of a Lagrangian flow trajectory), the transfer operator (a probabilistic method which seeks density distributions that remain coherent), the dynamic Laplace operator (an extension of the transfer operator method which explicitly includes diffusivity), fuzzy c-means clustering (grouping together collections of flow trajectories based on their consistent proximity) and coherent structure colouring (identifying coherent flow objects from how similarly groups of flow trajectories evolve as a flow advances). We compare the types of Lagrangian coherent structure each method is able to produce, and test how these methods react to the addition of stochastic noise to the velocity data which represents a flow. From our results, methods which detect two-dimensional coherent flow structures rather than the boundaries which separate them, such as coherent structure colouring, Lagrangian averaged vorticity deviation, stochastic sensitivity, the transfer operator and dynamic Laplace operator, are less sensitive to velocity uncertainty and give a more thorough picture of the most influential flow behaviour observable. We also perform a detailed analysis on the impact of spatial resolution in comparison to the size of coherent structures for each of the methods, both qualitatively by visually comparing the coherent structures produced and quantitatively using the absolute errors of various LCS quantities against a 'reference case' produced from the best velocity data resolution available.

Some of this research has been published in [1].

Reference

[1] A. Badza, T. W. Mattner and S. Balasuriya, 'How sensitive are Lagrangian coherent structures to uncertainties in data?', *Phys. D* (2022), Article no. 133580.

ALEKSANDAR BADZA, School of Mathematical Sciences,
University of Adelaide, Adelaide, South Australia 5005, Australia
e-mail: aleksbadza@gmail.com