

# Internal waves driven by stellar irradiation in a non-synchronized hot Jupiter

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**Abstract.** We investigate the dynamical response of a non-synchronized hot Jupiter to stellar irradiation. In our current model, the stellar radiation acts like a diurnal thermal forcing from the top of a radiative layer of a hot Jupiter. If the thermal forcing period is longer than the sound speed crossing time of the planet's surface, the forcing can excite internal waves propagating into the planet's interior. When the planet spins faster than its orbital motion, these waves carry negative angular momentum and are damped by radiative loss as they propagate downwards from the upper layer of the radiative zone. As a result, the upper layer gains the angular momentum from the lower layer of the radiative zone. Simple estimates of angular momentum flux are made for all transiting planets.

**Keywords.** diffusion, hydrodynamics, waves, planets and satellites: general

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## 1. Introduction

Infrared observations of transiting planetary systems with the Spitzer Space Telescope have revealed that the peak infrared brightness occurs before the secondary eclipse (Knutson *et al.* 2007). One interpretation of this phenomenon is that the atmosphere of a hot Jupiter is not tidally locked but exhibits a super-rotation state which advects the stellar heating downstream from the substellar point. Since a super-rotating atmosphere experiences day-night changes, internal waves can be excited by the periodic thermal forcing in the super-rotating flow. Furthermore, internal waves can carry angular momentum and therefore serve as a reasonable candidate for maintaining such a super-rotating flow. Here we employ linear analyses and investigate the possibility of wave excitation in a non-synchronized surface layer of a hot Jupiter driven by stellar irradiation.

## 2. Thermal tides in the non-rotating plane-parallel atmosphere

We initially consider a non-rotating plane-parallel atmosphere with uniform gravity  $\mathbf{g} = -g\mathbf{e}_z$ . The basic equations are

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \mathbf{g}, \quad (2.1)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \quad (2.2)$$

$$\frac{\partial p}{\partial t} + \mathbf{u} \cdot \nabla p + \gamma p \nabla \cdot \mathbf{u} = -(\gamma - 1) \nabla \cdot \mathbf{F}, \quad (2.3)$$

$$\mathbf{F} = -\frac{16\sigma T^3}{3\kappa\rho}\nabla T, \quad (2.4)$$

$$p = \frac{R\rho T}{\mu}. \quad (2.5)$$

For simplicity we also assume that  $\kappa$ ,  $\gamma$  and  $\mu$  are constant. We use the radiative diffusion approximation (2.4) throughout the atmosphere and apply the ‘Marshak’ boundary condition (cf. Pomraning 1973)

$$\sigma T^4 = \frac{1}{2}F_z + F_i \quad (2.6)$$

at  $z = +\infty$ , where  $F_i$  is the irradiating flux. For the equilibrium state we have

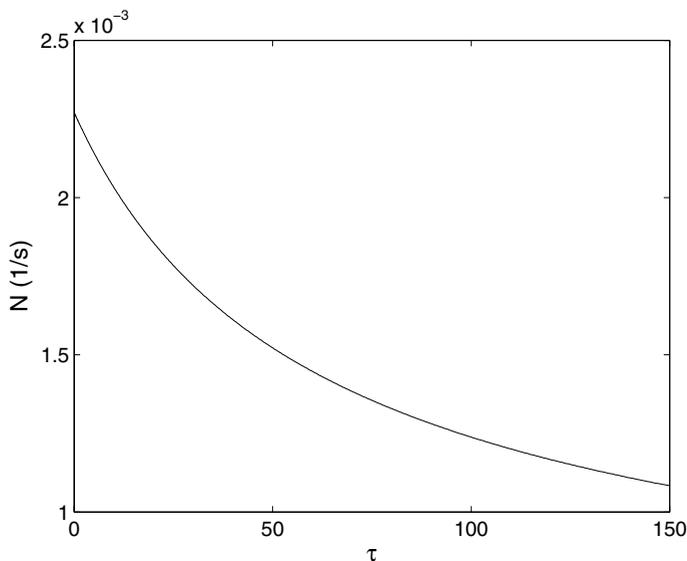
$$\frac{dp}{d\tau} = \frac{g}{\kappa}, \quad (2.7)$$

$$\frac{d}{d\tau}(\sigma T^4) = \frac{3}{4}F_z. \quad (2.8)$$

where  $F_z$  is the intrinsic radiative flux density of the planet and  $\tau$  is the optical depth.

The thermal tide is driven by a variation of the irradiating flux. Therefore we linearize equations(2.1)-(2.5) with Eulerian perturbations of the form  $\text{Re}[\mathbf{u}'(z) \exp(ik_x x - i\omega t)]$ , etc., where  $k_x$  is a real horizontal wavenumber and  $\omega$  a real frequency determined by the tidal forcing (thermal tides). The perturbed boundary condition at  $\tau = 0$  is  $4\sigma T^3 T' = \frac{1}{2}F'_z + F'_i$ , where  $F'_i$  is the thermal forcing term. The linearized equations admit a solution in the form of a internal wave in the limit of large  $\tau$ . In the case of diurnal thermal forcing, the magnitude of the horizontal wavelength  $2\pi/|k_x|$  is just the circumference of the planet’s surface  $2\pi R_p$ , where  $R_p$  is the planet’s radius.

We employ the following parameters for HD 209458b to be the input parameters for solving the linearized equations:  $R_p = 1.32R_J$ ,  $M_p = 0.69M_J$ ,  $a = 0.0474$  AU, and  $T_* = 6099\text{K}$ . We adopt  $F_z = 7 \times 10^6$  erg/cm<sup>2</sup> s (Bodenheimer *et al.* 2003),  $\kappa = 0.01$  gm/cm<sup>2</sup> (molecular opacity with no grains),  $\mu = 2$  gm/mol., and  $\gamma = 1.4$  for the gas in the radiative surface layer of the planet. The Brunt–Väisälä frequency  $N$  ranges from



**Figure 1.** The vertical profile of the Brunt–Väisälä frequency  $N$  for HD 209458b.

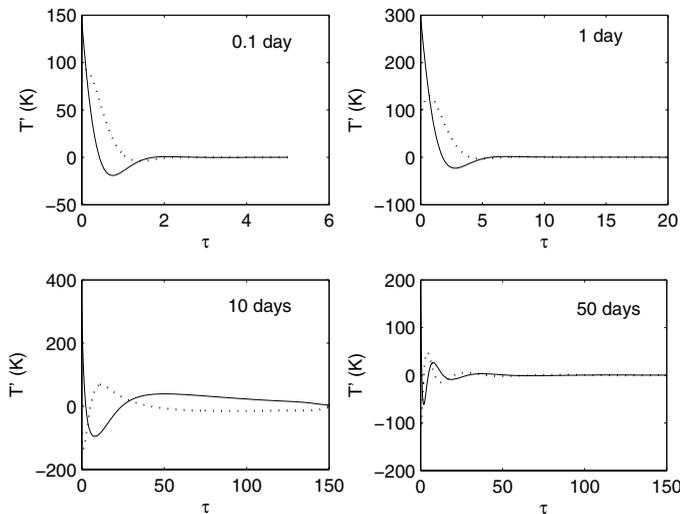
0.0022/s at  $\tau \approx 0$  to 0.001/s at  $\tau \approx 150$  (see Fig 1) and is far larger than the thermal forcing frequency  $\omega$  which is some fraction of the orbital frequency  $n = 2\pi/(\text{a few days})$ . The solutions are insensitive to the value of  $F_z$  because  $F_z$  is much smaller than the stellar irradiation  $F_i$ . We focus on the case that the planet is rotating faster than its orbital motion. Therefore the thermal forcing and the thermal tides propagate in a retrograde sense in the frame of the rotating planet.

Figure 2 shows the vertical structure of  $T'$  in units of K for different thermal forcing periods  $2\pi/\omega$ . The real and imaginary parts of  $T'$  are denoted by a solid and a dotted curve respectively. When the forcing periods are short (e.g. 0.1 and 1 day as shown in the top two panels),  $T'$  decays with the depth and the solutions behave like those to the thermal diffusion problem with the heat diffusing from the top of the atmosphere to a depth  $\propto 1/\sqrt{\omega}$ . Comparing the 0.1-day to the 1-day case, Figure 2 shows that  $T'$  can penetrate deeper in the 1-day case as a result of a longer forcing period and therefore a longer diffusion length.

On the other hand, when the forcing periods are long (e.g. 10 and 50 days as shown in the bottom panels of Figure 2), the vertical profiles of  $T'$  exhibit wavy-like solutions, meaning that waves are excited from the top of the atmosphere and propagate in. These waves are known as internal waves (i.e. g-mode).

### 3. Thermal tides in a rotating planetary atmosphere

We now consider the linearized dynamics of the form  $\text{Re}[u'(z)H_{\nu,m=1,n}(\theta)e^{im\phi-i\omega t}]$  in a thin uniformly rotating atmosphere ( $\Omega = \text{constant}$  & no winds), where  $H_{\nu,m=1,n}$  is the Hough function with  $m = 1$  (diurnal heating). Then the linearized equations remain the same except that  $k$  is replaced by  $\lambda/R_p^2$ , where  $\lambda$  is the Hough eigenvalue (Ogilvie & Lin 2004).  $\lambda$  is a function of  $2\Omega/\omega$ . We find that damped wave solutions are possible for even smaller  $\omega$  ( $2\pi/\omega > 50$  days) in the rotating case than in the non-rotating case ( $2\pi/\omega > 10$  days).



**Figure 2.** Temperature perturbation  $T'$  as a function of  $\tau$  for the cases of 4 different forcing periods:  $2\pi/\omega = 0.1, 1, 10,$  and  $100$  days. The real and imaginary parts of  $T'$  are denoted by a solid and a dotted curve respectively.

Assuming that  $2\pi/\omega = 100$  days, we find that  $\lambda \approx 0.11$  for the most dominant Hough heating. The linearized equations can then be solved. Fig 3 shows the angular momentum flux carried by inward propagating Hough waves through the equator ( $\theta = 90^\circ$ ) as a function of  $\tau$  for all transiting planets (Burrows *et al.* 2007). The positive value of angular momentum flux means that the angular momentum is transported outwards by Hough waves. The plot illustrates that for a given forcing period, the hot Jupiters which receive more stellar irradiation have larger angular momentum flux.

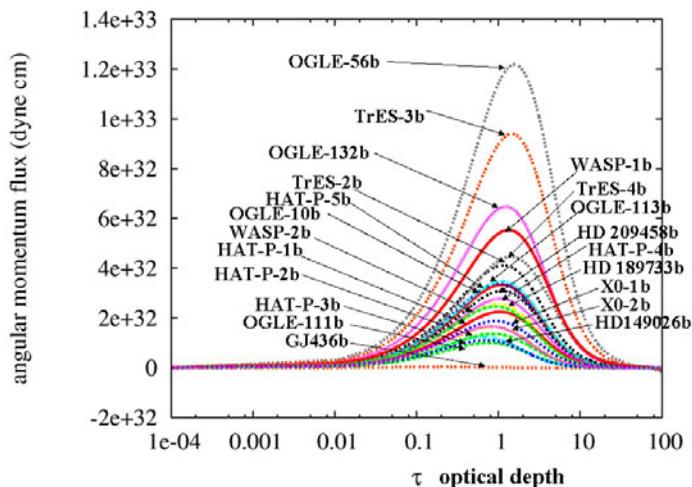
Although differential rotation and gravitational tides are not included in our analysis, we consider the following highly hypothetical scenario for the planetary spin based on our simple model. We assume that while the interior of the planet evolves to a synchronous state by both gravitational and thermal tides, the outer layer of the planet can maintain a super-synchronous state by the dissipation of Hough waves (thermal tides). Then as illustrated in Fig 3, the planet gas located below the photosphere ( $\tau \gtrsim 1$ ) keeps losing angular momentum to the planetary atmosphere and is therefore spinning down. We further assume that the spun-down gas below the photosphere due to the Hough-wave torque  $\tau_{Hough}$  can spin down the whole planetary interior against gravitational tides; i.e.,

$$\tau_{Hough} = \frac{9GM_*^2 R_p^5}{2a^6 Q'_p} \left( 1 - \frac{\Omega_{p,interior}}{n} \right), \quad (3.1)$$

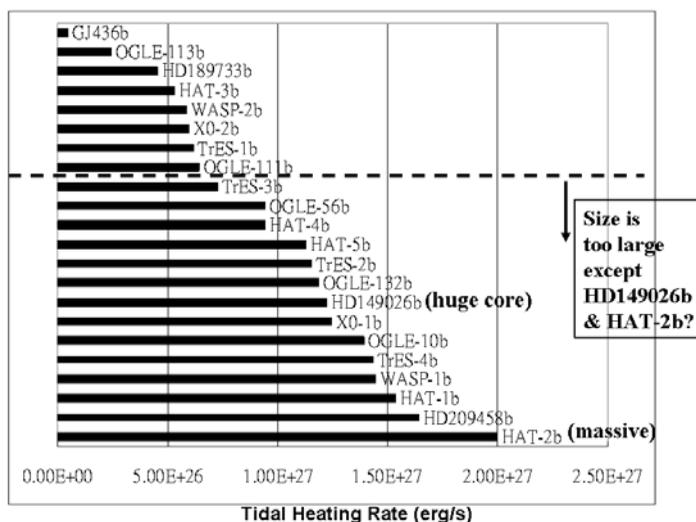
where  $M_*$  is the stellar mass,  $Q'_p$  is the tidal quality factor of the planet, and  $\Omega_{p,interior}$  is the spin rate of the planetary interior. Then the gravitational tidal heating rate  $\dot{E}_{grav}$  due to the sub-synchronous rotation of the planetary interior is generated (e.g. Eggleton *et al.* 1998):

$$\dot{E}_{grav} = \frac{9n^3 a^2 M_*}{2Q'_p} \left( \frac{R_p}{a} \right)^5 \left( 1 - \frac{\Omega_{p,interior}}{n} \right)^2. \quad (3.2)$$

The heating rates  $\dot{E}_{grav}$  in the case of  $2\pi/\omega = 100$  days &  $R_p = 1.67R_J$  for all transiting planets are illustrated in Fig 4: the resulting tidal heating rates  $\sim 10^{27}$  erg/s



**Figure 3.** The vertical profile of angular momentum flux at the equator of the transiting planets when  $2\pi/\omega = 100$  days.



**Figure 4.** The tidal heating rates of the transiting planets due to thermal tides when  $2\pi/\omega = 100$  days,  $R_p = 1.67R_J$ , and the tidal quality factor  $Q'_p = 10^6$ .

are high enough to inflate a hot Jupiter (Bodenheimer *et al.* 2003). Most of the hot Jupiters which gain more tidal heating (i.e. the ones below the dashed line in Fig 4) are larger in size except for HD 149026b and HAT-P-2b. However, HD 149026b may have an abnormally large core and HAT-P-2b is probably too massive to be inflated. In general, for the hot Jupiters which are less close-in, their interior is less synchronized (i.e.  $1 - \Omega_{p,interior}/n$  is larger) and therefore generates more internal tidal heating (see Eq(3.2)). It is because thermal tides dominates more over gravitational tides for less close-in hot Jupiters. We expect that the atmosphere will be spun up to a level that it spins too fast to excite waves as shown in the top panels of Fig 2. Then the atmosphere will spin down to a level that the thermal tides excite waves again as shown in the bottom panels of Fig 2 and another heating cycle begins.

#### 4. Summary

We show that when the surface layer of a hot Jupiter rotates a little faster than the synchronous state (i.e. small  $\omega$ ), the internal waves are excited on the top of the atmosphere and propagate downwards. When the planet spins faster than its orbital motion, the direction of the thermal forcing due to stellar irradiation exhibits a retrograde motion. The radiative damping on the retrograde waves causes the upward transport of angular momentum. We estimate the angular momentum flux for all transiting planets and find that for a given thermal forcing period, the atmosphere which receives more stellar irradiation transports more angular momentum flux upwards. If we assume that the upward transport of angular momentum can torque the whole planet against gravitational tides and therefore generate tidal heating, we find that most of the hot Jupiters which generate higher tidal heating rates in our scenario are observationally larger in size. This work is supported by the NSC grant in Taiwan through NSC 95-2112-M-001-073MY2.

**References**

- Bodenheimer, P., Laughlin, G., & Lin, D. N. C. 2003, *ApJ*, 592, 555
- Burrows, A., Hubeny, I., Budaj, J., Hubbard, W. B. 2007, *ApJ*, 661, 502
- Chapman, S. & Lindzen, R. 1970, *Atmospheric Tides: Thermal and Gravitational*, Gordon and Breach
- Eggleton, P. P., Kiseleva, L. G., & Hut, P. 1998, *ApJ*, 499, 853
- Knutson, H. A. *et al.*, 2007, *Nature*, 447, 183
- Ogilvie, G. I. & Lin, D. N. C. 2004, *ApJ*, 610, 477
- Pomraning, G. C. 1973, *The equations of radiation hydrodynamics*, Pergamon Press