Multiple Periodic Variable Stars
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ROTATION MIXING AND VARIABILITY IN A STARS

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The purpose of this paper is to contribute to answer the question : why among A stars are there variable and "constant" stars ? To do this we will first review the observational properties of these stars, then discuss the theoretical models based on the interplay between microscopic diffusion and rotation, and finally discuss the "unsolved" questions !

A Review of the properties of the A stars in the instability strip (i.e. on the main sequence from A2 to F0, and somewhat later types in giants).

1 Dwarf stars

We will first restrict the problem to the main sequence stars because there the situation is clearer and more quantitative work has been done on it i.e. there exists photometric calibration which permits to compute M_{bol}, $\log T_{eff}$, ...

1.1 On the main sequence among A stars the spectroscopists have recognized long ago the "normal stars" and the "metallic line stars" (1940). I do not want to discuss in detail the definition of the Am phenomenon.

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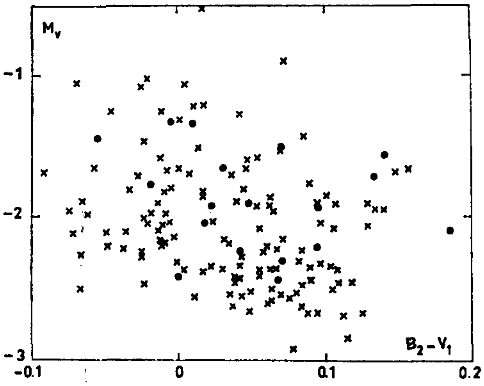

Figure 1.- Observational Hertzsprung-Russell diagram of Am and δ Scuti stars based on the photometric system of the Geneva Observatory. ✱ Am . δ Scuti from Baglin (1972).

Following Conti (1970) "the Am phenomenon is present in stars that have an apparent surface underabundance of Ca (and/or Sc) and/or apparent overabundance of the heavier (iron group) elements". The nature of the anomalies does not vary significantly with the temperature (Smith, 1973).

From extensive search of variability in the instability strip Breger (1970) has suggested that typical Am stars are not variable, and vice et versa. This result has been confirmed up to now.

Several doubtful cases have appeared, they have generally been solved in agreement with the preceding statement. Up to now, over more than 30 Am stars studied only three cases remain doubtful to my knowledge.

HR 5491 a typical Am star has been observed variable by Bessel and Eggen (1972). However the light curve seems to be very irregular. It has been reobserved since by several authors (Breger et al, 1972; Stobie and Eggen, 1973) and found constant.

32 Vir which has been classified sometimes as an Am (Cameron, 1967) but which also is considered as magnetic by Babcock (1958) has been shown to be variable by Bartolini et al (1972). However it is

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a very peculiar object; its spectrum is highly variable and the K line does not show the velocity variation of its orbital motion as the metallic lines do. It is probably a quite close binary composed of a normal A star and of an Am star (Breger, private communication).

ν^2 Dra has been proposed as a variable star by Mendoza and Gonzales (1974). However this variability needs to be checked.

1.2 The variable stars have a "normal spectrum" and normal solar abundances. With the present observational threshold (one thousandth of a magnitude) 34% of the "normal" stars have been detected as variable. I recall here Breger's result that the distribution of the amplitudes is consistent with the fact that all the normal stars are variable most of them having an undetectable amplitude.

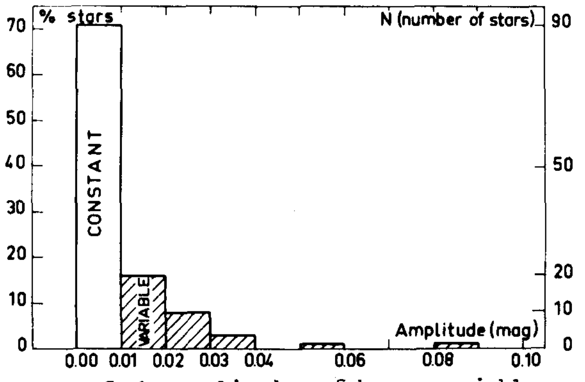

Figure 2.- Histogram of the amplitudes of known variables.
Undashed area : stars for which no variability can be established which lie in the instability strip.
From Baglin et al, 1973.

1.3 The main difference between "normal" and Am stars consists in the distribution of the rotationnal velocities. The first result of Slettebak (1955) has been confirmed by a more precise and extended work by Abt and Moyd (1973) :

$$\langle V \sin i \rangle A_m = 33 \text{ km/s}$$

$$\langle V \sin i \rangle A_{nor} = 141 \text{ km/s}$$

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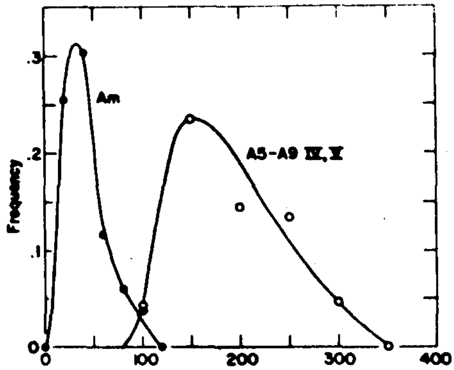

Figure 3 .- Distribution of the apparent rotational velocities in Am and normal A stars from Abt and Moyd (1973).

Smith (1971) has noted that statistically the degree of metallicism decreases as V_{Rot} increases.

As this question is of great importance for our purpose, let us comment on several difficulties.

i- There are normal stars which rotate slowly. Out of 34 variables from the catalogue of δ Scuti stars by Baglin et al (1973) , 3 have V sin i smaller than 50 km s^{-1} which is compatible with a random distribution of the inclination angles.

However HR 8584 which is a spectroscopic binary of period 2.3 days can hardly have a large inclination, its rotational velocity (32 km s^{-1}) is of the order of the synchronism velocity (25 km s^{-1}) (14 Aur, which is slightly evolved presents the same situation).

In the Pleiades the distribution of the rotational velocities is analogous to that of the field stars. However no Am has been detected. So the intrinsically slowly rotating A stars of the cluster are probably not Am.

ii- Some Am stars seem to rotate rapidly.

In the list of Abt and Moyd the fastest is HR 4646 with 95 km s^{-1}. Smith (1972) has analysed HR 7774 and showed that the most probable explanation of this system is : two rapidly rotating Am stars $(V \approx 120$ km $s^{-1})$ identical in a binary of period 61 days. However this very curious system needs more attention.

Note : As all the "normal" stars are variable, there can be some difficulty in defining the rotational velocity as measured from the "half width" (see Valtier, this conference).

1.4 Another parameter which is usually called upon to distinguish Am and "normal" stars is that Am stars are members of binaries.

The periods are extremely variable from 1 to more than 100 days, i.e. the binary systems are sometimes very close and sometimes so detached that no interaction exists between them.

However as shown by Zahn (1972) the binary motion through the dynamical tide produces a braking of the rotation of stars with a convective zone . The final state is that axes of intrinsic and binary motions are parallel and rotations are synchronized.

The time scale of this process for an A star is smaller than the evolutionary time scale if the period is less than 6 days. However, observations seem to show that this critical period is longer ...

In the following we will consider that the effect of the binary nature of A stars is to produce slow rotation.

2 Subgiants and giants

2.1 With respect to the "abundance" problem the situation is less clear than on the main sequence. The extension of the Am characteristics to this domain is under investigation. The Ca anomaly seems to decrease when the gravity decreases, though the Fe underabundance does not vary and Zr/Fe increases. In addition there seem to be a large variety of stars "with abnormal spectra" called by Breger the "cosmic garbage box" in which the spectroscopists have tried to distinguish

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classes : δ Del, F III p, λ Boo, ...

As seen from the discrepant spectral type determinations from different authors, the visual classification is hazardous. Probably there are other mechanisms than the variation of the abundances which alter the aspect of the spectrum i.e. luminosity, variability.

Now several detailed abundance analyses are available. From the most recent works (Kurtz cited by Breger, 1974b, and Ishikawa, 1973) the following results appear :

- the shape of the H and Ca lines are generally normal for their T_{eff} and g,
- the iron abundance is normal,
- the rare earth are enhanced by a factor ∿ 3.

2 No extended search for variability has been made on these stars so that no statistical results are available.

- In the "cosmic garbage box" 9 out of 13 studied are known pulsators, generally with large amplitudes.

TABLE 2

Interesting Delta Scuti Variables With Unusual Spectra

Metallicity Index (uvby)	Metal Abundance	Spectral Classifications		
		Cowley +Jaschek	Abt +Bidelman	Morgan +Abt
(high)	All up a bit	-	-	F3IIIp
(normal)	All down a bit	δ Del	δ Del	F0IVp
(high)	-	F2III	-	F3IIIp
(high)	All up a bit	-	-	F5IIp
(high)	All up a bit	-	-	F3III
normal	-	δ Del	-	-
(normal)	-	δ Del	Poss. Am	A9IV
normal	-	A8V and A5m:	Poss. Am	A9IV?

Figure 4 .- Interesting δ Scuti stars with abnormal spectra from Breger (1974b).

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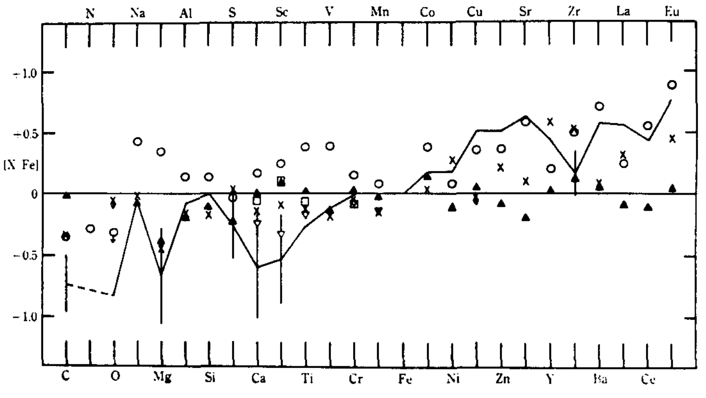

Figure 5 .- Individual abundances of δ Scuti stars from Ishikawa (1973).

- There are also pulsators with normal spectra, i.e. 14 Aur, HR 432.

2.3 Concerning the rotation parameter no definite statement can be made.

Among normal stars the pulsators can be fast or slow rotators : 14 Aur cannot have a very small sin i and V_R sin i = 30 km/s, HR 432 has broad lines corresponding to V_R sin i ≃ 110 km s^{-1}.

In the "cosmic garbage box" there seems to be a tendency to have essentially a small rotational velocity and a large amplitude.

However, if the anticorrelation between rotational velocity and amplitude close to the main sequence is still valid, the fast rotators should have small amplitude. As they are evolved, the period is longer (ex HR 432). These two conditions make them difficult to detect -

B Theoretical interpretation.

I recall the main result that the instability strip corresponds to sets of T_{eff}, g of the atmospheres for which the helium ionization zones are properly situated to trigger a vibrational instability of the whole star. The contribution of the hydrogen ionization zone is not yet well understood because it is located in the high outer layers where the motion is non linear. However, it seems that it is never the principal agent of instability (as proven by the good agreement between the observational and theoretical borders of the instability strip, which are calculated in the δ Scuti case at least, in the linear approximation).

This vibrational instability has been established for the normal cosmic helium abundance $y \simeq 0.3$ by weight.

The idea comes to attribute the disappearance of the instability (in the same (log T_{eff}, log g) region) to the disappearance of helium in these outerlayers, or at least to an important decrease of the He abundances. The influence of the helium content on the nature of the pulsation in the instability strip has been studied by Cox, King and Tabor (1973). The major result for the main sequence neighbourhood is that when y (in weight) becomes less than 0.2 the fundamental remains stable.

Note that the He abundance of Am stars is not known. For the hottest ones where some lines are visible, Smith (1974) has found that helium was deficient by a factor 3 to 5 in α Gem, θ Leo and Sirius. This last determination contradicts the previous analysis of Kohl (1964) who found He normal in Sirius.

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Then it becomes tempting to attribute to the same process the He and metals anomalies in Am stars. The microscopic diffusion as we will see can do the job.

The last question is then : why diffusion processes can occur in slow rotator stars and not in fast ones? We will see how fast rotation can give a meansof mixing the outerlayers of these stars.

Let us now go into the details of this model and then rediscuss the observational facts.

1 Microscopic diffusion

Let us recall the general equation of diffusion in a binary gas mixture as given by Chapman and Cowling (1960)

$$\omega_1-\omega_2= -D_{12}\left[\frac{1}{c_1}\,\frac{1}{c_2}\frac{\partial c_1}{\partial r}+\frac{m_2-m_1}{c_1m_1+c_2m_2}\,\frac{1}{P}\,\frac{\partial P}{\partial r}-\frac{m_1m_2(F_1-F_2)}{(c_1m_1+c_2m_2)kT}+\alpha_{12}\frac{1}{T}\,\frac{\partial T}{\partial r}\right] \quad (1)$$

ω_i, m_i, c_i, F_i, velocity in the F direction, mass,concentration,external forces on particles of species i ($i = 1, 2$)

α_{12} coefficient of thermal diffusion

D_{12} coefficient of diffusion

In the case of a test particle (of very small abundance) in completely ionized hydrogen Aller and Chapman (1960) computed the electric field and obtained the velocity of diffusion of an ionized test-particle assuming that the only forces acting are the electric and gravitational forces. This treatment is justified for A stars as the outerlayers are almost completely devoid of. neutral atoms.

$$\omega_2 = D_{12}\left[-\frac{1}{c_2}\frac{\partial c_2}{\partial r} + k_1\frac{1}{P}\frac{\partial P}{\partial r} + k_2\frac{1}{T}\frac{\partial T}{\partial r}\right] \qquad (2)$$

$k_1 = 2A - 2 - 1$

$k_2 = 2.54\, z^2 + 0.805\,(A - 2)$

$D_{12} = 6.62\ 10^9\ 5^{5/2}\ (n_1 M_2\, z^2\alpha)^{-1}$

n_1 is the number density of particles.

$$M_2 = \frac{m_2}{m_1 + m_2}$$

$$\alpha = \log_e\left(1 + \left(\frac{4d_D kT}{Ze^2}\right)^2\right) \simeq \log_e\left(\frac{4d_D kT}{Ze^2}\right)^2 \text{ slowly varying with } T$$

$$d_D = \left(\frac{kT}{4\pi\mu_e e^2}\right)^{1/2} \quad \text{so} \quad \alpha \simeq \log_e\left[6.4\ 10^{-16}\frac{T^3}{Z^2 P_A}\right].$$

In the outerlayers of A stars where $\frac{d\log T}{d\log P} \simeq 0.5$ the ratio of the pressure term to the thermal one is

$$\simeq 0.5\,\frac{k_2}{k_1} \simeq \frac{Z^2}{A}$$

i.e. of the order of 1 for helium and smaller for heavier elements.

So generally the pressure term dominates. In this case the diffusion velocity can be estimated by

$$\omega_2 \quad D_{12}\, k_1 \frac{1}{P}\frac{\partial P}{\partial r} = 10^{-22}\xi A T^{3/2} P^{-1} g \qquad (3)$$

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ξ is numerical coefficient close to 1 depending slightly on the composition (A, 2, μ)

In the case of helium the direct use of these formulae raises some difficulties because helium is not exactly a test particle.

The error due to the neglect of the collisions between test particles can be estimated using the second approximation of D_{12} from Chapman and Cowling (1960). For the Cosmic helium abundance $\frac{n_2}{n_1} \simeq 0.1$ it is of the order of several percents. However, the treatment of the electric field has been modified taking into account the electrons coming from ionization of helium. Work is in progress on that point by Pamjatnickh and Michaud and Montmerle (private communication).

2 Radiation pressure effect

The electric force is not always the only external force acting on the particles. Michaud (1970) introduced the idea that the radiation field coming from the interior of the star will push the different atoms upward. The strength of this effect will depend on the interaction between the radiation field and the species i.e. on the spectrum of the radiation, on the atomic structure of the species and probably on the abundance.

The upward acceleration on atoms of species 2 is

$$g_R = \frac{1}{m_2} \sum_{i,n} \frac{N_{i,n}}{N_e} \int_\nu \sigma_{i,n}(\nu) \frac{\phi(\nu)d\nu}{c} \qquad (4)$$

where i and n are the ionization and excitation states $\frac{1}{c}\phi(\nu)d\nu$ is the energy density and σ_i, n the absorption cross section of the atom in the state considered.

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In the case we are interested in i.e. outerlayers of A stars, for most species the only important contribution of g_R for heavy elements comes from the lines.

Then $m_2\, g_R$ has to be considered as an external force in the expression of ω_2, and after some modifications, using the perfect gas law approximation this term is added to the pressure term changing k_1 to

$$k_R = 2A\ (1 - \frac{g_R}{g}) - 2 - 1 \qquad (5)$$

assuming that no radiation pressure acts on protons.

On this expression one sees clearly that if in a layer g_R becomes larger than g the velocity of diffusion changes sign.

3 Diffusion in A stars

Using equation (3) one can estimate the diffusion velocity when knowing T and ρ. In A stars it is of the order of 10^{-5} cm s^{-1} and the corresponding time scale $t_D = \frac{H_p}{\omega_D}$ is approximately 10^5 to 10^6 years.*

Diffusion to take place needs a stable medium at least over this period. Convection which is a very powerful agent of mixing destroys immediately the effect of diffusion. Then, diffusion can take place only in radiative zone. But acting at the outerboundary of a convective region it is able to sort

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* This time scale corresponds to the sorting of the whole outerlayer down to the bottom of the convective zone, as its dimension is comparable to H_p.

a whole convective zone (which remains homogeneous) above.

In A stars the radiative zone intermediate between the two convective zones (see fig 6) is not stable and probably completely mixed by overshooting from the two convective zones, as proposed by Toomre et al (1975). In the homogeneous model it can take place only at the bottom of the convective zone.

But Vauclair, Vauclair and Pamjatnikh (1973) computed the influence of the diffusion of helium at the bottom of the convective zone on the structure of the models. They found that the outerlayers are deprived from helium in 10^6 years and that the second convective zone disappears (fig 6 et 7).

Then after 10^6 years, a stable star will be left with only a superficial convective zone. After this time has elapsed the diffusion processes will take place at the bottom of this zone.

Let us now examine the problems of metals for which under/over abundances have to be explained. This cannot be done with ordinary diffusion which will always have a tendency to deplete the surface of heavy elements. Then the influence of the radiation field becomes fundamental.

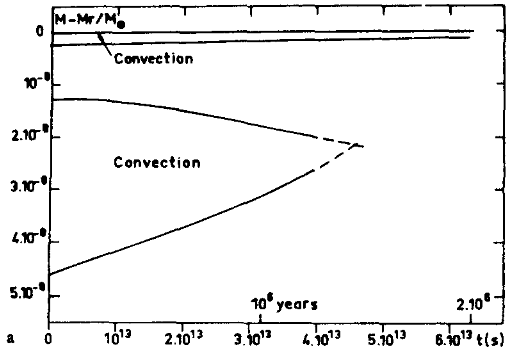

Figure 6 .- Evolution of the second helium convective zone, from Vauclair Vauclair, Pamjatnikh (1973).

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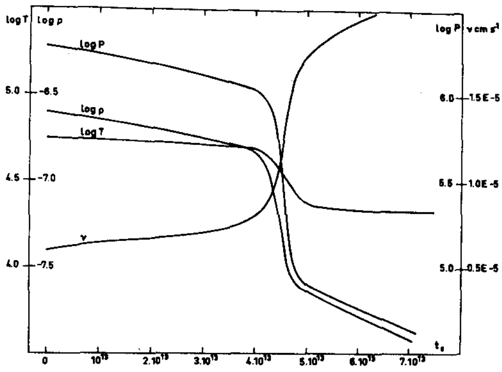

Figure 7 .- Variation of T, ρ, $V_{diffusion}$ at the lower boundary of the second helium convective zone during evolution from Vauclair, Vauclair, Pamjatnickh (1973).

At the bottom of the second ionization zone, the effect is small, the diffusion velocities are too low. For radiation pressure to be efficient the sorting has to take place at lower temperature. This is possible if the separation of metals takesplace after the disappearance of the second convective zone, at the bottom of the first one.

Detailed computations of g_R are now available from the work of Kobayashi and Osaki (1973) for several elements Sc, Sr, Y, Zr (fig 8). In these computations the gravity is 10^4 and the radiation pressure force is given as a function of depth, and of the abundance.

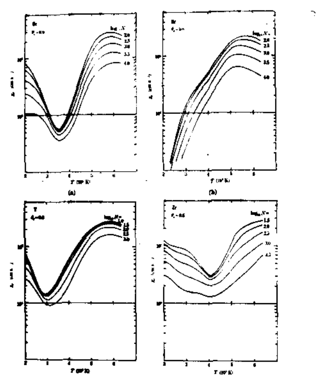

Figure 8 .- Upward acceleration due to the radiative force on Sc, Sr, Y, Zr as a function of the depth of the envelope for the case Θe = 0.6. Arrows indicate the location of the convective zones.

The authors found that "around 3.5 10^4 d°K" the radiation force acting on Sc is smaller than the gravitational force, while the radiation pressure forces acting on Sr, Y and Zr are larger than the gravitational force. Fig 9 illustrates the extreme sensitivity of the radiation force to the temperature; in particular the existence of a low g_R domain imposes drastic conditions on the temperature at the level where diffusion takes place.

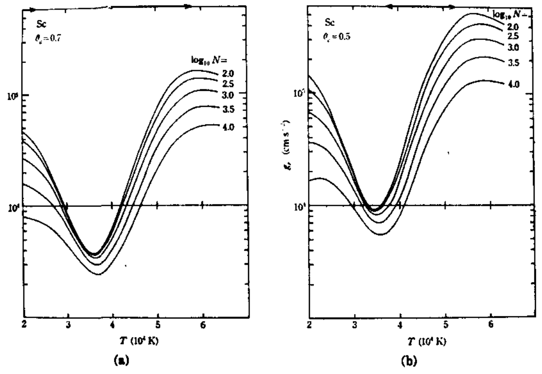

Figure 9 .- Same thing for Sc and different values of Te.

The case of calcium has not been treated in detail but due to its electronic structure it has a similar behaviour to Sc.

These results should be taken only qualitatively : the treatment needs still many improvements as for example the influence of the different ions of the same species present at the same time. In that case for which the coefficient of diffusion being proportionnal to Z^{-2} , Schatzman and Alecian (private communication) have shown that the diffusion velocity is the weighted sum of the diffusion velocities of the different ions

$$W_D = \sum_i x(i)\, w(i) \tag{6}$$

where x(i) is the degree of ionization of the ion i, and w(i) its diffusion velocity as computed from eq (2).

The underabundance of Sc (or Ca) is the most difficult to obtain and necessitates very drastic conditions on Te (and/or g) whereas the overabundances of

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heavy metals seem to be more easy to explain. The discussion of the abundances of the giants showed that, though the Ca deficiency seems to disappear, the heavy metals overabundance is very frequent, even in pulsating stars.

Time scale of sorting : Velocities of diffusion are 5 10^{-5} cm s^{-1} for Ca and Sc and 10^{-4} cm s^{-1} for Sr, Y, Zr, leading to time scales of the order of 10^5 to 10^6 years.

The age of Am stars seems to be highly variable. Smith (1972) searching for Am stars in clusters found some of them in very young clusters like Ori Ic which is 10^6 years old. Then this diffusion time scale is short enough to explain all the Am.

Predicted values of over(under) abundances : As there is enough time for diffusion to proceed, the equilibrium abundances should be reached. They satisfy $\omega_2 = 0$ i.e.

$$\frac{1}{c}\frac{\partial c}{\partial r} - A\left(2-\frac{g_R}{g}\right)\frac{1}{P}\frac{\partial P}{\partial r}=0 \quad . \tag{7}$$

As A is large for metals (≃ 100) the concentration gradient will be steep and the equilibrium abundance quite extreme.

This effect can be somehow reduced by the saturation of g_R which can be seen in fig 9.

When the abundance of the element is small the acceleration is independent of it. But when the element becomes a major contributor to the opacity, the process saturates and g_R is proportionnal to N^{-1}. Eventhough, the equilibrium abundances would be much more extreme than the observed ones (30 at most). Then as discussed in Schatzman (1969) a diffusion barrier is needed.

To sum up let us summarize what the theory of diffusion is probably able to explain in A stars.

Assuming that for some reason, (see sect. III) the outerlayers of Am stars are stable it predicts that :

1) the Am envelopes are deprived from helium.
This point explains why the Am do not pulsate. Vauclair, Vauclair and Pamjatnick (1973) have shown that after 10^6 years the second convective zone, responsible for the driving of the pulsation disappears. So, after this time has elapsed the pulsation cannot be fed anymore.

2) the sign of the abundances anomalies for the most important metals are explained, though some difficulties remain in the quantitative results.

3 Mixing

Let us examine now the question : why are the homogeneous and sorted stars in the same log T_{eff}, log g domain which is the theoretical equivalent of "why are there normal and Am stars in the same region of the HR diagram"

To have variable normal stars the diffusion of helium below the second convective zone has to be prevented by some "mixing agent", which should not be present in the slowly rotating Am stars.

The main difficulty in this kind of problem is that we do not know a general criterion for stability. What has been done up to now is to list some instabilities and study conditions under which they take place. We do not know either how to describe the state of motion produced by these instabilities. So we are left with very rough estimates quite unsatisfactory.

3.1 Meridional circulation

The well known Eddington-Sweet meridional circulation present in all rotating stars is certainly in itself a way of mixing. It is due to the thermal unbalance between pole and equator. The horizontal component drives away the elements from the surface to the interior and the vertical component produces a flux of matter through the convective zone. Only in the case of solid body rotation the velocity field (Mestel, 1965) can be computed easily; the boundary of the convective

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zone is a singularity for the velocity field. However the overshooting (over a scale height of the order of H_p) erases this singularity.

As remarked by Baglin (1972) the horizontal flow does not stop diffusion. But the time scale is lengthened. The lengthening depends on the exact topology of the motion in the convective zone like the shape of the cells, etc., ...

The time scale of the mixing of the outer zones can be estimated by the time needed to refill with new matter an outer layer of mass ΔM. Using the value of the vertical component as given by Mestel (1965)

$$V_r = \frac{\bar{\rho}}{\rho} \frac{R}{t_{KH}} \frac{\Omega^2 R}{g} \qquad (8)$$

where $\Omega, R, g, \rho, \bar{\rho}$ have their usual meaning and t_{KH} is the Kelvin Helmholtz time scale , this time is

$$t_{MC} = \frac{\Delta M}{M} t_{KH} \frac{\Omega^2 R}{g}^{-1} . \qquad (9)$$

This time scale is somewhat longer than the diffusion time. For this process to work, the meridional velocity field has to be completely stable and this does not conserve the solid body rotation.

3.2 Turbulent mixing from the shear of the meridional circulation

Below the surface, there is a shear due to the variation with depth of tangential velocity. Baglin (1972) studied the onset of turbulence due to this shear. For incompressible fluids the condition for a shear flow to become turbulent is that the Reynolds number Re becomes larger than a critical value of the order several thousands

$$Re = \frac{V\theta l}{\nu} > R_c . \qquad (10)$$

In stars, ν , the viscosity is dominated by the radiative viscosity in the physical conditions we are interested in.

$$\nu = \nu_o \, T^{7.5} \, \rho^{-3} \qquad \text{(with a Kramer opacity law).}$$

It is generally very small so that Reynolds numbers are high and shear instabilities are favoured.

Using for the approximation of the horizontal velocity field, the same as in eq 8, one defines the characteristic length of the shear

$$l = \left(\frac{\partial}{\partial r} \ln V_\theta\right)^{-1}.$$

And the condition for the onset of turbulence is then

$$T^{-6.5} \, \rho^2 \, V^2 > \text{Const.} \qquad (11)$$

which defines a critical velocity above which the flow becomes turbulent.

Using classical parameters for A stars Baglin (1972) obtained $V_{crit} \simeq 50$ km s^{-1} for turbulence below the second helium convective zone. However this result suffers quite important criticism.

This instability will determine the critical velocity of mixing only if it is the only one working at low rotational velocities.

3 Differential rotation

Zahn (1974) has shown that differential rotation in itself should be a very powerful agent of destabilization.

At least in the slow rotation regime the meridional circulation velocity field induces a differential rotation as a consequence of the conservation of angular momentum. In this process a gradient of angular velocity is built, for which the Reynold number becomes very rapidly larger than the Reynolds number associated to the shear.

$$Re_{diff\ rot} = Re_{shear} \times \Omega \, \Delta t \qquad (12)$$

over a time Δt.

This shows that the destabilization due to the building of differential rotation appears rapidly. However a detailed treatment of the diffusion of angular momentum is badly needed to settle the point and make quantitative estimates.

In gravitating media horizontal and vertical motions have different

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behaviours. Vertical motions are helped by the density stratification and they are submitted to the Richardson criterion of instability. However, taking into account the thermal diffusion, this criterion should be modified (as shown by Zahn 1974) strengthening the stability in the vertical direction

$$\left(\omega \frac{\partial \Omega}{\partial \omega}\right)^2 \gtrsim \sigma \ Rc \ \frac{g}{H_p} \left(\nabla_{ad} - \nabla_{rad}\right) \tag{13}$$

ω is the distance to the axis of rotation
σ the Prandtl number ratio viscosity over thermal conductivity.
Then, the following picture emerges.

Let us define two critical "equatorial velocities".

- V_1 corresponding to the onset of instability of the meridional circulation or differential rotation induced by it through the Reynolds criterion.
- V_2 corresponding to the instability in the vertical direction.

In the very low rotation regime $V < V_1$ the outerlayers are stable. Diffusion proceeds freely with the time scale computed precedingly.

In the high velocity regime $V > V_2$ a strong 3 dimensional turbulence sets in which mixes very rapidly the medium. This will be the regime of the variables.

In the intermediate regime $V_1 < V < V_2$ turbulence in the horizontal direction settles. It uniformizes the angular momentum over spheres. As motions are mostly horizontal they should not help diffusion.

3.4 Turbulent diffusion

Schatzman (1969) has shown that even in slowly rotating stars stellar spin-down or meridional circulation can produce a small turbulence sufficient to minimize the sorting. However in his model radiation pressure was not included and all estimates were made at the bottom of the second convective zone, which makes the numerical results difficult to use.

The "horizontal turbulence" which probably exists in the domain $V_1 < V < V_2$ might also act as a "diffusion barrier". However no estimate exists up to now of the corresponding structure of the motions.

III Discussion.

Let us list now the main difficulties encountered in this problem.

1- From the theoretical point of view, though we can give a rough sketch of the probable processes acting an exact treatment is far from our knowledge. The correct description of the hydrodynamical behaviour of the gas of a rotating star cannot be reached without a complete treatment of the three dimensional problem. Though several instability criteria can be derived the state of motion is unknown.

2- Fast rotators with Am abundances. Except if there is a strong agent which forces solid body rotation it is difficult to understand how instabilities could not develop. As usual a strong magnetic field can be invoked.

3- The slowly rotating variables. To interpret them one needs to find a process of mixing. Until we have an idea of the physical difference between these slow rotating variables and the corresponding slow rotating Am, it is difficult to attribute the mixing to a particular process, though we can imagine that other instabilities than those which have been studied here can exist.

These two last points probably mean that the situation is more than one parameter dependent and that other processes perturb the interplay between sorting and mixing, though up to now they are not known.

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4- The situation in the giant domain.

The abundances anomalies found here do not seem to be too difficult to understand, as the disappearance of the underabundance of Sc (or Ca) can be attributed to a slight variation of the physical parameters of the region of sorting.

However the fact that "constant" as well as "pulsators" seem to have the same kind of abundances-anomalies is difficult to understand.

The deepening of the ionization zones during the evolution from the main sequence to the giant branch should lengthen the diffusion times scales. If the diffusion time scale becomes larger than the evolutionary one (which is becoming shorter and shorter) the abundances could be only the fossil remnants of the abundances on the main sequence.

The interpretation of all these facts are to my point of view a very interesting field of investigation which will give us new clues to the hydrodynamical behaviour of the outerlayers of the stars.

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Discussion to the paper of BAGLIN

FITCH: If you need a mixing mechanism to explain the observed pulsation of some slow rotators, would tidally driven mixing currents help?

BAGLIN: Even in the case of 14 Aur, probably the perturbation is too small to be significant.

MOLNAR: You mentioned a problem with the diffusion mechanism is that it is too efficient. That is, overabundances should be larger in your stars than they are observed to be.

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One way to quench the diffusion mechanism (in middle A and late A stars) is the effect of increasing ultraviolet opacities. For these stars, a small metal overabundance quickly cuts off the ultraviolet radiation, halting the upward diffusion of the heavy elements. Because the ultraviolet radiation is the main driving force in a stellar atmosphere, future diffusion models should include the effects of increasing metal opacities.

BARTOLINI: I would like to show the results of my observations of 32 Vir, taken at the Bologna Observatory during the three consecutive nights of 20, 21, 22 March 1973. At the beginning of the first night the star was practically constant, but after $0\overset{d}{.}3$ it was clearly pulsating. During the second night the star was pulsating with increasing amplitude. Pulsations were present at the beginning of the third night but stopped at the end. From these observations it is possible to deduce two times of minimum amplitude; their difference of $1\overset{d}{.}8$ is a multiple of the beat period. I think the beat period should be $P_b \simeq 0\overset{d}{.}6$, while the fundamental period is $P_o = 0\overset{d}{.}076$.

FROLOV: I would like to point out that there is a δ Scuti star which is also an Am star, namely BS 3588 = FZ Vel.

BAGLIN: Before making a definite statement on such a star, it has to be studied in detail. Among A stars, binaries are frequent, and often difficult to distinguish spectroscopically because they are generally associated with another A star, as for example 32 Vir. On the other hand, as I have shown, the luminosity class is an important parameter.

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