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A STOCHASTIC ANALYSIS OF SCORING SYSTEMS

GRAHAM HILFORD POLLARD

Many scoring systems can be seen as statistical tests of hypotheses [1]. In tennis singles, for example, the scoring system can be seen as a test involving 2 binomial probabilities p_a and p_b where $p_a(p_b)$ is the probability player A (player B) wins a point initiated by player A (player B). Tennis singles is thus a *bipoints* game. The tennis scoring system is an inefficient test relative to the sequential probability ratio test ($SPRT$) based on pairs of these points. Miles showed that when $p_a + p_b > 1$ (the tennis context), an $SPRT$ based on the *play-the-loser* (PL) rule is super-efficient. Chapter 2 of this thesis shows that, when $p_a + p_b > 1$, there is in fact a spectrum of super-efficient tests (with even durations) based on *partial-PL (PPL)* rules. The most efficient tests within this spectrum, when $p_a + p_b > 1$, are the $SPRT$ based on the (full) PL rule. Chapter 3 extends this spectrum of tests to produce the total spectrum of tests (including those with odd durations).

Points within the tennis scoring system have different *importances* [2] whereas points within any member of the above (efficient) spectrum of PPL systems are seen to be equally *important* when $p_a = p_b$. Intuitively, the differing importances of the points within the tennis scoring system contribute to the inefficiency of that system. Chapter 4 establishes a relationship between the efficiency of a bipoints scoring system and the importances of the points within it; a relationship which is used in Chapter 5 to show that the $SPRT$ based on the PL (play–the–winner, PW) rule has an *optimal* efficiency property when $p_a+p_b > 1$ ($p_a+p_b < 1$). Thus Chapter 5 solves a well–known 2–sample binomial problem.

Chapter 6 shows that some complex $SPRT$ systems can be decomposed into smaller independent components called *modules* which can in turn be analysed to produce values from which the asymptotic efficiency of the complete $SPRT$ system can be evaluated. This module approach is used to give an intuitive explanation as to why the PL rule is more (less) efficient than the PW rule when $p_a + p_b > 1$ ($p_a + p_b < 1$). In another example, the module which produces the asymptotically most efficient $SPRT$ for the case in which α, the probability of a type I error and β, the probability of a

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type II error are equal ($\alpha = \beta$ is required for the *fairness* of a scoring system) is also the asymptotically most efficient for the case in which $\alpha \neq \beta$.

Chapter 7 uses the module approach to show that the super-efficiency of the PL rule carries over to the case of tennis doubles in which there are essentially 4 binomial probabilities p_{a1}, p_{a2}, p_{b1} and p_{b2} (provided $\overline{p}_a + \overline{p}_b > 1$ and $|p_{a1} - p_{a2}| = |p_{b1} - p_{b2}|$).

The particular scoring system currently used in tennis is analysed in Chapter 8 (see also [**3**]) and the methodology used is seen to be useful for analysing any *nested* scoring system (for example, tennis is 3-nested: points – games – sets). It was the study of this specific scoring system and its inherent inefficiency which lead to the theory of Chapters 2 to 7. A new tennis scoring system with a smaller variance of duration is proposed in Chapter 8. (See also [**5**].)

Chapter 9 contains a brief discussion of some of the characteristics that need to be considered by the designer of a scoring system. In particular, the roles played by the expected duration, the variance of duration and the efficiency of a system are discussed.

In Chapter 10 an exact relationship between the increased probability of winning a point or set of points and the increase in the probability of winning a match is given. An alternative approach shows that points or states can interact *positively* or *negatively*.

In Chapter 11 team play with associated countback rules is investigated. The general conclusion is that *upward-nested* countback systems (for example points – games – sets, in tennis) are preferable to *downward-nested* ones (sets – games – points).

In Chapter 12 it is shown that the classical scoring system used in multiple choice examinations can be considerably improved by modifying that scoring system and instructing the examinees to cross any boxes known to be incorrect when the correct box for that question is unknown (see also [**4**]). A scoring system which, it is argued, should remove random guessing completely is also given. (See also [**6**].)

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School of Information Sciences
Canberra College of Advanced Education
P.O. Box 1
Belconnon, A.C.T. 2616
Australia

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