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Two-variable bases for the laws of var $PSL(2, 2^n)$ and related topics

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In [1], Cossey, Macdonald and Street gave bases for the laws of the varieties generated by a number of finite groups. This thesis is a continuation of some of that work.

The central result is Theorem 3.1.1 of Chapter 3 which gives a two-variable basis for the laws of the variety generated by the finite simple group $PSL(2, 2^n)$, $n \geq 2$. Most of the material from this chapter has been published in the papers [3] and [4].

An important tool in finding the basis given in Theorem 3.1.1 is a result of [1] which says that if x and y are elements of some $SL(2, K)$, K a commutative ring with identity, then the trace of any word w in x and y is a polynomial in the traces of x, y, and xy, which we denoted by $\text{tr } x, \text{tr } y, \text{tr } xy$. This polynomial falls into one of four classes depending on the word w. Horowitz [2] gives a generalisation of this result to words in more than two variables. The trace of any word w in n variables, under an arbitrary mapping of those variables into $SL(2, K)$ is a polynomial in the $2^n - 1$ traces $t_{\sigma_1\sigma_2\ldots\sigma_m} = \text{tr } x_{\sigma_1}x_{\sigma_2} \ldots x_{\sigma_m}$, $1 \leq \sigma_1 < \sigma_2 < \ldots < \sigma_m \leq n$. Chapter 2 contains a new proof of this result, which establishes that the trace polynomial falls into one of 2^n classes, depending on the word w. Chapter 2 also contains examples of the calculation of trace polynomials of several three variable words, showing that these polynomials are not unique.

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Chapter 4 contains two-variable bases for the laws of the varieties generated by $PSL(2, 4)$, $PSL(2, 8)$, $PSL(2, 16)$, and $PSL(2, 64)$. These differ from the general basis given in Chapter 3, in the laws used to ensure that groups of odd exponent are abelian. In the case of $PSL(2, 64)$ the necessary laws were obtained from computer calculations.

References

[1] John Cossey, Sheila Oates Macdonald and Anne Penfold Street, "On the laws of certain finite groups", *J. Austral. Math. Soc.* 11 (1970), 441–489.

[2] Robert D. Horowitz, "Characters of free groups represented in the two-dimensional special linear group", *Comm. Pure Appl. Math.* 25 (1972), 635-649.

[3] Bruce Southcott, "A basis for the laws of a class of simple groups", *J. Austral. Math. Soc.* 17 (1974), 500-505.

[4] Bruce Southcott, "Two-variable laws for a class of finite simple groups", *Bull. Austral. Math. Soc.* 10 (1974), 85-89.

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