

# Modelling the deformability of magnetized neutron stars in the light of future continuous gravitational waves detection

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**Abstract.** Neutron stars are known to host extremely powerful magnetic fields. Among other effects, one of the consequences of harboring such fields is the deformation of the neutron star structure, leading, together with rotation, to the emission of continuous gravitational waves (CGWs). We present an extensive numerical study of magnetized neutron stars in GR with a large variety of different Equations of State (EoSs) and show that it is possible to find simple relations between the magnetic deformation of a neutron star, its mass and radius, that are mostly independent on the EoS or magnetic configuration. We discuss how these relations can be used in conjunction with possible future CGWs detection to set constraints on the EoS and magnetic configurations of NSs (e.g. the presence of a superconducting phase). By carrying out a population synthesis, we estimate the possible CGWs detectability of galactic millisecond pulsars, with third generation GW detectors.

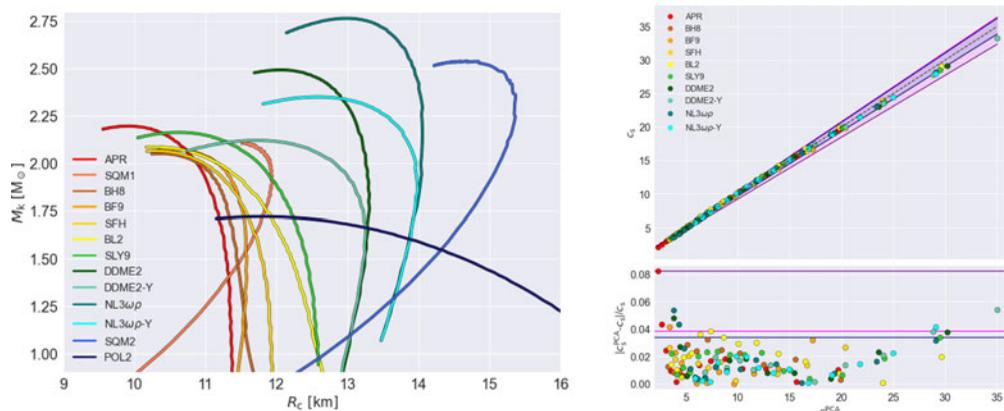
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## 1. Introduction

Neutron stars (NSs) are the most compact and dense material objects in the Universe. They are known to be rapidly spinning and to harbour strong magnetic fields. It is indeed the combination of fast rotation and strong magnetic field that makes them among the foremost high energy astrophysical sources. Their rapid rotation in the range 1-1000Hz, makes them ideal source of potentially detectable Continuous Gravitational Waves (CGWs) ([The LIGO Scientific Collaboration et al. 2022](#)). From this perspective, two classes of pulsars are particularly interesting: millisecond pulsars (MSPs), with typical rotation periods under  $\sim 20$ ms, and magnetars, possessing an internal magnetic field potentially of the order of  $10^{16}$ G.

Such strong magnetic fields are able to alter the NS shape inducing a large scale quadrupolar deformation, which will depend both of the Equation of State (EoS) and on the magnetic geometry ([Cutler 2002](#); [Oron 2002](#)). If the magnetic and spin axes are not aligned, the resulting time-varying quadrupole will lead to the emission of CGWs. For this reason it is important to understand the mutual interplay of the magnetic fields and the EoS in regard of the deformation of NSs. Particularly interesting is the existence of relations which are either truly independent or weakly dependent on the EoS (quasi-universal relations, e.g. [Breu & Rezzolla 2016](#)). Such relations can be used to infer more complex properties dependent on the internal structure and composition, like the



**Figure 1.** Left Panel: Mass-Radius relation for the various EoSs of our study. Right Panel: quasi-universal relation for the magnetic deformability, Eq. 2.1. Shaded regions (and relative purple lines) represent bounds within which the total and 90% of the results are found. Adapted from SB21.

deformability or moment of inertia, from more easily observable quantities like mass and radius.

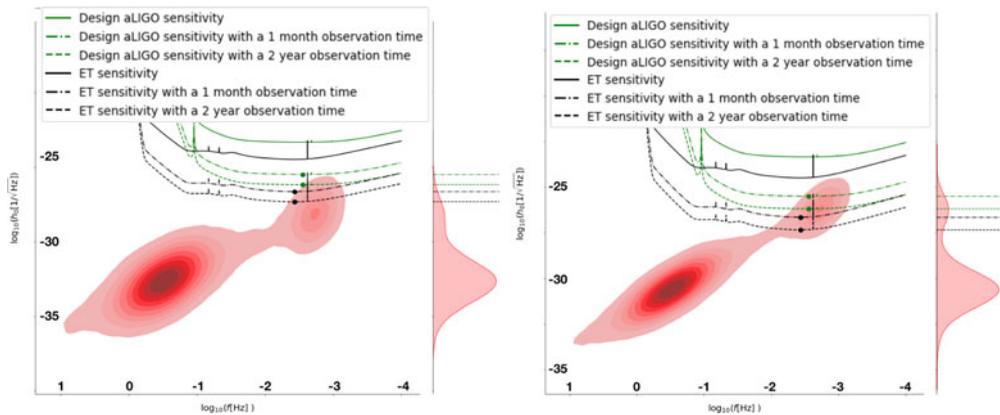
We review here the results of our recent investigation on the magnetic deformability of NSs, in particular we show the existence of a quasi-universal relation among the NS mass, radius, surface magnetic field and magnetic deformation, derived on a large set of EoSs (Soldateschi *et al.* 2021a [SBD21]; Soldateschi & Bucciantini 2021b [SB21]). We then proceed to illustrate how it can be used to assess the detectability of CGWs through the use of present and future GW detectors, both for the known sources of the ATNF catalogue, and for a generic Galactic population. Our approach shows that a minimal fraction of the MSPs in the Galaxy may be observable even with current detectors at design sensitivity, while canonical pulsars will be inaccessible even to 3rd generation ones.

## 2. Deformability

In order to evaluate the magnetic deformability of NSs, we select a large sample of EoS, computed under various physical assumptions, including purely nucleonic EoSs (Akmal *et al.* 1998; Bombaci & Logoteta 2018; Hempel & Schaffner-Bielich 2010; Fortin *et al.* 2016), hyperonic EoSs (Fortin *et al.* 2016), quarkionic EoSs (Baym *et al.* 2018; Baym *et al.* 2019) [see SBD21 for a detailed discussion on their properties]. Their mass-radius relation is shown in Fig. 1. For each EoS we computed several thousands fully 2D equilibrium GR models at different central densities and for different magnetizations using the XNS code (Bucciantini & Del Zanna 2011; Pili *et al.* 2014; Soldateschi *et al.* 2020). We then use these models to infer the so called distortion coefficient  $c_s$  relating the quadrupolar deformation  $e = Q/I$  (given as the ratio of quadrupole  $Q$  to the moment of inertia  $I$ ) to the surface magnetic field  $B_s$  (in units of  $10^{18}\text{G}$ ), as  $e = c_s B_s^2$ . By performing a Principal Component Analysis (PCA) we found that, independently of the EoS,  $c_s$  can be expressed as a function of the global NS parameters with a high level of accuracy (deviations  $< 4\%$ ) according to:

$$c_s \simeq c_s^{\text{PCA}} = 2.97 R_{10}^{4.61} M_{1.6}^{-2.80}, \quad (2.1)$$

where  $R_{10} = R_c/10\text{km}$ ,  $M_{1.6} = M_k/1.6M_\odot$ , and  $R_c$  and  $M_k$  are the circularisation radius and the Komar mass of the NS, respectively (see SBD21 for their formal definition). In Fig. 1 we show the relation between the magnetic deformability computed directly on



**Figure 2.** Left Panel: KDE density distribution of CGWs strains from a galactic PSR population, together with various sensitivity curves. On the right side the cumulative distribution and the detection thresholds. Right Panel: Same as left panel but assuming a superconducting core. Adapted from SBD21b.

the numerical models vs the relation given by Eq. 2.1. Here we report only the quasi-universal relation for a poloidal magnetic field in terms of the magnetic surface strength. Similar relations hold also for toroidal fields, and for parametrizations in terms of the magnetic energy (see SBD21).

### 3. Detectability of CGWs

Using the above findings we have computed the detectability of CGWs from Galactic PSRs (both a generic galactic population and the known population in the ATNF catalogue) for current and future detectors. To use Eq. 2.1 one needs to know the mass and radius (or alternatively the EoS) of each NS. The mass distribution is taken from Antoniadis et al. (2016), while the radius is computed from the mass by selecting a typical EoS, among the ones we have adopted. The rotation period distribution is derived from the ATNF catalogue, while the surface magnetic field is taken from the distribution by Faucher-Giguère and Loeb (2010) even for MSPs, under the assumption that the true field is buried below the surface. Pulsars are distributed in the galaxy either according to Lorimer et al. (2006) or to Kiel & Hurley (2009). These represent respectively an old and a young PSR population, such that a comparison between the two allows us to infer how relevant the PSR distribution in the Galaxy is to our findings. We have verified that neither the mass nor the galactic distribution affect significantly our results. The choice of the EoS (in the expected range of admissible EoSs) can at most change the strain by a factor of 2-3. We also considered the possibility of a superconducting core (Cutler 2002), that enhances the effect of magnetic field. For more details on the population synthesis we refer to SB21.

Our results are presented in Fig. 2, where we show the distribution of the expected strains computed with a Gaussian Kernel Density Estimator (KDE) over a population of few tens of thousands sources. In the absence of superconduction the chance of observing CGWs from MSPs is smaller than a few percent with current instruments and at most of the order of 30% with third generation ones. On the other hand the chance of detecting CGWs from MSPs in the presence of a superconducting core will be almost 100%. We recall that this is the more optimistic estimate of the detectability, because it assumes a purely poloidal magnetic field. The presence of an internal toroidal field would reduce the deformation and lower the total detectable strain.

It is obvious that with third generation detectors we will be able to assess the internal magnetic field geometry in MPSs. Canonical pulsars, on the other hand, rotate too slowly to produce a detectable signal even in the most optimistic scenario.

#### 4. Conclusions

The existence of quasi-universal relations that allow one to infer quantities like the deformability, that in principle is a function of the unknown EoS and internal structure of the NS and its magnetic field, from global and potentially observable quantities like mass, radius, or the surface magnetic field, opens the possibility to use GW observations to put constraints on the NS physics. In principle, as we have shown, the detection or non detection of CGWs by third generation instruments, could already be used to place constraints on the EoS, or more likely to the internal magnetic field geometry.

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