BULL. AUSTRAL. MATH. SOC. 34D05
VOL. 30 (1984), 435-448.

EXPONENTIAL DICHOTOMY OF STRONGLY DISCONTINUOUS SEMIGROUPS

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In this paper we give necessary and sufficient conditions for exponential dichotomy of a general class of strongly continuous semigroups of operators defined on a Banach space. As a particular case we obtain a Datko theorem for exponential stability of a strongly continuous semigroup of class C_0 defined on a Banach space.

1. Introduction

Let X be a real or complex Banach space. The norm on X and on the space $L(X)$ of all bounded linear operators from X into itself will be denoted by $\|\cdot\|$. $T(t)$ will stand for a semigroup of linear operators on X which is of class C_0 ; that is, $T(t)$ is strongly continuous on $\mathbb{R}_+ = [0, \infty)$ and $T(0)x = x$ for all x in X .

Throughout in this paper we suppose that the set

(1.1) $$X_1 = \{x \in X : T(\cdot)x \in L^\infty(X)\}$$

is a closed complemented subspace of X . Here $L^\infty(X)$ denotes the Banach space of X-valued functions f almost defined on $\mathbb{R}_+$, such that f is strongly measurable and essentially bounded. If X is a complementary

Received 19 June 1984.

subspace of X_1 then we denote by P_1 a projection along X_2 (that is, $P_1 \in L(X)$, $P_1^2 = P_1$, $\text{Ker } P_1 = X_2$) and by $P_2 = I - P_1$ a projection along X_1.

We also shall denote

$$(1.2) \qquad T_1(t) = T(t)P_1 \quad \text{and} \quad T_2(t) = T(t)P_2 .$$

DEFINITION 1.1. The C_0 semigroup $T(t)$ is said to be

(i) *exponentially stable* if and only if there are $N, \nu > 0$ such that

$$(1.3) \qquad \|T(t)\| \le Ne^{-\nu t} \quad \text{for all} \quad t \ge 0 ;$$

(ii) *exponentially dichotomic* if and only if there exist $N_1, N_2, \nu > 0$ such that

$$(1.4) \qquad \|T_1(t)x\| \le N_1 e^{-\nu(t-t_0)} \|T_1(t_0)x\|$$

and

$$(1.5) \qquad \|T_2(t)x\| \ge N_2 e^{\nu(t-t_0)} \|T_2(t_0)x\|$$

for all $t \ge t_0 \ge 0$ and $x \in X$.

Clearly, if $T(t)$ is exponentially dichotomic and $X_1 = X$ (that is, $P_2 = 0$) then $T(t)$ is exponentially stable. In this case is well known the following theorem due to Datko (see [5] and [6]).

THEOREM 1.1. *A necessary and sufficient condition that a strongly continuous semigroup* $T(t)$ *of class* C_0 *defined on a Banach space* X *be exponentially stable is that for some* $p \in [1, \infty)$ *the integral*

$$(1.6) \qquad \int_0^\infty \|T(t)x\|^p dt < \infty \quad \textit{for all} \quad x \in X .$$

In this note the above result is extended in a natural manner to the general class of exponentially dichotomic C_0 semigroups of linear

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operators defined on a Banach space X .

The case $T(t) = \exp(At)$, where A is a bounded linear operator has been considered in [2], [3], [4], [7] and [10]. The problem of exponential dichotomy of C_0-semigroup on Banach spaces has also been studied in [9], [11] and [12].

2. Preliminary results

The following simple lemmas will be needed in the sequel in proving the main results.

LEMMA 2.1. *Let* $f, g : \mathbb{R}_+ \to \mathbb{R}_+$ *be two continuous functions.*

(i) If

$$\inf_{t\geq 0} g(t) < 1 \quad \text{and} \quad f(t) \leq g(t-t_0)f(t_0) \quad \text{for all} \quad t \geq t_0 \geq 0 \tag{2.1}$$

then there are $N, \nu > 0$ *such that*

$$f(t) \leq Ne^{-\nu(t-t_0)}f(t_0) \quad \text{for all} \quad t \geq t_0 \geq 0 . \tag{2.2}$$

(ii) If

$$\sup_{t\geq 0} g(t) > 1 \quad \text{and} \quad f(t) \geq g(t-t_0)f(t_0) \quad \text{for every} \quad t \geq t_0 \geq 0 , \tag{2.3}$$

then there exist $N, \nu > 0$ *such that*

$$f(t) \geq Ne^{\nu(t-t_0)}f(t_0) \quad \text{for all} \quad t \geq t_0 \geq 0 . \tag{2.4}$$

Proof. See [7].

In the sequel for $p \in [1, \infty)$ we denote by

$$p' = \begin{cases} \infty & , \text{ if } p = 1 , \\ p/(p-1) & , \text{ if } p > 1 . \end{cases} \tag{2.5}$$

LEMMA 2.2. *For every* $a > 0$ *there exists* $b > 0$ *such that*

$$e^{at^{1/p}} \geq bt^{1/p'} \quad \text{for all} \quad t \geq 0 . \tag{2.6}$$

Proof. It is easy to see that for

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$$b = (ap'/p)^{p/p'}$$

the above inequality holds.

LEMMA 2.3. *Let* $\Delta = \left\{(t, t_0) \in \mathbb{R}_+^2 : t \ge t_0\right\}$, $p \in [1, \infty)$ *and let* $f : \Delta \to \mathbb{R}_+$ *be a continuous function with the property that there exist* $c, \alpha > 0$ *such that*

$$(2.7) \qquad \int_{t_0}^{t} f(s, t_0)ds \le c(t-t_0)^{1/p'} f(t, t_0)$$

and

$$(2.8) \qquad \int_{t}^{t+1} f(u, t)du \ge \alpha$$

for all $t \ge t_0$. *Then there are* $N, \nu > 0$ *such that*

$$(2.9) \qquad f(t, t_0) \ge Ne^{\nu(t-t_0)^{1/p}} \quad \text{for every } t \ge t_0 + 1 .$$

Proof. If we denote by

$$(2.10) \quad g(t, t_0) = \int_{t_0+1}^{t} f(s, t_0)ds \text{ and } h(t, t_0) = \frac{p}{c} \cdot (t-t_0)^{1/p}$$

then from the inequalities (2.7) and (2.8) we obtain

$$(2.11) \qquad \alpha + g(t, t_0) \le c(t-t_0)^{1/p'} \frac{\partial g(t,t_0)}{\partial t}$$

which implies

$$(2.12) \qquad \frac{\partial}{\partial t}\left(-\alpha e^{-h(t,t_0)}\right) \le \frac{\partial}{\partial t}\left(g(t, t_0)e^{-h(t,t_0)}\right) .$$

By integration on $[t_0+1, t]$ it follows that

$$(2.13) \qquad \alpha e^{-p/c} - \alpha e^{-h(t,t_0)} \le g(t, t_0)e^{-h(t,t_0)}$$

and hence using the inequality (2.11) we obtain

$$(2.14)\qquad \alpha e^{-p/c} e^{h(t,t_0)} \le \alpha + g(t, t_0) \le c(t-t_0)^{1/p'} \cdot f(t, t_0) \ .$$

From Lemma 2.2 and the preceding relation it follows that there exists $N > 0$ (independent of t and t_0) such that

$$(2.15)\qquad f(t, t_0) \ge Ne^{h(t,t_0)/2} \quad \text{for all} \quad t \ge t_0 + 1 \ .$$

The lemma is proved.

LEMMA 2.4. *If $T(t)$ is a C_0 semigroup on a Banach space X then there exist $M > 1$, $\omega > 0$ such that*

$$(2.16)\qquad \|T(t)\| \le Me^{\omega t} \quad \textit{for each} \quad t \ge 0 \ ,$$

$$(2.17)\qquad \|T(t_0+1)x\| \le Me^{\omega}\|T(t)x\| \le M^2e^{2\omega}\|T(t_0x)\| \ ,$$

$$(2.18)\qquad M^pe^{\omega p} \cdot \int_{t_0}^{t_0+1} \|T(t)x\|^p dt \ge \|T(t_0+1)x\|^p \ ,$$

and

$$(2.19)\qquad \|T_2(t_0)x\|^{-p} \le M^pe^{\omega p} \cdot \int_{t_0}^{t_0+1} \|T_2(t)x\|^{-p} dt \ ,$$

for all $t_0 \ge 0$, $x \in X$, $t \in [t_0, t_0+1]$ *and* $p \in [1, \infty)$.

Proof. It is well known (see, for example, [1], pp. 165-166) that there are $M > 1$ and

$$(2.20)\qquad \omega \ge \inf_{t\ge 0} \frac{\ln T(t)}{t}$$

such that (2.16) holds.

From

$$(2.21)\qquad \|T(t_0+1)x\| \le \|T(t_0+1-t)\|\|T(t)x\| \le Me^{\omega}\|T(t)x\|$$

and

$$(2.22)\qquad \|T(t)x\| \le \|T(t-t_0)\|\|T(t_0)x\| \le Me^{\omega}\|T(t_0)x\|$$

the relation (2.17) results.

The inequalities (2.18) and (2.19) follow immediately from (2.17).

LEMMA 2.5. *Let* $T(t)$ *be a* C_0 *semigroup on the Banach space* X *and let* P_1 *respectively* P_2 *be the projection along the closed complemented subspace* X_1 *defined by* (1.1) *respectively* $X_2 = X \ominus X_1$. *Then we have that*

(2.23) $$T_1(t) = P_1 T_1(t) \text{ for every } t \geq 0 ,$$

(2.24) $$T_2(t)x \neq 0 \text{ for all } t \geq 0 \text{ and } x \notin X_1 ,$$

and

(2.25) *if* $T_1(t)x \neq 0$ *for every* $t \geq 0$ *then* $T(t)x \neq 0$ *for all* $t \geq 0$.

Proof. For (2.23) it is sufficient to prove that the subspace X_1 is an invariant subspace for $T(t)$.

Indeed, if $x \in X_1$ and $t \geq 0$ then from

(2.26) $$\|T(s)T(t)x\| = \|T(t+s)x\| \leq \|T(t)\|\|T(s)x\| \leq Me^{\omega t} \sup_{s\geq 0} \|T(s)x\|$$

it follows that $T(t)x \in X_1$.

If there exist $t \geq 0$ and $x \notin X_1$ such that $T_2(t)x = 0$ then from

(2.27) $$T(s)x = T_1(s)x \text{ for all } s \geq t$$

and $T_1(\cdot)x \in L^\infty(X)$ it follows that $x \in X_1$. This contradiction proves the property (2.24).

The implication (2.25) is obvious from the equality

(2.28) $$X_1 \cap X_2 = \{0\} .$$

3. The main results

We are now ready to prove the following

THEOREM 3.1. *Let* $T(t)$ *be a strongly continuous semigroup of*

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operators of class C_0 *defined on the Banach space* X *. Then* $T(t)$ *is exponentially dichotomic if and only if there exist* $c, p \geq 1$ *such that*

(3.1) $$\int_0^t \|T_1(t-s)\|^p ds \leq c^p$$

and

(3.2) $$\int_t^\infty \|T_2(u)x\|^{-p} du \leq c^{-p} \cdot \|T_2(t)x\|^{-p}$$

for all $t \geq 0$ *and* $x \in X$ *.*

Proof. Necessity. We omit the simple verification (using Definition 1.1 (ii)) that if $T(t)$ is exponentially dichotomic then it satisfies the above inequalities (3.1) and (3.2).

Sufficiency. Suppose that the C_0 semigroup $T(t)$ has the properties (3.1) and (3.2).

Let $t_0 \geq 0$, $x \in X$ be fixed.

(i) Firstly, we suppose that

(3.3) $$T_1(t)x \neq 0 \quad \text{for all} \quad t \geq 0 .$$

Let $f : \Delta \to \mathbb{R}_+$ be the function defined by

(3.4) $$f(t, t_0) = \frac{1}{\|T_1(t-t_0)\|} .$$

From Lemma 2.4 it follows that there exist $M, \omega > 0$ such that

(3.5) $$f(u, t) \geq e^{-\omega}/M\|P_1\| = \alpha \quad \text{for all} \quad u \in [t, t+1] \quad \text{and} \quad t \geq t_0 .$$

Hence

(3.6) $$\int_t^{t+1} f(u, t)du \geq \alpha ,$$

that is, the inequality (2.8) from Lemma 2.3 holds.

From Lemmas 2.4 and 2.5, using Hölder's inequality, we have

$$(3.7)\quad \|T_1(t-t_0)\| \int_{t_0}^{t} f(s,\ t_0)ds = \int_{t_0}^{t} \|T_1(t-s)T_1(s-t_0)\|f(s,\ t_0)ds$$

$$\leq \int_{t_0}^{t} \|T_1(t-s)\|ds \leq \left(\int_{t_0}^{t} \|T_1(t-s)\|^p ds\right)^{1/p} (t-t_0)^{1/p'} \leq c(t-t_0)^{1/p'} .$$

This shows that the inequality (2.7) from Lemma 2.3 is verified.

By Lemma 2.3 there are $M_1,\ \lambda_1 > 0$ such that

$$(3.8)\quad \|T_1(t-t_0)\| \leq M_1 e^{-\lambda_1 (t-t_0)^{1/p}} \quad \text{for all} \quad t \geq t_0 + 1 .$$

From this inequality and

$$(3.9)\quad \|T_1(t)x\| \leq \|T_1(t-t_0)\| \cdot \|T_1(t_0)x\|$$

we obtain that there is $N > 0$ such that

$$(3.10)\quad \|T_1(t)x\| \leq Ne^{-\lambda_1 (t-t_0)^{1/p}} \|T_1(t_0)x\| ,\ \text{for all} \quad t \geq t_0 .$$

(ii) Suppose now that

$$(3.11)\quad \text{there exists} \quad s_0 > 0 \quad \text{such that} \quad T_1(s_0)x = 0 .$$

Then

$$(3.12)\quad T_1(s)x = T_1(s-s_0)T_1(s_0)x = 0 \quad \text{for all} \quad s \geq s_0 .$$

Let $t_x > 0$ such that $T_1(t_x)x = 0$ and $T_1(t)x \neq 0$ for every $t < t_x$.

If $t \geq t_0 \geq t_x$ or $t \geq t_x \geq t_0$ then $T_1(t)x = 0$ and hence the inequality (3.10) holds.

If $t_x \geq t \geq t_0 \geq 0$ then from the preceding case (3.10) is also verified.

From Lemma 2.1 it follows that there exist $N_1,\ \nu_1 > 0$ such that

$$(3.13)\quad \|T_1(t)x\| \leq N_1 e^{-\nu_1 (t-t_0)} \|T_1(t_0)x\|$$

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for all $t \geq t_0 \geq 0$ and $x \in X$. This shows that the semigroup $T_1(t)$ is exponentially stable.

For $T_2(t)$ we consider the function $g : [t_0, \infty) \to \mathbb{R}_+$ defined by

(3.14) $$g(t) = \int_t^\infty \|T_2(u)x\|^{-p} du .$$

The inequality (3.2) shows that

(3.15) $$c^p g(t) \leq - \frac{dg(t)}{dt}$$

and hence, by integration, we obtain

(3.16) $$g(t) \leq g(t_0) \cdot e^{(t_0-t)c^p} \quad \text{for all} \quad t \geq t_0 ,$$

which implies that

(3.17) $$g(t)e^{(t-t_0)c^p} \leq g(t_0) \leq c^{-p}\|T_2(t_0)x\|^{-p} .$$

Therefore

(3.18) $$\int_t^{t+1} \|T_2(u)x\|^{-p} du \cdot e^{(t-t_0)c^p} \leq c^{-p}\|T_2(t_0)x\|^{-p} ,$$

for every $t \geq t_0$.

If we denote by $\alpha = Me^\omega$ then from Lemma 2.4 it follows that

(3.19) $$\alpha^{-p}\|T_2(t)x\|^{-p}e^{(t-t_0)c^p} \leq c^{-p}\|T_2(t_0)x\|^{-p}$$

and hence there is $N_2, \nu_2 > 0$ such that

(3.20) $$\|T_2(t)x\| \geq N_2 e^{\nu_2(t-t_0)}\|T_2(t_0)x\| \quad \text{for all} \quad t \geq t_0 \geq 0 \quad \text{and} \quad x \in X .$$

If $\nu = \min\{\nu_1, \nu_2\}$ then from (3.13) and (3.20) it follows that the inequalities (1.4) and (1.5) hold and hence $T(t)$ is exponentially dichotomic.

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THEOREM 3.2. *The C_0 semigroup $T(t)$ is exponentially dichotomic if and only if there are $c, p \geq 1$ such that*

$$\int_0^t \|T_1(t-s)x\|^p ds \leq c^p \|x\|^p \tag{3.21}$$

and

$$\int_t^\infty \|T_2(u)x\|^{-p} du \leq c^{-p} \|T_2(t)x\|^{-p} , \tag{3.22}$$

for all $t \geq 0$ and $x \in X$.

Proof. Necessity is obvious from the preceding theorem.

Sufficiency. From the hypothesis (3.21) it results that

$$\int_0^\infty \|T_1(s)x\|^p ds \leq c^p \cdot \|x\|^p \quad \text{for all } x \in X . \tag{3.23}$$

From Theorem 1.1 and (2.23) it follows that $T_1(t)$ is an exponentially stable semigroup. Hence there is $N_1, \nu_1 > 0$ such that

$$\|T_1(t)x\| \leq \|T_1(t-t_0)\| \cdot \|T_1(t_0)x\| \leq N_1 e^{-\nu_1(t-t_0)} \|T_1(t_0)x\| \tag{3.24}$$

for all $t \geq t_0 \geq 0$ and $x \in X$.

Then using this inequality and the proof of the preceding theorem we obtain that $T(t)$ is exponentially dichotomic.

As a particular case $\left(\text{when } P_2 = 0\right)$ we obtain Datko's result:

COROLLARY 3.1. *Let $T(t)$ be a C_0 semigroup of linear operators defined on the Banach space X . The following statements are equivalent:*

(i) $T(t)$ is exponentially stable;

(ii) there are $c, p \geq 1$ such that

$$\int_0^\infty \|T(t)\|^p dt \leq c^p ; \tag{3.25}$$

(iii) there exist $c, p \geq 1$ such that

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(3.26) $$\int_0^\infty \|T(t)x\|^p dt \le c^p \cdot \|x\|^p \quad \textit{for all} \quad x \in X .$$

Proof. Is obvious from Theorems 3.1 and 3.2.

REMARK 3.1. In the proofs of Theorem 3.1 and that of the equivalence *(i)* $\Leftrightarrow$ *(ii)* from the preceding corollary we have not used Datko's theorem.

THEOREM 3.3. *A necessary and sufficient condition for the* C_0 *semi-group* $T(t)$ *to be exponentially dichotomic is the existence of positive constants* m, c *and* $p \ge 1$ *such that*

(3.27) $$\int_t^\infty \|T_1(u-t)\|^p du \le c^p ,$$

(3.28) $$\|T_2(t+1)x\| \ge m\|T_2(t)x\| ,$$

and

(3.29) $$\int_0^\infty \|T_2(s)x\|^p ds \le c^p \|T_2(t)x\|^p ,$$

for all $t \ge 0$ *and* $x \in X$.

Proof. Necessity is a simple verification.

Sufficiency. From

(3.30) $$\int_0^t \|T_1(t-s)\|^p ds = \int_0^t \|T_1(s)\|^p ds \le \int_t^\infty \|T_1(u-t)\|^p du \le c^p$$

and the proof of Theorem 3.1 it follows that the inequality (3.13) holds.

Let $t_0 \ge 0$ and $x \in X$. Let now f be the real function

(3.31) $$f : \mathbb{R}_+ \to \mathbb{R}_+ , \quad f(t) = \int_0^t \|T_2(s)x\|^p ds .$$

From the above inequality (3.29) we have that

(3.32) $$f(t) \le c^p \cdot \frac{df(t)}{dt}$$

and hence by integration it follows

(3.33) $$e^{-c^{-p}} e^{(t-t_0)/c^p} f(t_0+1) \le f(t) \le c^p \cdot \|T_2(t)x\|^p$$

for every $t \geq t_0 + 1$.

On the other hand from (2.18) and (3.29) it results that there exists $m > 0$ such that

$$(3.34) \qquad f(t_0+1) \geq \int_{t_0}^{t_0+1} \|T_2(s)x\|^p ds \geq \alpha^{-p}\|T_2(t_0+1)x\|^p$$

$$\geq m^p\alpha^{-p}\|T_2(t_0)x\|^p \text{ , where } \alpha = Me^{\omega} .$$

Finally, we obtain

$$(3.35) \qquad \|T_2(t)x\| \geq N_3 e^{\nu_2(t-t_0)} \cdot \|T_2(t_0)x\| ,$$

for all $t \geq t_0 + 1$ and $x \in X$, where

$$(3.36) \qquad N_3 = \frac{m}{c} \cdot e^{-c^{-p}/p} \text{ and } \nu_2 = \frac{1}{pc^p} .$$

If $t_0 \leq t \leq t_0 + 1$ then from (2.17) and (3.35) we obtain

$$(3.37) \quad \|T_2(t)x\| \geq \frac{\|T_2(t_0+1)x\|}{\alpha} \geq \frac{N_3 e^{\nu_2}}{\alpha} \|T_2(t_0)x\| \geq \frac{N_3}{\alpha} e^{\nu_2(t-t_0)} \cdot \|T_2(t_0)x\|$$

and hence

$$(3.38) \quad \|T_2(t)x\| \geq \frac{N_3}{\alpha} e^{\nu_2(t-t_0)} \|T_2(t_0)x\| \text{ for all } t \geq t_0 \geq 0 \text{ and } x \in X .$$

If $N_2 = N_3/\alpha$ and $\nu = \min\{\nu_1, \nu_2\}$ then (1.4) and (1.5) are satisfied and hence $T(t)$ is exponentially dichotomic.

COROLLARY 3.2. *Let $T(t)$ be a C_0 semigroup of linear operators defined on a Banach space X. Then $T(t)$ is exponentially dichotomic if and only if there exist $m, c > 0$ and $p \geq 1$ such that*

$$(3.39) \qquad \int_t^\infty \|T_1(u-t)x\|^p du \leq c^p \cdot \|x\|^p ,$$

$$(3.40) \qquad \int_0^t \|T_2(s)x\|^p ds \leq c^p \cdot \|T_2(t)x\|^p ,$$

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and

$$\|T_2(t+1)x\| \geq m\|T_2(t)x\| \; , \tag{3.41}$$

for all $t \geq 0$ *and* $x \in X$.

Proof. Similar to the proof of Theorem 3.2.

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