SOME INFINITE FACTOR GROUPS OF BURNSIDE GROUPS

Dedicated to the memory of Hanna Neumann

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Let $B_d(e)$ denote the Burnside group with $d \geqq 2$ generators $a_1, a_2, \cdots, a_d$ and exponent $e > 0$, i.e., the free group of rank d of the Burnside variety of exponent e. It is known that $B_d(e)$ is infinite for all sufficiently large odd values of e; cf. Novikov and Adyan [3] or Britton [1]. In particular $B_d(p^k)$, where p is an odd prime, is infinite for all sufficiently large k. It is not known whether or not $B_d(2^k)$ is infinite for all sufficiently large k; infiniteness would imply that $B_d(n)$ is infinite for all sufficiently large n, as has been conjectured by Novikov [2].

Now let $J_2(p^k)$, where p is prime, be the factor group of $B_2(p^k)$ obtained by adding the defining relations $a_1^p = 1$, $a_2^p = 1$.

THEOREM 1. (i) *If p is odd, $J_2(p^k)$ is infinite for all sufficiently large k.*
(ii) *If $p = 2$, $J_2(p^k)$ is finite for all k.*

The proof of (ii) is trivial since the relations $a_1^2 = a_2^2 = (a_1a_2)^{2^k} = 1$ define a dihedral group of order 2^{k+1}. (i) is a special case of Theorem 2 below.

Let $\Pi = G_1 * G_2 * \cdots * G_r$ be a free product of $r \geqq 2$ finite groups of odd otder. An element $X \in \Pi$, $X \neq 1$, whose normal form is

$$x_1 x_2 \cdots x_n \qquad (x_i \in G_{f(i)},\ i = 1, 2, \cdots, n)$$

is called *externally reduced* if $n \geqq 2$ and $f(1) \neq f(n)$; let J_0 be the set of all such elements. Let $\Gamma^e(S)$, where $e > 0$ and S is any subset of J_0, be the group obtained from Π by adding the defining relation

$$X^e = 1 \quad (X \in S)$$

THEOREM 2. $\Gamma^e(J_0)$, *hence* $\Gamma^e(S)$, *is infinite for all sufficiently large odd values of e.*

PROOF. This follows from the author's paper [1] in view of the second sentence of Section 2. of Chapter II.

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Note that $\Gamma^e(J_0)$ is finitely generated and has a (non-zero) exponent. Of course, if all G_i are cyclic of order e then $\Gamma^e(J_0)$ is $B_r(e)$.

To prove (i) of Theorem 1 take $r = 2$, G_i cyclic of order $p (i = 1, 2)$, $e = p^k$; then $\Gamma^e(J_0)$ is $J_2(p^k)$.

References

[1] J. L. Britton, *The existence of infinite Burnside groups, article in: Word problems Studies in Logic and the Foundations of Mathematics* (ed. by Boone, W. W., Cannonito, F. B., Lyndon, R. C.). Amsterdam. North-Holland). (1972).

[2] P. S. Novikov, 'On periodic groups', (Russian) *Dokl. Akad. Nauk SSSR* (1959) 127 749–752,

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