

Abstracts of Australasian PhD theses

An algebraic approach to Runge-Kutta methods

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Methods are studied for solving an initial value problem of the form $y'(x) = f(y(x))$, $y(x_0) = y_0$ by the use of Runge-Kutta processes. For a process of this type, the function f must be evaluated a certain number of times during the execution of a step, this number depending on the particular method being used. New methods are introduced which yield greater accuracy than existing Runge-Kutta methods for the same number of function evaluations per step.

The problem is approached in a manner suggested by the work of Butcher [2]. The main results of Butcher's work are used extensively and a summary of them is given.

The concept of effective order introduced in [1] is examined. This work is extended to higher order methods and generalised to cater for the possibility of changing the integration step length. Also, the cyclic composition of methods is investigated. Certain advantages in accuracy and efficiency over classical methods are obtained.

Finally, some problems associated with error estimation are discussed. The numerical solutions of certain problems are included. These results are obtained by using newly developed methods as well as methods already in existence, giving some indication of the relative merits of the new methods.

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References

- [1] J.C. Butcher, "The effective order of Runge-Kutta methods", *Conference on the numerical solution of differential equations, Dundee, 1969*, 133-139 (Lecture Notes in Mathematics, 109. Springer-Verlag, Berlin, Heidelberg, New York, 1969).
- [2] J.C. Butcher, "An algebraic theory of integration methods", *Math. Comp.* 26 (1972), 79-106.